Relaxation times for magnetized plasma - a parametric study[1]

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Abstract

We have considered the relaxation towards equilibrium of a charged test-particle weakly interacting with a uniform magnetized plasma. A set of analytic expressions are presented for relaxation times, depending of the form of the (long-range) inter-particle interactions as well as the bulk equilibrium configuration. Considering a uniform magnetic field, Debye interactions and a Maxwellian background, explicit new expressions are computed. All quantities are functions of the strength of the field $\mathbf{B}$, the temperature T and the t.p. velocity components [1].

A parametric study is envisaged in terms of these parameters [2]. The numerical study presented here reveals the mechanism of influence of the magnetic field on relaxation towards equilibrium, through an interplay between collision and gyration length/time scales. This fact seems to be in agreement with arguments appearing in [3], yet seems to contradict the 'standard'nam description used in the past, where the influence of the magnetic field on the collision term is either under-estimated [4] or neglected [5] when discussing the physical - transport - properties of plasma.

[1] *29th EPS Conference on Plasma Physics and Controlled Fusion*, Montreux, 17-21/06/2002; Contribution reference P4.008.

1 Introduction

In an earlier paper [1] we have undertaken a study of the dynamics of a charged test-particle (t.p.) interacting with a magnetized background plasma in equilibrium. Starting from first microscopic principles, a markovian Fokker-Planck-type kinetic equation (F.P.E.) was derived and analytical expressions for the coefficients were obtained. Emphasis was made on the magnetic field dependence of the collision integral, as well as on the effect of non space-uniformity of the t.p. distribution function $f(\mathbf{x}, \mathbf{v}; t)$. This new F.P.E. was thus suggested as a basis for the study of the influence of a magnetic field on the kinetic properties of plasma in various parameter regions and regimes - (as compared, that is, to the standard *unmagnetized* Landau description).

In the following, we summarize these results and then carry on by explicitly evaluating the diffusion coefficients by considering a Maxwellian reservoir background state and a Debye-type interaction law. The aim of this brief report is to present a set of exact computable expressions for the diffusion coefficients and actually point out their dependence on, among other parameters, the magnitude of the magnetic field.

2 The model

We consider a test-particle (t.p.) Σ (of species $\alpha_\Sigma = \alpha$; charge $e_\Sigma^\alpha = e$, mass $m_\Sigma^\alpha = m$) surrounded by (and weakly coupled to) a homogeneous background plasma (the reservoir 'R': N particles, of species $\alpha' \in \{\alpha_j\} = \{e, i, ...\}, \quad j = 1, 2, \ldots, N$). The whole system is subject to a *uniform* stationary magnetic field along $\hat{z}$. The equations of motion for the t.p. read:

$$\dot{\mathbf{x}} = \mathbf{v}\,; \qquad \dot{\mathbf{v}} = \frac{1}{m}\,[\frac{e}{c}(\mathbf{v} \times \mathbf{B}) \,+\, g\,\mathbf{F_{int}}(\mathbf{x}, \mathbf{v}; \mathbf{X_R}; t)] \tag{1}$$

where $\mathbf{X} = (\mathbf{x}, \mathbf{v}) \equiv (\mathbf{x_\Sigma}, \mathbf{v_\Sigma})$ and $\mathbf{X_R} \equiv \{\mathbf{X_j}\} = (\mathbf{x_j}, \mathbf{v_j})$ denote the coordinates of the test- (Σ) and reservoir (R) particles respectively. The *interaction* force $\mathbf{F_{int}}(\mathbf{x}, \mathbf{v}; \mathbf{X_R}; t) = -\frac{\partial}{\partial \mathbf{x}} \sum V(|\mathbf{x} - \mathbf{x_j}|)$ ('tagged' by g), represents the sum of random interactions between Σ and the heat bath (assumed in equilibrium); it is actually a stationary Gaussian process with zero mean-value.

The zeroth-order (in g^0) problem of motion yields the well-known (helicoidal) solution:

$$\mathbf{x}(t) = \mathbf{x}(0) + \mathbf{N}(t)\,\mathbf{v}(0) \qquad\qquad \mathbf{v}(t) = \mathbf{N}'(t)\,\mathbf{v}(0)$$

where

$$\mathbf{N}_j^{\alpha_j}(t) = \Omega^{-1} \begin{pmatrix} \sin\Omega t & s\,(1-\cos\Omega t) & 0 \\ s\,(\cos\Omega t - 1) & \sin\Omega t & 0 \\ 0 & 0 & \Omega t \end{pmatrix} \tag{2}$$

$\Omega = \Omega^{\alpha_j} \equiv \frac{|e_{\alpha_j}|B}{m_{\alpha_j}c}$ is the gyro-frequency of particle j and $s = s_{\alpha_j} = \frac{e_{\alpha_j}}{|e_{\alpha_j}|} = \pm 1$ is the *sign* of e_j (the subscript will be omitted where Σ is understood); finally $\mathbf{N}'(t) = d\mathbf{N}(t)/dt$.

