# **Dust-acoustic Mach cones in magnetized electron-dust plasmas** of Saturn

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[1] We discuss the possibility of the Mach cone formation involving the modified dust-acoustic waves in a dusty magnetoplasma composed of electrons and positively charged dust grains. It is shown that the modified dustacoustic waves, whose wavelength is of the order of (or shorter than) the electron gyroradius, are one of the viable candidates for the formation of Mach cones in Saturn's INDEX TERMS: 6275 Planetology: Solar System dusty rings. Objects: Saturn; 7807 Space Plasma Physics: Charged particle motion and acceleration; 7827 Space Plasma Physics: Kinetic and MHD theory; 7867 Space Plasma Physics: Wave/particle interactions; 7899 Space Plasma Physics: General or miscellaneous. Citation: Mamun, A. A., and P. K. Shukla (2004), Dust-acoustic Mach cones in magnetized electron-dust plasmas of Saturn, Geophys. Res. Lett., 31, L06808, doi:10.1029/ 2003GL018684.

### 1. Introduction

[2] The structure and dynamics of Saturn's rings continue to present surprises and challenges to celestial mechanics. The first surprise came in October 1980 when Voyager 1 sped past Saturn and sent back lots of information and images of Saturn's rings [Smith et al., 1981; Hill and Mendis, 1981; Morfill et al., 1983]. The CASSINI spacecraft, which will go into the orbit around Saturn on 1 July 2004, promises to yield even more detailed information on the plasma and optical properties of Saturn's dusty rings. Since a direct probing of Saturn's dense rings is not practically possible due to the danger of collisions, one must resort to remote sensing to investigate the physical conditions within such dense rings. One can do this by the stellar occultation measurements, or through the observations of absorption of high-energy particles or radio-waves or of light scattering by dust particles at different angles. It has been proposed that the characteristics (viz. opening angle) of Mach cones in dusty plasmas of Saturn's rings might play an interesting role as potential diagnosis method [Havnes et al., 1995, 2001; Brattli et al., 2002] and can be used for deducing information regarding the physical state of the ambient dusty plasma of Saturn's rings, since Mach cones can be directly viewed from outside the plasma system. On this point of view, Mach cones by dust-associated waves (viz. dust-acoustic and dust-magnetoacoustic waves) in unmagnetized [Havnes et al., 1995] and magnetized [Mamun et al., 2003; Shukla and Mamun, 2003] dusty

plasmas containing negatively charged dust particles, ions, and electrons have been investigated.

