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Electrostatic solitary structures in non-thermal plasmas

R.A. Cairns[1], A.A. Mamum[1], R. Bingham[2], R. Boström[3], R.O. Dendy[4], C.M.C. Nairn[5], P.K. Shukla[6]

Abstract. Solitary electrostatic structures involving density depletions have been observed in the upper ionosphere by the Freja satellite [*Dovner et al.,* 1994]. If these are interpreted as ion sound solitons, the difficulty arises that the standard Korteweg-de Vries description predicts structures with enhanced rather than depleted density. Here we show that the presence of non-thermal electrons may change the nature of ion sound solitary structures and allow the existence of structures very like those observed.

Introduction

Recent observations made from the Freja satellite have detected structures with density depletions of the order of 10%, see Fig.1, associated with electric fields which suggest that they are most likely electrostatic in nature [*Dovner et al.,* 1994]. Similar structures have been observed previously by the Viking satellite [*Boström et al.,* 1988, *Boström* 1992]. There are basically two types of density structures observed by these satellites. The Freja satellite observed density cavi- known as lower-hybrid cavitons [*Erikson et al.,* 1994]. The first observations of lower-hybrid cavities in the auroral zone were reported by Vago et al., [1992]. The other structures observed are density depletions in the absence of lower-hybrid waves [*Dovner et al.,* 1994]. As noted above similar structures without associated lower-hybrid waves have also been observed by the Viking satellite. Various theories have been advanced to explain these density depletions. The theory of wave-collapse of the lower-hybrid mode was developed by Shapiro et al., [1993,1995]. These collapsing lower-hybrid cavitons are also responsible for accelerating the ions perpendicular to the magnetic field. Mälkki et al., [1989] put forward a BGK mode model for the pure density depletions. Other theories have assumed them to be associated with weak double layers while others have considered the possibility of producing ion acoustic potential structures by plasma currents [*Maslov* 1990], and have associated them with solitary wave structures. Our aim here is to propose a possible theoretical explanation of these structures, which we take to be large amplitude ion-sound waves. We show that in the presence of a distribution of electrons which is non-thermal, with an excess of energetic particles, the nature of ion sound solitary structures changes and that it is possible to obtain solutions with density depletions and dimensions roughly in agreement with those observed. A similar result was obtained by Nishihara and Tajiro

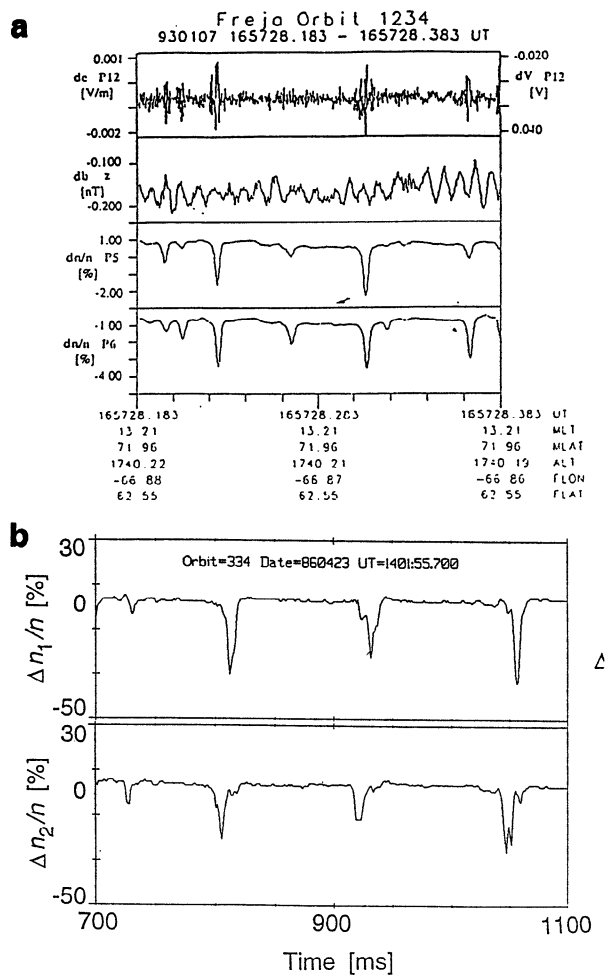

Figure 1. Examples of density and lower-hybrid cavities. 1a observed by the Freja satellite [Dovner et al.,: (1994)] 1b observed by the Viking satellite [R Boström, (1992)]

[1]School of Mathematical and Computional Sciences, University of St. Andrews, St. Andrews, Fife KY16 9SS, United Kingdom.
[2]Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom.
[3]Swedish Institute of Space Physics, Uppsala Division, S-755 90 Uppsala, Sweden.
[4]AEA Fusion, Culham Laboratory, Abingdon, Oxon OX14 3DB,
[5]Umeå University, Box 812, 981 28 Kiruna, Sweden.
[6]Faculty of Physics and Astromony, Ruhr-University of Bochum, D-44780 Bochum, Germany.