3 Statistical formulation - a kinetic equation

The test-particle's reduced distribution function is $f(\mathbf{x}, \mathbf{v}; t) = (I, \rho)_R \equiv \int_{\Gamma_R} d\mathbf{X_R}\, \rho$, where $\rho = \rho(\{\mathbf{X}, \mathbf{X_R}\}; t)$ denotes the total phase-space d.f., normalized to unity: $\int d\mathbf{X}\, \rho = 1$. By assuming interactions to be weak ($g \ll 1$), the BBGKY hierarchy of equations can be truncated to 2nd order in g; neglecting initial correlations, f is thus found to obey a Non-Markovian Master Equation. Following an approach developed in the past in the theory of open statistical mechanical systems [3], the latter was shown in [1] to lead to a Markovian Fokker-Planck-type equation of the form:

$$\begin{aligned} \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{e}{mc}(\mathbf{v}\times\mathbf{B})\frac{\partial f}{\partial \mathbf{v}} &= \Big[\Big(\frac{\partial^2}{\partial v_x^2} + \frac{\partial^2}{\partial v_y^2}\Big)\Big[D_\perp(\mathbf{v})f\Big] + \frac{\partial^2}{\partial v_z^2}[D_\parallel(\mathbf{v})f] \\ &+2\,s\,\Omega^{-1}\Big[\frac{\partial^2}{\partial v_x \partial y} - \frac{\partial^2}{\partial v_y \partial x}\Big]\Big[D_\perp(\mathbf{v})f\Big] + \Omega^{-2}\,[D_\perp^{(XX)}(\mathbf{v})](\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})f \\ &-\frac{\partial}{\partial v_x}\Big[\mathcal{F}_x(\mathbf{v})\,f\Big] - \frac{\partial}{\partial v_y}\Big[\mathcal{F}_y(\mathbf{v})\,f\Big] - \frac{\partial}{\partial v_z}\Big[\mathcal{F}_z(\mathbf{v})\,f\Big] \\ &+ s\,\Omega^{-1}\,\mathcal{F}_y(\mathbf{v})\,\frac{\partial}{\partial x}f - s\,\Omega^{-1}\,\mathcal{F}_x(\mathbf{v})\,\frac{\partial}{\partial y}f \end{aligned} \tag{3}$$

where $f = f(\mathbf{x}, \mathbf{v}; t)$ [4]. Note that all coefficients are functions of $\{v_\perp, v_\parallel\}$. Therefore, by integrating over position $\{\mathbf{x}\}$, one recovers a reduced F.P.E., describing the evolution of $f_{loc}(\mathbf{v}; t) = \int d\mathbf{x} f(\mathbf{x}, \mathbf{v}; t)$. This equation can be viewed as a 'linearized' version of a kinetic equation which has appeared in earlier works [5], [6]. In velocity space cylindrical coordinates it reads:

$$\begin{aligned} \frac{\partial f}{\partial t} - s\,\Omega\,\frac{\partial f}{\partial \theta} &= \Big(\frac{\partial^2}{\partial v_\perp^2} + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} + \frac{1}{v_\perp^2}\frac{\partial^2}{\partial \theta^2}\Big)\Big[D_\perp(\mathbf{v})f\Big] + \frac{\partial^2}{\partial v_\parallel^2}[D_\parallel(\mathbf{v})f] \\ &-\Big(\frac{\partial}{\partial v_\perp} + \frac{1}{v_\perp}\Big)\Big[\mathcal{F}_\perp(\mathbf{v})\,f\Big] + \frac{1}{v_\perp}\frac{\partial}{\partial \theta}\Big[\mathcal{F}_\theta(\mathbf{v})\,f\Big] - \frac{\partial}{\partial v_\parallel}\Big[\mathcal{F}_\parallel(\mathbf{v})\,f\Big] \end{aligned} \tag{4}$$

Notice that, for a gyrotropic distribution function $f = f(v_\perp, v_\|) \neq f(\theta)$, the field appears *only* in the coefficients in the rhs. Note the definitions of the *dynamical friction vectors*:

$$\mathcal{F}_\perp = (1 + \frac{m}{m_{\alpha'}})\,\frac{\partial D_\perp}{\partial v_\perp}, \qquad \mathcal{F}_\theta = (1 + \frac{m}{m_{\alpha'}})\,\frac{\partial D_\angle}{\partial v_\perp}, \qquad \mathcal{F}_\| = (1 + \frac{m}{m_{\alpha'}})\,\frac{\partial D_\|}{\partial v_\|}$$

