Response to “Comment on 'Instability of the Shukla mode in a dusty plasma containing equilibrium density and magnetic field inhomogeneities’ and ‘New resonance and cutoff for low-frequency electromagnetic waves in dusty magnetoplasmas’” [Phys. Plasmas 11, 4154 (2004)]

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It is shown that the Comment of Rudakov on the papers by Shukla et al. and Mamun et al. is misleading. © 2004 American Institute of Physics. [DOI: 10.1063/1.1770845]

The purpose of this Response is to point out that Rudakov's claims are not justified to criticize the work of Shukla et al. and Mamun et al. We shall therefore demonstrate that his criticisms are irrelevant.

Rudakov claims that Shukla et al. have revisited the magnetic drift wave instability, a topic addressed about 35 years ago in the context of electron magnetohydrodynamics (EMHD), and then in dusty plasma physics. We refute this claim on the grounds given below.

The EMHD describes wave phenomena (magnetic electron drift waves and obliquely propagating electron whistlers) on time scales much longer than the ion plasma and ion gyroperiods. Thus, ions form a neutralizing background and the ion fluid velocity is zero. In such a situation, the nonlinear wave motions in a collisionless, cold electron plasma is governed by

$$\frac{\partial}{\partial t} \left( B - \frac{c^2}{\omega_{pe}} \nabla^2 B \right) - \nabla \times \left[ v_e \times \left( B - \frac{c^2}{\omega_{pe}} \nabla^2 B \right) \right] = 0,$$

where $B$ is the magnetic field, $c$ is the speed of light in vacuum,

$$v_e = -\frac{c}{4 \pi n_e e} \nabla \times B$$

is the electron fluid velocity, $\omega_{pe} = (4 \pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, $n_e$ is the electron number density, $e$ is the magnitude of the electron charge, and $m_e$ is the electron mass. Equation (1) admits obliquely propagating electron whistlers which have the frequency

$$\omega = \frac{k^2 c^2 \omega_{ce} \cos \theta}{\omega_p^2 + k^2 c^2}$$

in a uniform magnetoplasma. Here, $k$ is the wave number, $\theta$ is the angle between the wave vector $k$ and the external magnetic field $B_0$, $\hat{z}$ is the unit vector along the $z$ axis, $\omega_{ce} = eB_0/m_e c$ is the electron gyrofrequency, and $\omega_p$ is the unperturbed electron plasma frequency. In a nonuniform magnetoplasma, one can also have the compressional magnetic electron drift mode $\omega = k_e c B_0/4 \pi e n_e L_n(1 + k^2 \lambda_d^2)$, as reported in Ref. 5. Here, $k_e$ is the $y$ component of the wave vector $k$, $L_n$ is the electron density gradient scale length, and $\lambda_d = c/\omega_p$ is the electron skin depth. However, the magnetic drift waves in an electron plasma are quite different from the Shukla mode, which exists in an electron-ion plasma containing stationary charged dust grains. The Shukla mode is a low-frequency (in comparison with the Rao cut-off frequency) compressional dispersive wave in a nonuniform magnetoplasma, and it has not been reported in Ref. 6. In his present (revised) Comment, Rudakov suggests that the frequency in Ref. 6 is $\omega = -k_e c B_0/4 \pi e Z_d n_d L_{nd}(1 + k^2 \lambda_d^2)$, where $Z_d$ is the number of charges residing on the dust grain, $n_d$ is the dust number density, $L_{nd} = d \ln(Z_d n_d)/dx$ is the inverse scale length of the dust density gradient, $\lambda_d = d n_d/Z_d n_d$, and $\omega_p$ is the unperturbed ion plasma frequency. However, it should be noted that in the above Rudakov is just copying down the Shukla mode frequency, as the latter cannot be derived from Eq. (18) in Ref. 6 which does not contain the density inhomogeneity term [note that the third term in the left-hand side of Eq. (18) is nonlinear since $b = B(x,y)/B_0$]. Furthermore, Ref. 6 assumes that the wave frequency $\omega$ is much smaller than the ion gyrofrequency $\omega_{ci}$, which is a significantly different condition than that of Ref. 8 which supposes that $\omega \ll \omega_{ci}/n_e$. Rudakov claims that in Ref. 6 the dust component is unmagnetized because of inertia and collisions with a neutral gas, but the electrons and ions are magnetized. In the Shukla mode, both electrons and ions are magnetized and the dust grains are stationary. Clearly, Rudakov is misleading the readers.