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(1981) for a two Maxwellian plasma. Here, however we demonstrate that the effect is possible with distribution functions similar to what would be produced by quasi-linear flattening due to a spectrum of lower hybrid waves. Structures which are, at least superficially, similar have been obtained for example by Schamel [1986], but these rely on a distribution of ions trapped in the potential well of the structure. Our solutions involve only a simple fluid response of the ions and do not require that the ion distribution function take a carefully tailored form. Observations by the Viking satellite [*Lundin et al.,* 1987] of counterstreaming electron distribution functions indicate that they are a common feature of the auroral zone. Simulations by Retterer et al., [1986] show that these distributions can result from lower-hybrid turbulence. The AMPTE satellite found similar features of counterstreaming electrons [Hall et al., 1991] in the dayside boundary layers and attributed these energetic electrons to acceleration by lower-hybrid turbulence at altitudes of the Viking spacecraft.

Since the ion hole structures are sometimes found to be correlated with lower-hybrid waves we infer that a possible scenario is that lower-hybrid waves collapse in configuration space producing cavitons [*Shapiro et al.,* 1993]. These lower-hybrid waves can be damped by generating fast electrons producing the nonthermal counterstreaming electron tails. The damping of the lower-hybrid waves may leave behind an ion hole which can be self consistently sustained well after the lower-hybrid waves have been damped. Experimental observations of this phenomenon are shown in fig. 1 indicating the existence with and without the presence of lower-hybrid waves.

Details of the model

Assuming that some solitary structure exists, we look at it in a frame in which it is at rest and take the potential to be $\phi(x)$. For the moment we consider a one-dimensional structure, though a brief discussion of the three-dimensional case will be given later. To model an electron distribution with a population of fast particles we take

$$f_e(\nu) = \frac{n_o}{(3\alpha+1)\sqrt{2\pi\nu_e^2}}\left(1+\frac{\alpha\nu^4}{\nu_e^4}\right)\exp\left(-\frac{\nu^2}{2\nu_e^2}\right). \quad (1)$$

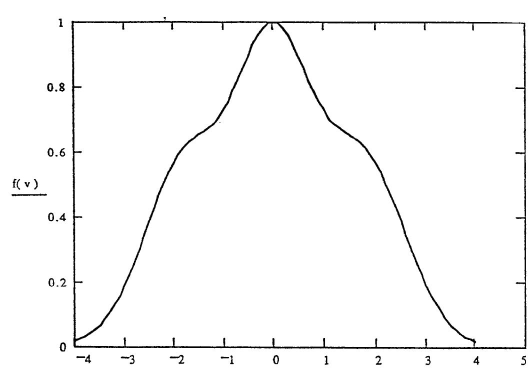

Figure 2. Nonthermal electron distribution function $f_e(\nu)$ given by Eq.1.

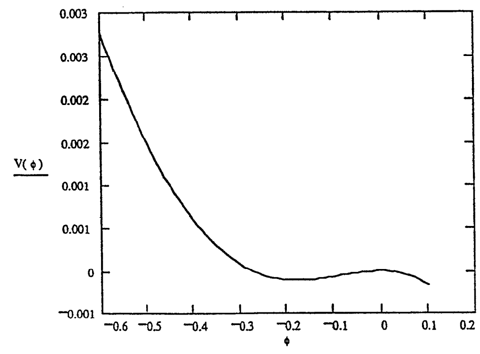

Figure 3. The Sagdeev potential $V(\Phi)$ defined by Eq.(5) for $M^2 = 2.1$ showing the negative potential structures Φ

This distribution function is shown in Fig.2, for $\alpha = 0.2$, a value which we use throughout this work. It has been chosen as a convenient form with which to illustrate the theory, rather than as a precise model of what is observed.