4 Coefficients

The diffusion coefficients in (3) are defined by:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} D_\perp \\ D_\angle \\ D_\perp^{(XX)} \end{array} \right\} \\ D_\| \end{array} \right\} = \sum_{\alpha'} \frac{1}{m_{\alpha'}^2} \int_0^t d\tau \left\{ \begin{array}{c} \left\{ C_\perp^{\alpha,\alpha'} \right\} \\ C_\|^{\alpha,\alpha'} \end{array} \right\} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{2}\cos \Omega^\alpha \tau \\ (-s^\alpha)\,\frac{1}{2}\sin \Omega^\alpha \tau \\ (1 + \frac{1}{2}\cos \Omega^\alpha \tau) \end{array} \right\} \\ 1 \end{array} \right\} \tag{5}$$

where $C^{\alpha,\alpha'}_{\{\perp,\|\}}(v_\perp, v_\perp; \Omega)$ are elements of the (diagonal) force-correlation matrix $\mathbf{C}(\tau) = \langle \mathbf{F_{int}}(t)\ \mathbf{F_{int}}(t-\tau)\rangle_R$; they are given by:

$$C_{\{\perp,\|\}} = n_{\alpha'}(2\pi)^3 \int d\mathbf{v_1}\,\phi^{\alpha'}_{eq}(\mathbf{v_1}) \int d\mathbf{k}\,\tilde{V}_k^2\, e^{ik_n N^\alpha_{nm}(\tau) v_m}\, e^{-ik_n N^{\alpha'}_{nm}(\tau) v_{1,m}}\, k^2_{\{\perp,\|\}} \tag{6}$$

(a summation over n, m is understood) where v_i $(v_{1,i})$, $i = 1, 2, 3$ denote the velocity coordinates of the test- (R-) particle and $\tilde{V}_k$ stands for the Fourier transform of $V(r)$; remember that $V = V(|\mathbf{r}|) = V(r)$ implies $V = \tilde{V}(|\mathbf{k}|) \equiv \tilde{V}_k$. The dynamical friction terms in (3) are given by:

$$\begin{aligned} \mathcal{F}_x &= (1+\mu)\,(\frac{\partial D_\perp}{\partial v_x} + \frac{\partial D_\angle}{\partial v_y}) \qquad \mathcal{F}_y = (1+\mu)\,(-\frac{\partial D_\angle}{\partial v_x} + \frac{\partial D_\perp}{\partial v_y}) \\ \mathcal{F}_z &= (1+\mu)\,\frac{\partial D_\|}{\partial v_z} \end{aligned} \tag{7}$$

$(\mu = m_\alpha/m_{\alpha'})$ [1]. Note the explicit dependence on the magnetic field as well as on the form of the reservoir equilibrium d.f. $\phi_{eq} = \phi_{eq}(v_\perp, v_\|)$ and the interaction potential $V(r)$.

As a matter of fact, expressions (4), (5) correspond to the formulae which appear in [1], [5]. In seek of an asymptotic form for the kernel (i.e. $t \to \infty$) the authors therein have chosen to straightforward carry out the time-integration first and thus obtain a set of final expressions in terms of an infinite series of Bessel functions (as expected from the cylindrical symmetry of the problem). That result is exact, yet quite delicate to manipulate. Since the test-particle formulation permits a more analytically tractable treatment, we have chosen not to adopt this procedure but rather try to advance the analytic computation as far as possible, still tracing the time dependence till the end, instead.

4.1 Velocity integrals

The v_1- integration in (5) can be carried out at this stage, once one assumes an analytic form for ϕ_{eq}. Here, it will be explicitly taken to be a Maxwellian of the form:

$$\phi_{Max}^{\alpha'}(v_1) = \prod_{i=1,2,3} \phi_0^{(i,\alpha')}\, e^{-v_{1,i}^2/\sigma_i^{\alpha'}} \tag{8}$$

where $\phi_0^{(i)} = (\frac{m_{\alpha'}}{2\pi T_{\alpha'}^{(i)}})^{1/2} \equiv \frac{1}{\sqrt{\pi\sigma_i^{\alpha'}}}$; $\sigma_i^{\alpha'} \equiv 2\, {v_{i,th}^{\alpha'}}^2 \equiv \frac{2T_i^{\alpha'}}{m_{\alpha'}}$ $\forall i \in \{1,2,3\} \equiv \{x,y,z\}$; Considering a *single-species* (i.e. $\alpha' = \alpha$) plasma and assuming $\sigma_1 = \sigma_2 = \sigma_\perp$, $\sigma_3 = \sigma_\parallel$, we obtain:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} D_\perp \\ D_\angle \\ D_\perp^{(XX)} \end{array} \right\} \\ D_\parallel \end{array} \right\} = \frac{n}{m^2}(2\pi)^4 e^{-v_\parallel^2/\sigma_\parallel} \int_0^t d\tau \int_0^\infty dk_\perp \Big[\int_{-\infty}^\infty dk_\parallel\, k_\parallel^{\{0,2\}} e^{-\sigma_\parallel \left(k_\parallel \tau - i \frac{2v_\parallel}{\sigma_\parallel}\right)^2/4}\, \tilde{V}_k^2 \Big]$$