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in the low-frequency limit \(\frac{\partial}{\partial t} + v_e \cdot \nabla\) the magnitude of the Rao cutoff frequency \(\Omega_R(q_d \eta_d/en_i)\omega_c,\) where \(q_d\) is the dust charge (equals \(-Z_d e\) for negatively charged dust grains and \(Z_d e\) for positively charged), \(\omega_c = eB_0/m_i c\) is the ion gyrofrequency, and \(m_i\) is the ion mass. It should be stressed that Eq. (4), which contains the Coriolis-like force \(\Omega_R v_e \times z\) due to the separation of charges in the presence of stationary charged dust grains, has been obtained by substituting the electric field from the inertialess electron momentum equation into the usual ion momentum equation, and then by eliminating the electron fluid velocity by using Ampère’s law and the quasineutrality condition \(en_i = en_e - q_d\eta_d.\) Thus, the effect of immobile charged dust in Eqs. (4) and (5) appears through the latter as \(n_i \neq n_e\) in dusty plasmas. Furthermore, contrary to the assertion of Ref. 1, we emphasize that Eq. (4) was not obtained by Smith and Brice\(^{10}\) who considered the propagation of electromagnetic waves in a multi-ion magnetoplasma containing electrons and multiple positive ions, and showed that for each ionic species for waves propagating perpendicular to the magnetic field, there exists a multiple-ion resonance and a multiple-ion cutoff. In multi-ion magnetoplasmas one of the ion species cannot be assumed immobile due to the narrow range of ion charge to ion mass distributions. In dusty plasmas, this separation is allowed as \(Z_d/m_d \ll Z_i/m_i,\) and the dust grains can be assumed stationary as done in deriving Eqs. (4) and (5). Here, \(m_d\) is the dust mass and \(Z_i\) is the ion charge state. Mamun et al.\(^{11}\) have presented a new cut-off frequency for low-frequency electromagnetic waves in a multi-ion magnetoplasma with stationary charged dust grains.

Shukla et al.\(^{2}\) have considered the instability of the flute-like Shukla mode in the presence of equilibrium density and magnetic field inhomogeneities. They have presented a self-consistent plasma equilibrium in crossed dc electric and magnetic fields. Their equations (3) and (4) relate the dc electric field with a constant magnetic field gradient in a self-consistent manner. Such an equilibrium is absent in the present work of Rudakov as well as in Ref. 6. Furthermore, Shukla et al.\(^{2}\) have shown that the magnetic field inhomogeneity provides a linear coupling between the compressional magnetic field and the electron density perturbation, see, for example, Eqs. (11)–(13) in Ref. 2. In the local approximation, which holds for wavelengths much smaller than the scale lengths of the density and magnetic field inhomogeneities, Shukla et al.\(^{2}\) found their Eqs. (15) and (16), and that the equilibrium plasma is unstable against the modified Shukla mode provided that the equilibrium magnetic field and plasma density gradients oppose each other. Physically, instability arises because of a finite value of the divergence of the product of the density perturbation and the equilibrium ion drift in an inhomogeneous magnetic field. The electron density and magnetic field gradients in combination with the sheared wave electric field produce an electron flux, which breaks the frozen-in-field lines. The resulting phase shift between the density and compressional magnetic field perturbations, in turn, produces instability of the modified disper- sive Shukla mode, governed by Eq. (15) in Ref. 2 which contains the dispersive term \(k_z^2 \rho_d^2,\) where the modified ion skin depth \(\rho = \lambda_d\) differs from \(\lambda_i\). We thus do not need the inclusion of the dust dynamics for producing instability, contrary to the suggestion of Rudakov.\(^1\)