The ions are assumed to be cold. As long as the ion temperature is not so high that a significant number of ions can be trapped in the potential well of the solitary wave the results are not changed in any essential way by a finite ion temperature. It is convenient to define a Mach number M for the velocity of the structure relative to the ion rest frame in terms of the velocity $\sqrt{\frac{m_e\nu_e^2}{m_i}}$ and to define a dimensionless potential $\Phi = \frac{e\phi}{m_e\nu_e^2}$. In the rest frame of the structure it is then a simple matter to see from the energy and particle conservation equations for ions that the ion density is

$$n_i = n_o\frac{1}{\sqrt{1-\frac{2\Phi}{M^2}}} \quad (2)$$

Assuming that the velocity of the structure is small relative to the electron thermal velocity, we may neglect the effect of the streaming velocity on the electrons, since in the steady state the electron distribution is a function of the electron energy we find the distribution in the presence of a non-zero potential can simply be found by replacing $\frac{\nu^2}{\nu_e^2}$ in (1) with $\frac{\nu^2}{\nu_e^2} - 2\Phi$. Integrating over the resulting distribution gives the electron density to be

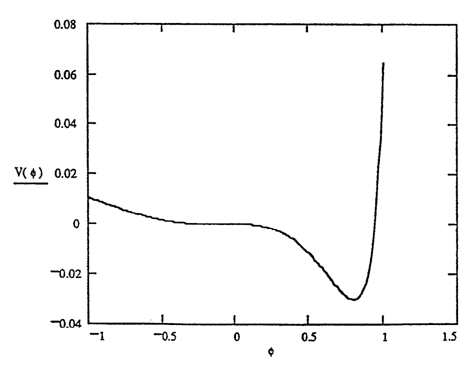

Figure 4. Same as Fig.3 with a change of scale to show the positive potential structure.

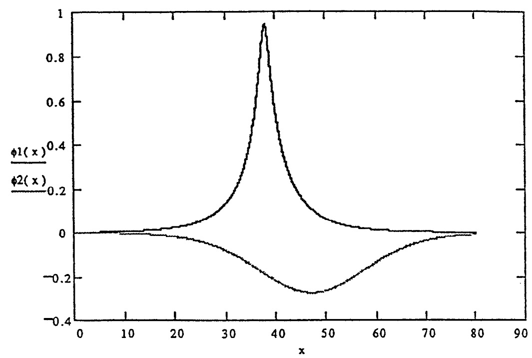

Figure 5. Potential profiles for the two solitary wave solutions of Eq.(5).

$$n_e = n_o \exp(\Phi)\frac{4\alpha\Phi^2 - 4\alpha\Phi + 3\alpha + 1}{3\alpha + 1} \quad (3)$$

We now use (2) and (3) in Poisson's equation. Taking lengths in terms of $\frac{v_e}{\omega_{pe}}$, that is the Debye length determined by the thermal component of the electrons, we obtain, in the one-dimensional case,

$$\frac{d^2\Phi}{dx^2} = -\frac{1}{\sqrt{1 - \frac{2\Phi}{M^2}}} + \exp(\Phi)\frac{4\alpha\Phi^2 - 4\alpha\Phi + 3\alpha + 1}{3\alpha + 1} \quad (4)$$

The qualitative nature of the solutions of this are most easily seen by introducing the Sagdeev potential [*Sagdeev* 1966], defined so that (4) takes the form

$$\frac{d^2\Phi}{dx^2} = -\frac{dV(\Phi)}{d\Phi}. \quad (5)$$

The problem is then analogous to particle motion in the potential V. For our present problem

$$V(\Phi) = -M^2\sqrt{1 - \frac{2\Phi}{M^2}} - \exp(\Phi)\frac{4\alpha\Phi^2 - 12\alpha\Phi + 15\alpha + 1}{3\alpha + 1} \quad (6)$$

If we take $\alpha = 0$, then for $M^2 < 1$, the origin is a stable fixed point in the phase plane of (5) and perturbations give small oscillations which are just standard ion sound waves. For $M^2 > 1$ there is a potential well on the positive Φ side and, in the phase plane of the differential equation there is a homoclinic orbit which approaches the origin as $x \to \pm\infty$ and represents a solitary wave with positive potential and positive density perturbation. No solitary wave solution exists with a negative potential. In the next section we describe how this changes in the case of the non-thermal distribution.

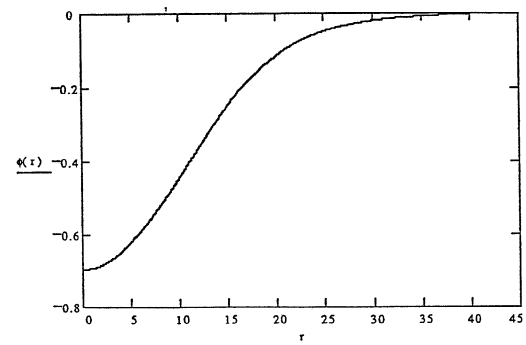

Figure 7. The radial potential profile for a spherically symmetrical solitary structure.