$$k_\perp^{\{3,1\}}\, e^{-\sigma_\perp \frac{k_\perp^2}{\Omega^2} \sin^2 \frac{\Omega\tau}{2}} J_O(2\frac{k_\perp v_\perp}{\Omega} \sin\frac{\Omega\tau}{2}) \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{2}\cos\Omega\tau \\ (-s)\,\frac{1}{2}\sin\Omega\tau \\ 1 + \frac{1}{2}\cos\Omega\tau \end{array} \right\} \\ 1 \end{array} \right\} \tag{9}$$

Obviously, m (n) in $\{m,n\}$ correspond to the upper (lower) i.e. $\perp$ ($\parallel$) parts respectively. Note that the trivial angle integration has also been carried out in (9), since neither $\tilde{V}_k$ nor the rest of the integrand depends on the angle variable in Fourier space (as expressed in polar coordinates).

4.2 Fourier integrals

In fact, relation (9) holds as it stands for *any* particular form of (long-range) central interaction potential $V(r)$. Let us now explicitly consider a Debye potential: $V(r) = e^2 \frac{e^{-k_D r}}{r}$ i.e. $\tilde{V}_k = \frac{e^2}{2\pi^2}\frac{1}{k^2+k_D^2}$ ($\lambda_D = k_D^{-1}$ is the Debye length [6]; obviously $k^2 = k_\perp^2 + k_\parallel^2$).

The coefficients in (9) (actually functions of $\{v_\perp, v_\|, t;\, \sigma_\perp, \sigma_\|, \Omega\}$) now become:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} D_\perp \\ D_\angle \\ D_\perp^{(XX)} \end{array} \right\} \\ D_\| \end{array} \right\} = \frac{n}{m^2} 4e^4 \int_0^t d\tau \int_0^\infty dk_\perp \, e^{-\sigma_\perp \frac{k_\perp^2}{\Omega^2} \sin^2 \frac{\Omega\tau}{2}} J_O(2\frac{k_\perp v_\perp}{\Omega} \sin \frac{\Omega\tau}{2})$$

$$\Big(1 - \frac{k_D^2}{k_D^2 + k_\perp^2}\Big)^{\{3/2,1/2\}} \left\{ \begin{array}{c} \left\{ \, F_\perp \, \right\} \\ F_\| \end{array} \right\} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{2} \cos \Omega\tau \\ \frac{-s}{2} \cos \Omega\tau \\ 1 + \frac{1}{2} \cos \Omega\tau \end{array} \right\} \\ 1 \end{array} \right\} \tag{10}$$

J_0 is a Bessel function of the first kind; the functions $F = F_{\{\perp,\|\}}(k_\perp, v_\|, \tau;\, \sigma_\|)$ are given by:

$$\begin{aligned} F_{\{\perp,\|\}} &= \pm \frac{\sqrt{\pi}}{2} \sqrt{\sigma_\|}\, \tilde{k}_\perp \, \tau \, e^{-v_\|^2/\sigma_\|} \\ &+ \frac{\pi}{4} e^{\sigma_\| \tilde{k}_\perp^2 \tau^2/4} \sum_{s=+1,-1} \Big[e^{s\tilde{k}_\perp v_\| \tau} \, (1 \mp \sigma_\| \tilde{k}_\perp^2 \tau^2/2 \mp s\tilde{k}_\perp v_\| \tau)\, Erfc(\frac{1}{2}\sqrt{\sigma_\|}\tilde{k}_\perp \tau + s\frac{v_\|}{\sqrt{\sigma_\|}}) \Big] \end{aligned} \tag{11}$$

the upper (lower) signs corresponding to the $\perp$ ($\|$)- parts respectively; $\tilde{k}_\perp = (k_\perp^2 + k_D^2)^{1/2}$. $Erfc(x)$ is the *complementary* error function:

$$Erfc(x) = 1 - Erf(x) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Note that the integrand vanishes at infinity i.e. at $\tilde{k}_\perp \to \infty$ (and also at $\tau \to \infty$). Futhermore, notice that there is *no divergence* at $k_\perp = 0$, as the limit of the integrands at $k_\perp \to 0$ is finite (and the same holds for $\tau \to 0$). For the sake of clarity and briefness in presentation, details concerning the (tedious but straightforward) derivation of (8) - (10) has been omitted here; they will be reported soon in a more detailed account of our work [7].

