Nonlinear Aspects of Quantum Plasma Physics: Nanoplasmonics and Nanostructures in Dense Plasmas

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Eugene Paul Wigner (1902–1995). Nobel Prize in Physics in 1963 "for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles" (Quantum plasma physics: Wigner-Moyal transform, Wigner function)

David Joseph Bohm (1917–1992) (Bohm-diffusion, the Bohm sheath criterion, the quantum Bohm potential. Also the Aharonov-Bohm effect where a charged particle is affected by electromagnetic fields in regions from which the particle is excluded)

David Pines (1924–) Awarded two Guggenheim Fellowships, the Feenberg Medal, Friemann, Dirac, and Drucker Prizes. (Theory of many-body systems and theoretical astrophysics, early works on collective effects in quantum plasma.) Prof. Em. at University of Illinois at Urbana–Champaign.
Introduction

- Quantum plasmas are ubiquitous in:
  - ultrasmall electronic devices and micromechanical systems
  - intense laser–solid density plasma interaction experiments
  - microplasmas
  - superdense astrophysical objects (neutron stars and white dwarfs)

- Quantum mechanical effects can be important when the de Broglie wavelength of the charge carriers (electrons, positrons) is comparable to:
  - the dimension of the system → tunneling effects
  - the mean distance between particles → overlapping of wave functions, quantum statistics
Introduction (Continued)

- Classical vs. Quantum plasmas
  - Classical plasmas have low density and high temperature.
  - Quantum plasmas have high density and low temperature

- Quantum forces due to the
  - strong electron/positron (hole) density correlations (the Bohm potential),
  - the quantum statistical description for a Fermi plasma yields a new pressure law owing to the Fermi-Dirac statistics.

- Nonlinear waves and structures
  - dark quantum solitons and vortices
  - nonlinear interaction with electromagnetic waves
Classical Plasmas vs. Quantum Plasmas

- Quantum effects can be measured by the thermal de Broglie wavelength of the particles composing the plasma

\[ \lambda_B = \frac{\hbar}{mV_T}, \quad V_T = \sqrt{\frac{k_B T}{m}} \]

which roughly represents the spatial extension of a particle’s wave function due to quantum uncertainty.

- For classical regimes, the de Broglie wavelength is so small that particles can be considered as point-like, and therefore there is no overlapping of the wave functions and no quantum interference.
Classical Plasmas vs. Quantum Plasmas (Continued)

Quantum effects start playing a significant role when

- the de Broglie wavelength is similar to or larger than the average interparticle distance \( n^{-1/3} \), i.e. when

\[
    n\lambda_B^3 \gtrsim 1,
\]

or, the temperature is comparable or lower than the Fermi temperature \( T_F = E_F/k_B \), where

\[
    E_F = \frac{\hbar^2}{2m} (3\pi^2)^{2/3} n^{2/3}
\]

is the Fermi energy for electrons, so that

\[
    \chi = \frac{T_F}{T} = \frac{1}{2} (3\pi^2)^{2/3} (n\lambda_B^3)^{2/3} \gtrsim 1
\]
Properties of Dense Quantum Plasmas

For an ultracold plasma, the Fermi screening scalelength

$$\lambda_F = V_F / \omega_p$$

is the quantum analogue of the Debye radius, where the Fermi speed

$$V_F = (2E_F/m)^{1/2} = \frac{\hbar}{m}(3\pi^2n)^{1/3}.$$

is the speed of an electron at the Fermi surface.

The quantum coupling parameter

$$G_q = \frac{E_{int}}{E_F} \sim \left(\frac{1}{n\lambda_F^3}\right)^{2/3} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2,$$

is analogous to the classical one when $$\lambda_F \rightarrow \lambda_D.$$
Anti-symmetric wave function — Pauli exclusion principle

Model for the quantum N body problem: The Schrödinger equation for the N-particle wave function \( \psi(q_1, q_2, \ldots, q_N, t) \) where \( q_j = (r_j, s_j) \) (space, spin) of particle \( j \).