- [3] Recently, positively charged dust grains have been observed in space [Havnes et al., 1996; Horányi, 1996; Fortov et al., 1998; Howard and Horányi, 2001; Fortov et al., 2003] as well as in laboratory experiments [Samarian et al., 2001], and have received a considerable interest in understanding their important role in different collective processes in space and laboratory dusty plasmas. There are three principal mechanisms by which a dust grain becomes positively charged. These are i) photoemission in the presence of a flux of ultraviolet (UV) photons [Fortov et al., 1998; Rosenberg et al., 1999; Fortov et al., 2003], ii) secondary emission of electrons from the surface of the dust grains [Chow et al., 1994], and iii) thermionic emission induced by radiative heating [Goertz, 1989].
- [4] The dust grains in space are charged positively mainly by photoemission in the presence of a flux of UV photons with energy larger than the work function of the grains, but lower than the ionization potential of the background gas [Rosenberg et al., 1999]. The amount of photo-electrons, i.e., positive charges on a dust grain depends on i) the wavelength of the incident photons, ii) the surface area of the dust grain, and iii) the properties of the dust grain material. We note that various metals typically have photo-electric work function  $W_f < 5$  eV, such as Ag  $(W_f = 4.46 \text{ eV})$ , Cu  $(W_f = 4.45 \text{ eV})$ , Al  $(W_f = 4.2 \text{ eV})$ , Ca  $(\widetilde{W}_f = 3.2 \text{ eV})$ , and Cs  $(W_f = 1.8 \text{ eV})$ . There are also a number of low work function materials, e.g., carbides (binary compounds of carbon and more electro-positive metals) with work functions  $W_f \simeq 2.18-3.50$  eV, borides (binary compounds of boron and more electro-positive metals) with work functions  $W_f = 2.45-2.92$  eV, metal oxides with work functions ranging from  $W_f = 1$  eV (Cs) to  $W_f = 4 \text{ eV}$  (Zirconium). The space dust grains are composed of low-work function materials, and can therefore be easily positively charged by photoemission in the presence of a flux of UV photons [Rosenberg et al., 1999]. It has been shown by Rosenberg et al. [Rosenberg et al., 1999] that as a result of only the photoemission process a dust grain of few micron size can acquire a positive charge of the order of  $10^2 - 10^5$  proton charges.
- [5] It is important to mention here some features of an electron-dust (ED) plasma [Shukla, 2000; Khrapak and Morfill, 2001], which significantly differ from those of an electron-ion-dust (EID) plasma. The characteristic length of an ED plasma without an external magnetic field is the electron Debye radius  $\lambda_{De}$ , contrary to the effective Debye radius  $\lambda_D = \lambda_{De} \lambda_{Di} / (\lambda_{De}^2 + \lambda_{Di}^2)^{1/2}$  in an EID plasma, where  $\lambda_{Di}$  is the ion Debye radius. The scalelength in a magnetized ED plasma is  $\lambda_{De}$  or the electron thermal gyroradius  $\rho_e$  =

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 $V_{Te}/\omega_{ce}$ , contrary to the ion thermal gyroradius  $\rho_i = V_{Te}/\omega_{ci}$ or the ion-acoustic gyroradius  $\rho_s = C_s/\omega_{ci}$  in a magnetized EID plasma, where  $V_{Te}$  ( $V_{Ti}$ ) is the electron (ion) thermal speed,  $C_s$  is the ion-acoustic speed, and  $\omega_{ce}$  ( $\omega_{ci}$ ) is the electron (ion) gyrofrequency. On the other hand, the dustacoustic wave (DAW) frequency in unmagnetized ED and EID plasmas are  $\sim k\lambda_{De}\omega_{pd}$  and  $\sim k\lambda_{D}\omega_{pd}$ , respectively, where k is the wavenumber and  $\omega_{pd}$  is the dust plasma frequency. In a magnetized EID plasma, the frequency spectra have a broad range [Shukla and Mamun, 2002, 2003]. In this Letter, we investigate the formation of Mach cones in a magnetized ED plasma associated with the modified DAWs which involve magnetized hot electrons and unmagnetized cold positively charged dust grains. Thus, spatial and temporal scales for the Mach cone formation in an ED magnetoplasma are significantly different from those in an EID magnetoplasma.

#### 2. Modified Dust-Acoustic Waves

[6] Let us consider the propagation of low-frequency  $(\omega \ll \omega_{ce}, \omega_{ce} = eB_0/m_ec, B_0$  is the magnitude of the external magnetic field  $\hat{\mathbf{z}}B_0$ ,  $m_e$  is the electron mass, e is the magnitude of the electron charge, c is the speed of light in vacuum, and  $\hat{\mathbf{z}}$  is the unit vector along the z-direction) electrostatic waves in an ED plasma composed of electrons and positively charged dust particles [Shukla, 2000; Khrapak and Morfill, 2001; Fortov et al., 2003]. Thus, at equilibrium we have  $n_{e0} \simeq Z_d n_{d0}$ , where  $n_{e0} (n_{d0})$  is the equilibrium electron (dust) number density and  $Z_d$  is the number of proton charges residing onto the dust grain surface.