4.3 Non-dimensional expressions

4.3.1 Correlations

Notice that the above relations imply a set of expressions for the force correlation functions $C^\alpha_{\{\perp,\|\}}(\tau)$, readily obtained by comparing (9), (10) to (5). The integration variable $k_\perp$ therein can be re-scaled to the non-dimensional variable: $x = \frac{\tilde{k}_\perp}{k_D} = (1 + \frac{k_\perp^2}{k_D^2})^{1/2}$; relation

(6) thus becomes:

$$C^{\alpha}_{\{\perp,\|\}}(\tau) \;=\; 4\,n\,e^4\,k_D \int_1^{x_{max}} dx\; e^{\lambda^2\,(1-x^2)\,\sin^2\frac{\Omega\tau}{2}} \left(1-\frac{1}{x^2}\right)^{\{1,0\}} J_O(2\lambda\,\sqrt{x^2-1}\,\tilde{v}_\perp\,\sin\frac{\Omega\tau}{2})\;\tilde{F}_{\{\perp,\|\}} \quad (12)$$

$\tilde{F} = \tilde{F}(\phi(x,\tau), \tilde{v}_\|)$ is given by:

$$\tilde{F}^{\alpha'}_{\{\perp,\|\}} = \pm\sqrt{\pi}\,\phi + \frac{\pi}{4}\sum_{s=+1,-1}\Big[(1\mp 2\,\phi^2 \mp s2\,\phi\,\tilde{v}_\|)\,e^{(\phi+s\,\tilde{v}_\|)^2}\,Erfc(\phi+s\,\tilde{v}_\|)\Big] \quad (13)$$

where

$$\phi = \frac{1}{\sqrt{2}}\,\omega_{p,\alpha}\,\tau\,x\,, \qquad \tilde{v}_\| = v_\|/\sqrt{\sigma}$$

$$\tilde{v}_\perp = v_\perp/\sqrt{\sigma}\,, \qquad \lambda = \sqrt{\sigma}\,\frac{k_D}{\Omega} = \cdots = \sqrt{2}\,\frac{\omega_p}{\Omega}$$

(having set $\sigma_\perp = \sigma_\| = \sigma$ for simplicity). Remember that $\sigma_\alpha = 2\,k_B\,T_\alpha/m_\alpha = 2\,v^2_{th,\alpha}$ is related to the thermal velocity (i.e. the temperature), $\Omega_\alpha = e_\alpha\,B/m_\alpha\,c$ is the cyclotron (gyroscopic) frequency, $k_D = \frac{4\pi e^2_\alpha n_\alpha}{k_B T_\alpha}$ is the Debye wave-number and $\omega_{p,\alpha} = (\frac{4\pi e^2_\alpha n_\alpha}{m_\alpha})^{1/2}$ is the plasma (Langmuir) frequency (so $\omega_p = \sqrt{\sigma\,k_D/2}$). Notice the interplay of collision and magnetic field scales through $\lambda \approx \frac{T_{gyro}}{T_{coll}} \equiv \frac{v_{thermal}}{v_{Alfven}}$.

4.3.2 Diffusion coefficients

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As we saw, the final formulae for the diffusion coefficients can be simplified by rescaling the integration variables $k_\perp$ (Fourier wave-number) and τ (time) therein to the non-dimensional variables $x \equiv \frac{\tilde{k}_\perp}{k_D} = (1+\frac{k^2_\perp}{k^2_D})^{1/2}$ and $\tau' = \Omega\tau$. The diffusion coefficients $D_*(t)$ are thus given by:

$$\left\{ \begin{array}{c} \left\{ \begin{array}{c} D_\perp \\ D_\angle \\ D^{(XX)}_\perp \end{array} \right\} \\ D_\| \end{array} \right\} = \frac{2\sqrt{2}\,n\,e^4}{m^2\,\sqrt{k_B T}}\,\lambda \int_0^t d\tau' \int_1^{x_{max}} dx\; e^{\lambda^2\,(1-x^2)\,\sin^2\frac{\tau'}{2}} \left(1-\frac{1}{x^2}\right)^{\{1,0\}} e^{-\tilde{v}^2_\|}$$

$$J_O(2\lambda\,\sqrt{x^2-1}\,\tilde{v}_\perp\,\sin\frac{\tau'}{2})\;\tilde{F}_{\{\perp,\|\}} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{2}\cos\tau' \\ (-s^\alpha)\,\frac{1}{2}\,\sin\tau' \\ (1+\frac{1}{2}\cos\tau') \end{array} \right\} \\ 1 \end{array} \right\} \quad (14)$$