Identical Fermions: The Slater determinant

\[
\psi(q_1, q_2, \ldots, q_N, t) = \frac{1}{\sqrt{N!}} \begin{vmatrix}
\psi_1(q_1, t) & \psi_2(q_1, t) & \cdots & \psi_N(q_1, t) \\
\psi_1(q_2, t) & \psi_2(q_2, t) & \cdots & \psi_N(q_2, t) \\
\vdots & \vdots & \ddots & \vdots \\
\psi_1(q_N, t) & \psi_2(q_N, t) & \cdots & \psi_N(q_N, t)
\end{vmatrix}
\]

Anti-symmetric under odd numbers of permutations. The Pauli exclusion principle: \( \psi \) vanishes if two rows are identical.

Example (\( N = 2 \)):
\[
\psi(q_1, q_2, t) = \frac{1}{\sqrt{2}} [\psi_1(q_1, t)\psi_2(q_2, t) - \psi_1(q_2, t)\psi_2(q_1, t)] \text{ so that }
\]
\[
\psi(q_2, q_1, t) = -\psi(q_1, q_2, t) \text{ and } \psi(q_1, q_1, t) = 0.
\]
Quantum kinetic model

The quantum analogue to the Vlasov-Poisson system is the Wigner-Poisson model

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = -\frac{iem_e^3}{(2\pi)^3 \hbar^4} \]

\[ \times \int \int d^3 \lambda d^3 \mathbf{v}' e^{i m (\mathbf{v} - \mathbf{v'}) \cdot \lambda / \hbar} \left[ \phi \left( \mathbf{x} + \frac{\lambda}{2}, t \right) - \phi \left( \mathbf{x} - \frac{\lambda}{2}, t \right) \right] f(\mathbf{x}, \mathbf{v}', t) \]

and

\[ \nabla^2 \phi = 4\pi e \left( \int f \, d^3 \mathbf{v} - n_0 \right) \cdot \]

Note that the Wigner equation converges to the Vlasov equation when \( \hbar \to 0 \)

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{e}{m_e} \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} \]
Quantum Hydrodynamical (QHD) Model

We take the moments of the Wigner equation and obtain for the quantum-electron fluid

\[ \frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0 \]

\[ m \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = e \nabla \phi - \frac{1}{n} \nabla P + F_Q, \]

where \( \phi \) is determined from \( \nabla^2 \phi = 4\pi e (n - n_0) \), and for the FD plasma we have

\[ P = \frac{m V_F^2}{3n_0^2} n^3 \quad \text{and} \quad F_Q = \frac{\hbar^2}{2m} \nabla \left( \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) \equiv -\nabla \phi_B. \]

Manfredi & Haas, Phys. Rev. B 64, 075316 (2001),
Electrostatic electron waves

Linearization of the NLS-Poisson Eqs. yields the frequency of EPOs

\[ \omega_k = \left( \omega_{pe}^2 + k^2 V_{TF}^2 + \frac{\hbar^2 k^4}{4m_e^2} \right)^{1/2}, \quad V_{TF} = \sqrt{\frac{k_B T F e}{m_e}} \]

Two distinct dispersive effects:

- Long wavelength regime: \( V_{TF} \gg \frac{\hbar k}{2m_e} \)
- Short wavelength regime: \( V_{TF} \lesssim \frac{\hbar k}{2m_e} \)
- Critical wavenumber:

\[ k_{crit} = \frac{2\pi}{\lambda_{crit}} = \frac{\pi \hbar}{m_e V_{TF}} \sim n^{-1/3} \]

Effective nonlinear Schrödinger equation

Introduce the effective wave function

$$\psi(r, t) = \sqrt{n(r, t)} \exp(iS(r, t)/\hbar)$$

where $S$ is defined according to $mu = \nabla S$ and $n = |\psi|^2$. It is easy to show that the QHD equations are equivalent to

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi + e\phi\psi - \frac{mV_F^2}{2n_0^2} |\psi|^{4/D} \psi = 0$$

and

$$\nabla^2 \phi = 4\pi e (|\psi|^2 - n_0)$$

Manfredi & Haas, Phys. Rev. B 64, 075316 (2001),
Nonlinear Structures at Quantum Scale