[7] When the wavelength of the electrostatic wave is shorter than or comparable to the electron thermal gyroradius  $\rho_e$ , one must employ a kinetic theory for calculating the electron density perturbation  $n_{e1}$ . Thus, using a kinetic theory [Brambilla, 1998] in the presence of such a low-frequency ( $\omega \ll \omega_{ce}$ ) electrostatic wave potential  $\phi$ , the electron density perturbation  $n_{e1}$  is given by  $n_{e1} = (k^2/4\pi \ e)$   $\chi_e \phi$ , where the electron susceptibility  $\chi_e$  for two-dimensional electron motion in a plane perpendicular to  $\hat{\mathbf{z}}$  is

$$\chi_e \simeq \frac{1}{k^2 \lambda_{De}^2} \left[ 1 - \Gamma_0(b_e) + 2\Gamma_1(b_e) \frac{\omega^2}{\omega_{ce}^2} \right],$$
(1)

where  $\Gamma_{0,1} = I_{0,1} \exp(-b_e)$ ,  $I_0(I_1)$  is the modified Bessel function of zero (first) order, and  $b_e = k^2 \rho_e^2$ .

[8] We now suppose that the perturbation wave frequency is much larger than the dust gyrofrequency, so that positively charged dust grains are unmagnetized. Accordingly, one obtains the dust number density perturbation  $n_{d1} = (k^2/4\pi Z_d e)\chi_d \phi$ , where the dust susceptibility is

$$\chi_d \simeq -\frac{\omega_{pd}^2}{\omega^2 - 3k^2 V_{Td}^2},\tag{2}$$

where  $V_{Td} = (T_d/m_d)^{1/2}$  is the dust thermal speed.

[9] Using equations (1) and (2) in  $1 + \chi_e + \chi_d = 0$ , the dispersion relation involving the perturbation wave phase speed  $V_p = \omega/k$  can be expressed in the form

$$[\alpha - \Gamma_0(b_e)]V_p^2 + 2\Gamma_1(b_e)\frac{k^2V_p^4}{\omega_{ce}^2} = C_D^2,$$
 (3)

where  $C_D = \lambda_{De} \omega_{pd} = (Z_d T_e/m_d)^{1/2}$  is the dust acoustic speed [Shukla, 2000] and  $\alpha = 1 + k^2 \lambda_{De}^2$ . In deriving equation (3), we assumed that  $\omega^2 \gg 3 \ k^2 V_{Td}^2$ . Equation (3) is the general dispersion relation for the modified DAWs [Rao et al., 1990; Shukla, 2000] in a magnetized ED plasma including finite electron gyroradius effect. This is valid for arbitrary (short or long) wavelength modified DAWs. In the short wavelength limit (viz.  $b_e \gg 1$ ), we have a Boltzmann electron density perturbation  $n_{e1} = n_{e0} e \phi / T_e$ , which dictates that unmagnetized electrons in the DAW potential follow a straightline orbit across  $\hat{\mathbf{z}}$ . In such a situation, equation (3) reduces to  $V_p = C_D / \sqrt{\alpha}$ . We note that a Boltzmann electron response also appears for  $b_e \ll 1$  and  $\omega \ll k_z V_{Te}, \omega_{ce} k_z / k_\perp$  when  $k_z \neq 0$ , where  $k_z$  is the magnetic field aligned wavenumber.

### 3. Dust Dynamics

[10] We consider a positively charged dust particle of mass  $m_d$  and charge  $Z_{de}$  moving in a field which includes Keplerian gravity, corotating planetary magnetic field (taken to be aligned centered dipole) with concomitant induced electric field [Mendis et al., 1982; Howard et al., 1999]. We examine the motion of a single dust particle and neglect the radiation pressure, plasma drag, planetary oblateness, charge fluctuations, and collective effects. The dynamics of such a positively charged dust grain is governed by the combined gravitational, magnetic, and electric forces. The orbital angular velocity  $\omega_d$  of the positively charged dust particle can therefore be expressed as [Mendis et al., 1982; Howard et al., 1999]