where all quantities in the rhs except $\frac{2\sqrt{2}\, n\, e^4}{m^2 \sqrt{k_B T}} \equiv D_0$ are non-dimensional; J_0 is a Bessel function of the first kind; $\tilde{F} = \tilde{F}(\phi(x, \tau'), \tilde{v}_{\|})$ is given by:

$$\tilde{F}^{\alpha'}_{\{\perp,\|\}} = \pm\sqrt{\pi}\, \phi + \frac{\pi}{4} \sum_{s=+1,-1} \Big[(1 \mp 2\, \phi^2 \mp s 2\, \phi\, \tilde{v}_{\|})\, e^{(\phi + s\, \tilde{v}_{\|})^2}\, Erfc(\phi + s\, \tilde{v}_{\|}) \Big] \tag{15}$$

where

$$\phi = \frac{1}{2} \lambda\, \tau'\, x\,, \qquad \lambda = \sqrt{2}\, \frac{\omega_p}{\Omega}\,, \qquad \tilde{v}_* = (\frac{m v_*^2}{2 k_B T})^{1/2}\,, \qquad * \in \{\perp, \|\}$$

Remember that $k_D = (\frac{4\pi e_\alpha^2 n_\alpha}{k_B T_\alpha})^{1/2}$ is the Debye wave-number and $\omega_{p,\alpha} = (\frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha})^{1/2}$ is the plasma (Langmuir) frequency. Notice the interplay of collision and gyration scales through $\lambda \approx \frac{T_{gyro}}{T_{coll}} \equiv \frac{v_{thermal}}{v_{Alfven}}$.

Therefore, for a given set of parameter values one only has to determine the values of ω_p, Ω and then λ; the above formulae for $C(\tau)$ can then be evaluated as functions of τ (or, rather, $\Omega\tau$), by carrying out the integration in x numerically; by integrating in τ, one can then study the behaviour of the diffusion coefficients $D_{\perp,\angle,\|,\dots}(t)$ (defined as a definite integral in τ, from 0 to t; cf.(4)) with respect for velocity components $v_\perp, v_\|$ and the magnitude of the magnetic field (through the cyclotron frequency Ω). In the following we shall limit ourselves to examining out the dependence of certain coefficients on the magnitude of the magnetic field. A detailed numerical study will be reported elsewhere [8].

5 A numerical parametric study

We have chosen a set of typical values, i.e. a temperature of $T = 10\, KeV$ and a particle density of $n = 10^{14}\, cm^{-3} = 10^{20}\, m^{-3}$, implying a plasma frequency $\omega_{p,e} = 5.64 \cdot 10^{11}\, s^{-1}$ and a (gyro-)frequency of: $\Omega_e = 1.76\, 10^{11} \times B \quad s^{-1}$ (B expressed in Tesla).

In figure 1, we have represented all coefficients against time t (measured in cyclotron periods), for $B = 1T$. The diffusion coefficients start from zero and soon evolve towards a final asymptotic value which remains practically constant after a few gyration periods. Notice the short "jumps" every cyclotron period, in fact a consequence of the thin 'spikes' in the correlations (cf. fig 1b). Notice the short peaks appearing every gyration period, actually smoothed out very fast as time goes by. Eventually, particle interactions seem to be completely decorrelated after a few gyration periods.

The velocity dependence of the coefficients qualitatively reproduces the unmagnetized result [7]: see figure 2; the diffusion coefficients take lower values for faster particles.

In figure 3a we have depicted $D_\perp$ versus λ. Above $\lambda \approx 1$ (i.e. for $\rho_L \cong r_D$ or higher) the field slightly enhances relaxation [6]: the higher its magnitude B, the higher the value of $D_\perp(\tau)$. We can see that the asymptotic value of the diffusion coefficient $D(t)$ (as $t \to \infty$) depends on the magnetic field: in fact, the higher the field, the higher the final value $D_\perp(\infty)$. Remember that the diffusion coefficients $D_{\perp,\|}(t)$ are related to the *inverse* of the time needed for relaxation towards equilibrium [9]. Physically speaking, this fact reflects particle confinement by the magnetic field, since particles 'stick' to their helicoidal trajectory around the magnetic field lines and thus 'feel' each other for longer periods of time.

The friction vector $\mathcal{F}_\perp \sim \partial D_\perp / \partial v_\perp$ behaves in a similar way (fig. 3b).

However, their $\|$ – counterparts (fig. 3c, d) are practically time- (and field-) independent.