Rudakov’s analysis of a nondispersive compressional mode, given by his Eq. (4), is based on inertialess electrons and ions [e.g., Eqs. (1) in Ref. 1], contrary to the modified Shukla mode\(^2\) which employs inertialess electrons but inertial magnetized ions in the low-frequency limit in which the ion fluid velocity is controlled by the Coriolis-like force of Rao [the first term in the right-hand side of our Eq. (4) above], which is dependent on the presence of immobile charged dust grains in an electron-ion plasma. Hence, Rudakov’s stability argument based on his Eq. (3) is useless, since the situation described in Ref. 2 is quite different from what Rudakov presents in his Comment.\(^1\)

Incidentally, the linear and nonlinear low-frequency (in comparison with the ion gyrofrequency) electromagnetic waves in a dusty magnetoplasma with mobile dust grains and inertialess electrons and ions are described by the dust H-MHD equations\(^{12}\) consisting of the dust continuity equation

\[
\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d v_d) = 0, \tag{6}
\]

the dust momentum equation

\[
\left( \frac{\partial}{\partial t} + v_d \cdot \nabla \right) v_d = \frac{(\nabla \times B) \times B}{4 \pi \rho_d}, \tag{7}
\]

and Faraday’s law

\[
\frac{\partial B}{\partial t} = \nabla \times \left[ v_d - \frac{c \nabla \times B}{4 \pi \rho_d \eta_d} \right] \times B, \tag{8}
\]

where \(\rho_d = n_d m_d \) is the dust mass density. Equations (6)–(8) admit dust Alfvén waves on temporal and spatial scales that are of the order of the dust gyroperiod and the dust skin depth. Thus, our Eqs. (6)–(8) above are much more appropriate for wave studies than Eqs. (1)–(3) of Rudakov,\(^1\) which completely neglect the dust fluid velocity in the induction equation (3).

We now turn our attention to the work of Mamun et al.\(^3\) who presented the dispersion properties of low-frequency (in comparison with the ion gyrofrequency) dust-electromagnetic waves in a four component uniform magnetoplasma. Their model consists of inertialess cold electrons and ions [e.g., Eqs. (1) and (2) in Ref. 3] and two components of inertial and magnetized charged dust grains, which obey Eqs. (3) and (4) in Ref. 3. They showed that the simul-
taneous presence of positive and negative inertial dust grains introduces a new resonance and cutoff for dust-electromagnetic waves, and a sharp forbidden-frequency-band in between the two branches of the propagating dust-electromagnetic waves. This is a completely different scenario as compared to that of the Buchsbaum’s resonance in a magnetized plasma with two ion species without charged dust grains. Buchsbaum found that the presence of two-ion species introduces an additional resonance for electrostatic waves propagating across the static magnetic field at a frequency between the two-ionic gyrofrequencies. Hence, the work of Mamun et al. and that of Buchsbaum cannot be compared, and Eqs. (6) and (7) of Rudakov are irrelevant as they do not incorporate the dusty plasma model of Mamun et al. We stress that neither Smith and Brice nor Buchsbaum have charged dust grains in their descriptions. Finally, it is inexplicable that Rudakov goes on suggesting that the wavelength of the dust Alfvén waves should be of the order of 10 km for laboratory plasma conditions, which is totally unrealistic. The theory of Mamun et al. would work in magnetized laboratory dusty discharges for waves with $k \gg \omega_p/\omega_{pd}$. The latter is satisfied for measurable waves in magnetized dusty plasmas.

In conclusion, the criticism of Rudakov in Refs. 2 and 3 is peculiar, as it does not contain relevant information. The materials presented in Ref. 1 as well as the history descriptions are misleading to the readers of Physics of Plasmas.

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