$$\omega_d = \frac{1}{2r^3} \left[ \omega_{cd} \pm \sqrt{\omega_{cd}^2 + 4r^3 \left(\Omega_k^2 - \omega_{cd}\Omega_p\right)} \right],\tag{4}$$

where r is the dust particle position normalized by the planet radius  $R_p$ ,  $\omega_{cd} = Z_d e B_0/m_d c$  is the dust gyrofrequency evaluated at a point on the planetary equator,  $B_0$  is the magnetic field strength on the planetary equator,  $\Omega_k = (GM_p/R_p^3)^{1/2}$  is the Kepler frequency evaluated at a point on the planetary equator,  $M_p$  is the planet mass, G is the universal gravitational constant, and  $\Omega_p$  is the angular rotation of the planet. We note that in deriving equation (4) the planetary magnetic field  $\mathbf{B}$  is assumed to be dipolar, i.e.,  $\mathbf{B} = \mathbf{B}_0/r^3$  is used in the definition of  $\omega_{cd}$  for r > 0.

[11] The + (-) sign in equation (4) represents the prograde (retrograde) motion of the dust particle. We are interested here in the prograde motion of the dust particle. We can consider equation (4) in both magnetic and Keplerian regimes [Howard et al., 1999]. Equation (4) implies that in a magnetic (Keplerian) regime the dust grain radius  $r_d$  must be smaller (larger) than a critical value  $r_{d0}$ , where  $r_{d0} = 1.563(\Omega_p Z_d B_0/\Omega_k^2 \rho_m)^{1/3}$  nm. Here  $\Omega_p$  and  $\Omega_p$  are in units of rad/s,  $B_0$  is in units of G, and  $\rho_m$  (dust material mass density) is in units of gm/cm<sup>3</sup>. It is found that  $r_{d0} \simeq 42$  nm for Saturn, where  $\Omega_p = 1.691 \times 10^{-4}$  rad/s,  $\Omega_k = 4.16 \times 10^{-4}$  rad/s,  $\Omega_k = 0.2$  G,  $Z_d = 100$ , and  $\Omega_p = 1$  gm/cm<sup>3</sup>. Therefore, the dust boulders, whose gyrofrequency is obviously negligible compared to their Kepler frequency, move at the Keplerian velocity. Hence, a dust boulder and a small dust particle will move at difference speeds. The difference in speeds is

$$V_d = rR_p \left(\omega_d - r^{-3/2}\Omega_k\right). \tag{5}$$

When  $V_d$  is larger than the modified DAW phase speed  $V_p$ , i.e.,  $V_d/V_p > 1$ , the effect of the dust boulder will be equivalent to that of a body moving through the medium with a super-dust-acoustic speed [Havnes et al., 1995].

[12] To approximate  $\omega_d$  in equation (4) for small (micron, even submicron sized) dust particles we take  $\omega_{cd} \ll \Omega_k$ ,  $\Omega_p$ , which is well satisfied for the dusty plasma parameters ( $Z_d = 100$ ,  $B_0 = 0.2$  G,  $r_d = 0.5$   $\mu m$ , i.e.,  $\omega_{cd} \simeq 6 \times 10^{-7}$  rad/sec) in Saturn. Hence, equation (4) can be approximated as  $\omega_d = r^{-3/2}\Omega_k + \omega_{cd}r^{-3}(1 - r^{3/2}\Omega_p/\Omega_k)/2$ , which can be substituted into equation (5) to obtain

$$V_d = \frac{\omega_{cd}R_p}{2r^2} \left( 1 - \frac{\Omega_p}{\Omega_k} r^{3/2} \right). \tag{6}$$

It is obvious from equation (6) that  $V_d = 0$  for  $r = r_0 = (\Omega_k / \Omega_p)^{2/3}$ , and for a particular planet  $|V_d| > 0$  for both  $r < r_0$  and  $r > r_0$ , where  $r_0$  is known as the synchronous distance. It is found that  $r_0 = 1.822$  for Saturn.