We therefore see that the magnetic field slightly *favours* thermalization (i.e. relaxation of the distribution function towards a maxwellian state). Once more, this seems to agree with physical intuition (the more 'confined' the particles, the more they influence each other and the more efficient collisions are towards relaxation). This fact seems to be in agreement with arguments appearing in [11], yet seems to contradict the standard description used in the past, where the influence of the magnetic field on the collision term is either under-estimated [10] or neglected [6] when discussing the physical - transport - properties of plasma.

6 Conclusions

In conclusion, we have reported new exact formulae for the diffusion coefficients in magnetized plasma. These formulae suggest an explicit dependence on both particle velocity and physical parameters such as plasma temperature, density, and - the point we wanted to focus upon - the magnitude of the magnetic field. A more extended study will be reported soon [8].

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Acknowledgements

The author would like to acknowledge many discussions with Prof. A. Grecos, who actually suggested this investigation, as well as Drs. B. Weyssow and D. Carati. This work has been supported by a fellowship of the *David & Alice Van Buuren Foundation.* It has also been partially supported by a contract of the EURATOM - Belgian State association. The content of the publication is the sole responsibility of the author and does not necessarily represent the views of the Commission or its services.

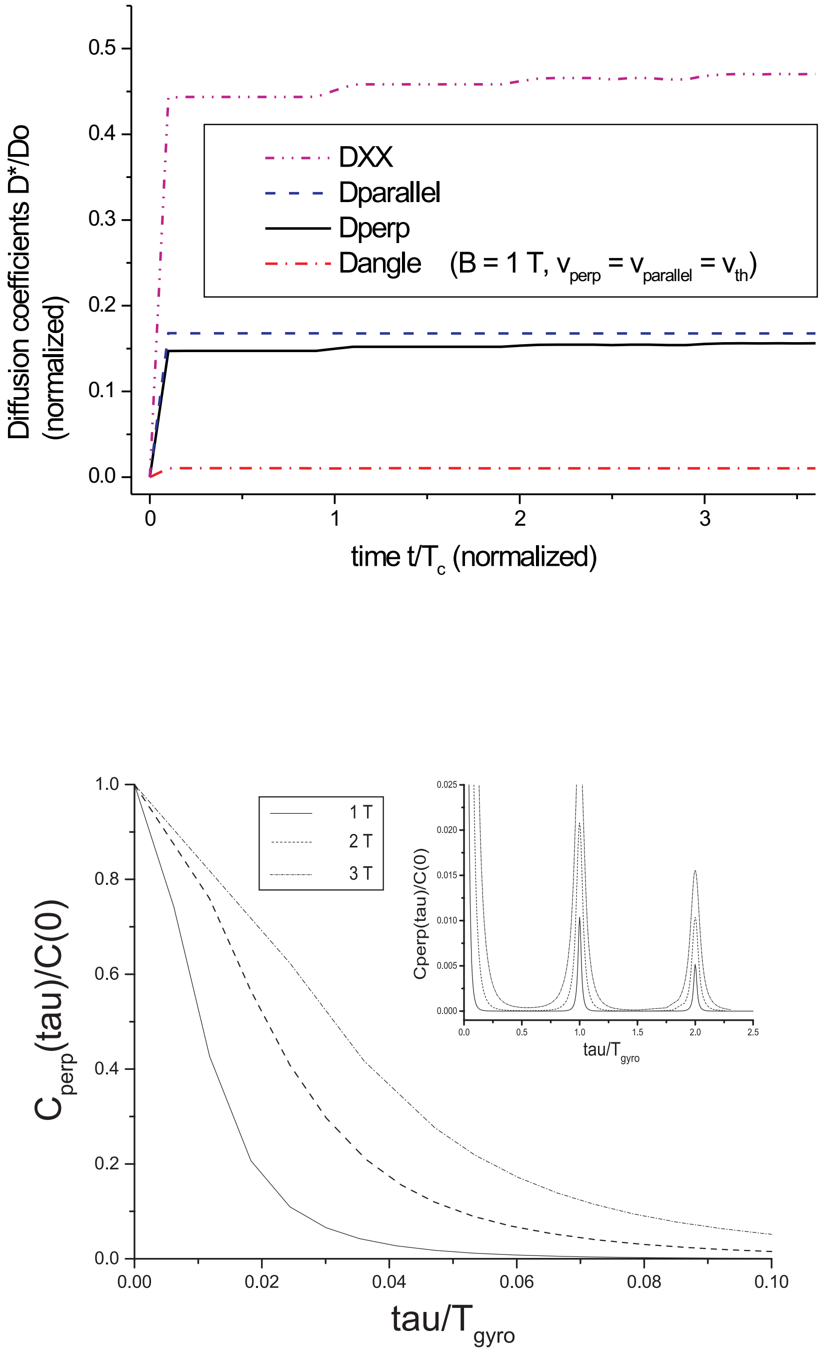


Figure 1: (a) Diffusion coefficients plotted against time t for $B = 1T$. The small 'kinks' at every gyration reflect the form of $C_\perp$ (cf. fig. 1b).