Solitary Waves in non-thermal plasma

We now take non-zero α in our electron distribution function (more specifically we take the value 0.2 for purposes of illustration). This changes the long wavelength limit of the small amplitude wave velocity to a value corresponding to

$$M^2 = \frac{1 + 3\alpha}{1 - \alpha},$$

that is $M^2 = 2$ for our particular case. Below this value the origin is a stable point and only small amplitude oscillations exist. In Fig.3 we plot the Sagdeev potential for a value of $M^2 = 2.1$. It will be seen that there is a well on the negative potential side, allowing a solitary wave solution with a negative potential and a negative density perturbation. The larger scale view of the potential shown in Fig.4 shows that these is also a well on the positive side, which is much larger and deeper. Thus the equation allows solitary wave solutions with both positive and negative potentials, Fig.5 shows solutions for these two solitary waves and Fig.6 shows the

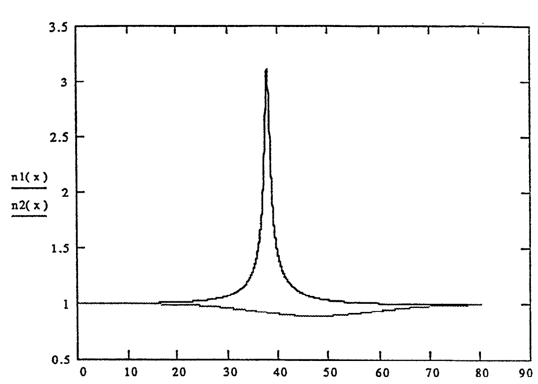

Figure 6. Density profiles corresponding to potentials shown in Fig.5. There is a possibility of obtaining a density hump as well as a density hole.

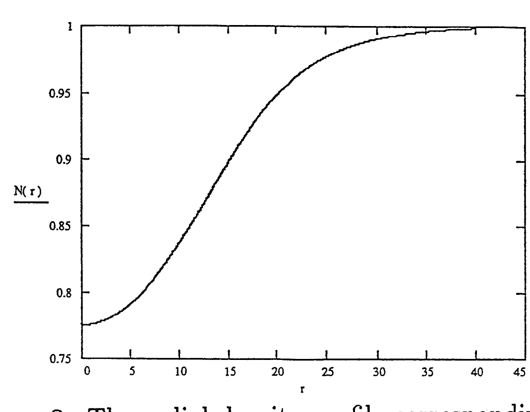

Figure 8. The radial density profile corresponding to Fig.7.

corresponding densities. It can be seen that the structure with a density depletion is a fairly shallow wave, with a density depletion of the order of that which is observed, and also a length scale, of the order of 50 Debye lengths, which is of the order of observed length scales. The positive structure, on the other hand, is highly nonlinear and would, presumable, require a very large initial perturbation to produce it. This may by why only density depletions have been observed. The changed properties of the solitons arise from the change in the way that electron density depends on potential.

Finally we give brief consideration to three dimensional structures, since the structures observed in space are certainly not infinite in two directions. If we assume that the structures move along the magnetic field, and that the ion Larmor radius is small compared to the size of the structures, then the ions stream along the field lines through the structure and their density is as before, in terms of the potential. The electron density is also the same, it if is assumed that the distribution function given previously is the result of integrating over the velocities perpendicular to the magnetic field, so the only difference is that the left hand side of (4) is a Laplacian acting on Φ. In Figs 7 and 8 we show the potential and density in a spherically symmetrical solution of this equation. The most obvious change is that there is a larger dip in the density for the same parameters.

Discussion

Motivated by the observations of solitary structures with density depletions made by the Freja satellite, we have shown that the presence of a population of energetic electrons changes the properties of ion sound solitary waves. If a simple fluid response of the ions is taken, so we do not allow trapped ion distributions, then the only solitary wave structure in a thermal plasma has an enhanced density. However, with a non-thermal electron population, we have shown that solitons with both positive and negative density perturbations can exist. The weakly nonlinear solution which might be expected to be set up in response to a moderate perturbation of the plasma is that with a density depletion. The structure with a density enhancement which can exist under the same conditions is highly nonlinear and would be expected to require a large initial perturbation to set it up. In a magnetic field, the one dimensional structures would have to propagate along the field. However, we have also demonstrated that three dimensional structures can exist and these, of course, are the type of structures which may be relevant to observations.

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R A Cairns and A A Mamum, School of Mathematical and Computational Sciences,University of St Andrews, St. Andrews, Fife KY16 9SS, UK.

R Bingham, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon. OX11 0QX, UK

R Boström, Swedish Institute of Space Physics, Uppsala Division, S-755 90 Uppsala, Sweden

R O Dendy, AEA Fusion. Culham Laboratory, Abingdon, Oxon OX14 3DB

C M C Nairn, Umeå University, Box 812, 981 28 Kiruna, Sweden

P K Shukla, Faculty of Physics and Astronomy, Ruhr-University of Bochum, D-44780 Bochum, Germany

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