Normalized system of equations for plasmons

\[ i \frac{\partial \Psi}{\partial t} + A \nabla^2 \Psi + \varphi \Psi - |\Psi|^{4/D} \Psi = 0, \]

\[ \nabla^2 \varphi = |\Psi|^2 - 1, \]

where \( A \) represents the quantum coupling strength. Conserved quantities:

\[ N = \int |\Psi|^2 d^3x \]

\[ P = -i \int \Psi^* \nabla \Psi \, d^3x \]

\[ L = -i \int \Psi^* \mathbf{r} \times \nabla \Psi \, d^3x \]

\[ E = \int \left[ -\Psi^* A \nabla^2 \Psi + |\nabla \varphi|^2 / 2 + |\Psi|^{2+4/D} \right] \, d^3x \]

Shukla & Eliasson, PRL 96, 245001 (2006)
1D Quantum Electron Hole (Dark Soliton)

Shukla & Eliasson, PRL 96, 245001 (2006)
Dynamics of Quantum Electron Holes

Electron density (left) & electrostatic potential (right)

Shukla & Eliasson, PRL 96, 245001 (2006)
2D Quantum Electron Vortices

Introduce $\Psi = \psi(r) \exp(i n \theta - i \Omega t)$

$$\Omega \psi + A \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{n}{r^2} \right) \psi + \varphi \psi - |\psi|^2 \psi = 0,$$

and

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \varphi = |\psi|^2 - 1,$$

with $\Omega = 1$ (due to $\psi = 1$ and $\varphi = d\psi/dr = 0$ at $r = \infty$) and vortex charge states $n = 0, \pm 1, \pm 2, \ldots$. 
2D Quantum Electron Vortices

\[ |\psi|^2 \]

- - - - - \( n = 1 \)
- - - - - \( n = 2 \)
- - - - - \( n = 3 \)

Shukla & Eliasson, PRL 96, 245001 (2006)
**Interacting 2D Quantum Vortices**

Single charge states \((n = 1)\)

Shukla & Eliasson, PRL 96, 245001 (2006)
Interacting 2D Quantum Vortices

Double charge states \((n = 2)\)

Shukla & Eliasson, PRL 96, 245001 (2006)
Nonlinear Photon–Plasmon Interactions

- Photons get trapped into quantum electron holes. The governing dynamical equations are the Schrödinger equations for the photons and plasmons, which are respectively

$$2i\Omega_0 \left( \frac{\partial}{\partial t} + V_g \frac{\partial}{\partial x} \right) A_\perp + \frac{\partial^2 A_\perp}{\partial x^2} - \left( \frac{|\psi|^2}{\sqrt{1 + |A_\perp|^2}} - 1 \right) A_\perp = 0,$$

and

$$iH_e \frac{\partial \psi}{\partial t} + \frac{H_e^2}{2} \frac{\partial^2 \psi}{\partial x^2} + (\varphi - \sqrt{1 + |A_\perp|^2} + 1) \psi = 0,$$

where $H_e = \hbar \omega_{pe}/m_e c^2$, and $\varphi$ follows from the Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = |\psi|^2 - 1$$

Shukla & Eliasson, PRL 99, 096401 (2007)
Localized Excitations for Different $H_e$

Shukla & Eliasson, PRL 99, 096401 (2007)
Collapse of Photons into Solitary Structures, $H_e = 0.1$.

Shukla & Eliasson, PRL 99, 096401 (2007)
Collapse of Photons into Solitary Structures, $H_e = 0.5$. 

Shukla & Eliasson, PRL 99, 096401 (2007)
Summary & Discussions

- We have summarized some properties of quantum plasmas.

- Presented quantum models for collective motion of quantum plasmas.
  - Localized electrostatic structures in the form of quantum electron holes and 2D quantum electron vortices
  - Discussed relativistic localization of EM waves in a quantum plasma.

- Future works
  - Validation of nonlinear quantum fluid equations
  - Strongly relativistic effects — Dirac equation
  - Collective effects in strong magnetic fields — Landau quantization