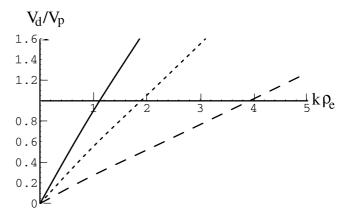
## 4. Formation of Mach Cones

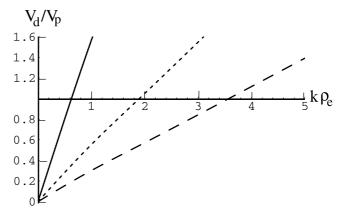
[13] The Mach cones can be formed by any perturbation (e.g., modified DAWs in our case) if the perturbing object (viz. a dust boulder in our case) speed  $V_d$  is larger than the modified DAW phase speed  $V_p$ , i.e.,  $V_d/V_p > 1$ . If this condition is satisfied, the Mach cone opening angle is  $\theta = \sin^{-1}(V_p/V_d)$ , where  $V_d$  is given by equation (5) or (6) and  $V_p = \omega/k$  is defined by equation (3). It is obvious that Mach cones can only be formed by those dust particles which are either inside  $(r < r_0)$  or outside  $(r > r_0)$  the synchronous distance  $r_0$ .

[14] To analyze the possibility for the formation of Mach cones associated with the modified DAWs defined by equation (3), we have numerically analyzed the relative speed  $V_d$  defined by equation (5), and the DAW phase speed  $V_p$  defined by equation (3) for typical ED plasma parameters of Saturn [Mendis et al., 1982; Howard et al., 1999; Shukla and Mamun, 2002; Verheest, 2000]:  $T_e = 100$  eV,  $n_{d0} = 10 \text{ cm}^{-3}$ ,  $Z_d = 100$ ,  $B_0 = 0.2 \text{ G}$ ,  $M_p = 5.688 \times 10^{26} \text{ kg}$ ,  $R_p = 60300$  km, and for dust particles inside  $(r < r_0)$  the synchronous distance  $r_0$ . The numerical results are displayed in Figure 1. The upper plot of Figure 1 shows that for  $r_d = 1 \mu m$ , the maximum wavelength  $(\lambda_m)$  of the modified DAWs, for which the Mach cones are formed, are  $\sim 6.81$  m,  $\sim 3.94$  m, and  $\sim 1.92$  m for r = 1.5, r = 1.6, and r = 1.7, respectively. The lower plot of Figure 1 indicates that for r = 1.6, the maximum wavelength are  $\sim 10.7$  m,  $\sim 3.94$  m, and  $\sim 2.08$  m for  $r_d = 0.5$   $\mu$ m,  $r_d = 1$   $\mu$ m, and  $r_d = 1.5 \mu m$ , respectively.

[15] We have also analyzed the Mach cone formation at locations outside  $(r > r_0)$  the synchronous distance  $r_0$ . Our analysis reveals that for  $r_d = 1~\mu\text{m}$ , the maximum wavelength  $(\lambda_m)$  of the modified DAWs, for which the Mach cones are formed, are  $\sim 1~\text{m}$ ,  $\sim 2.14~\text{m}$ , and  $\sim 3~\text{m}$  for r = 1.9, r = 2.0, and r = 2.1, respectively. We have also found that for r = 2, the maximum wavelength are  $\sim 6~\text{m}$ ,  $\sim 2.14~\text{m}$ , and  $\sim 1.15~\text{m}$  for  $r_d = 0.5~\mu\text{m}$ ,  $r_d = 1~\mu\text{m}$ , and  $r_d = 1.5~\mu\text{m}$ , respectively.

[16] Figure 1 also shows how the Mach cone opening angle  $\theta = \sin^{-1}(V_p/V_d)$  varies with  $k\rho_e$ , r, and  $r_d$ . We have estimated the Mach cone opening angle  $\theta$  associated with





**Figure 1.** Showing  $V_d/V_p$  versus  $k\rho_e$  curves for  $r < r_0 = 1.822$  with parameters (given in the text), and for different values of r and  $r_d$ . The upper plot, where  $r_d = 1$   $\mu$ m, r = 1.5 (solid curve), r = 1.6 (dotted curve), and r = 1.7 (dashed curve), shows the effect of r on  $V_d/V_p$  versus  $k\rho_e$  curve. The lower plot, where r = 1.6,  $r_d = 0.5$   $\mu$ m (solid curve),  $r_d = 1$   $\mu$ m (dotted curve), and  $r_d = 1.5$   $\mu$ m (dashed curve), shows the effect of  $r_d$  on  $V_d/V_p$  versus  $k\rho_e$  curve.

the modified DAWs of wavelength  $\sim$ 7.5 m (corresponding to  $k\rho_e \simeq 1$ , since  $\rho_e \simeq 1.193$  m for  $T_e = 100$  eV and  $B_0 = 0.2$  G) in both cases of  $r = 1.5 < r_0$  and  $r = 2.5 > r_0$  for  $r_d = 0.5$   $\mu$ m. This is found to be  $\sim$ 25° and  $\sim$ 29° in the cases of  $r = 1.5 < r_0$  and  $r = 2.5 > r_0$ , respectively.

## 5. Discussion

[17] We have discussed the possibility of the Mach cone formation involving the modified dust-acoustic waves, with a perpendicular wavelength of the order of (or shorter than)  $\rho_e$ , in a magnetized ED plasma. We have numerically estimated the maximum wavelength  $(\lambda_m)$  of the modified DAWs for which Mach cones are formed in Saturn's rings. We have found that  $\lambda_m$  increases as r decreases (increases) in the cases of  $r < r_0$  ( $r > r_0$ ), and  $\lambda_m$  increases as  $r_d$  decreases in both the cases of  $r < r_0$  and  $r > r_0$ . We have estimated the Mach cone opening angle  $\theta$  associated with the modified DAWs of wavelength  $\sim$ 7.5 m (corresponding to  $k\rho_e \simeq 1$ ) in both the cases of  $r = 1.5 < r_0$  and  $r = 2.5 > r_0$  for  $r_d = 0.5$   $\mu$ m and dusty plasma parameters corresponding to Saturn's rings. This is found to be  $\sim$ 25° and  $\sim$ 29° in the cases of  $r = 1.5 < r_0$  and  $r = 2.5 > r_0$ , respectively.

- [18] We have shown that in magnetized dusty plasmas of Saturn's rings, Mach cones are formed due to resonance interactions between the modified DAWs and the relative motion of the small dust particles to that of a dust boulder. The relative motion is caused by the balance between the gravitational force and the Lorentz force associated with the planetary magnetic field. The speed of small dust particle relative to that of the dust boulder depends on i) the sign of the dust, ii) the size of the dust  $r_d$ , iii) the location of the dust r, and iv) whether the dust is inside ( $r < r_0$ ) or outside  $(r > r_0)$  the synchronous distance  $r_0$ . Our present investigation has clearly shown how the relative speed  $V_d$  of positively charged dust particles (participating in the formation of Mach cones) varies with the size as well as with the position of the dust particle in both inside  $(r < r_0)$  and outside  $(r > r_0)$  the synchronous distance.
- [19] We expect that the NASA/ESA space probe CASSINI can make direct observations of Mach cones involving the modified dust acoustic waves, which we have reported in this Letter. The opening angles of the corresponding Mach cones, which we have estimated herein, can be used for obtaining the dust mass density, dust charge, dust composition, and the optical depth of Saturn's dense rings.

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