(b) The perpendicular force correlation function $C_\perp(\tau; v_\perp, v_\|, B)$ (normalized over $C_\perp(\tau = 0)$) as a function of time τ (scaled over a cyclotron period T_c). In ascending order, the magnitude of the magnetic field is set to $B = 1, 2, 3 \quad T$ respectively. Both velocity components are taken equal to $v_{th} = (T/m)^{1/2}$. $C_\perp$ can be seen to decrease very fast in time, still bearing a 'tail' of gradually smoothed out peaks every gyration period (actually a signature of the magnetic field; see embedded figure 1b).

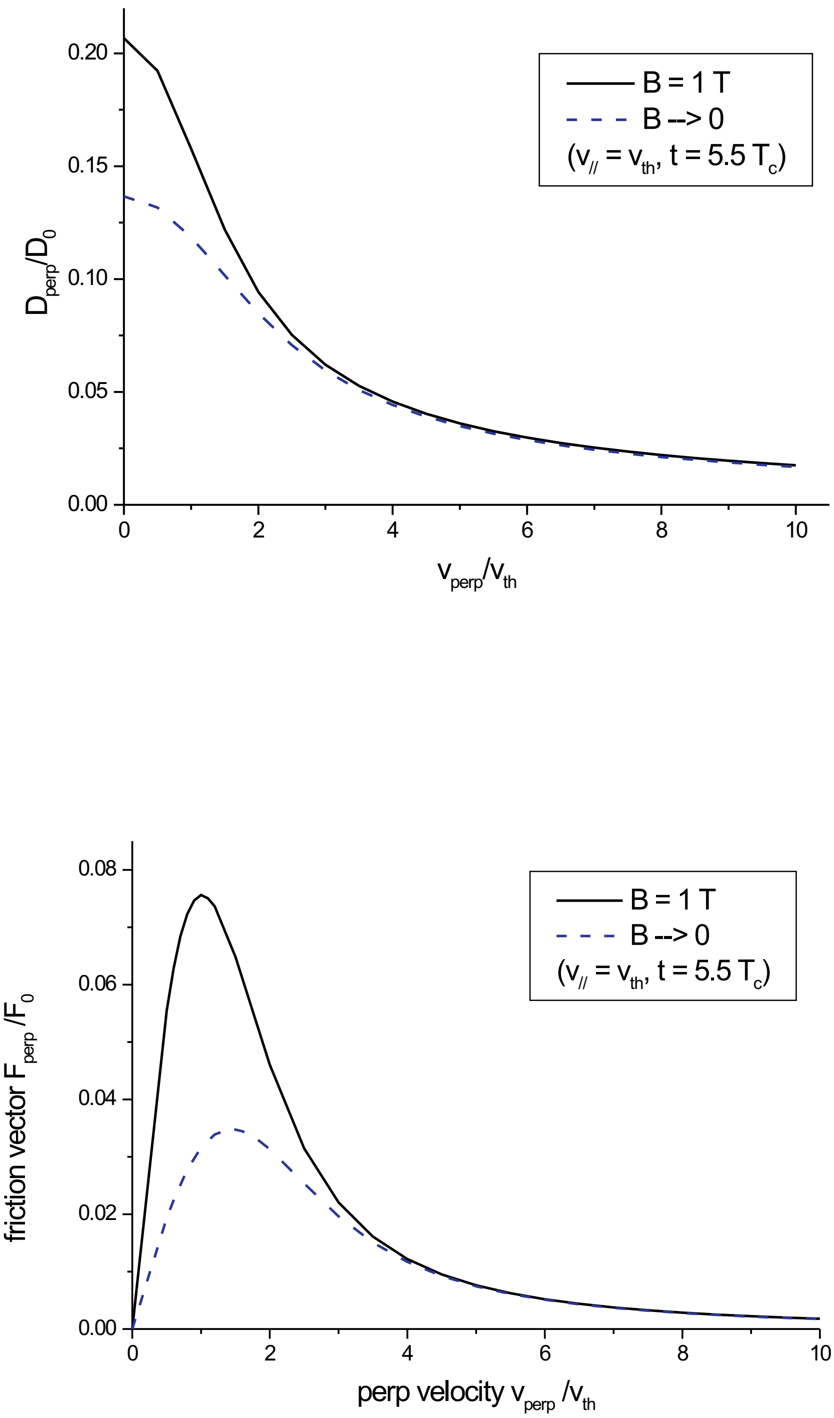
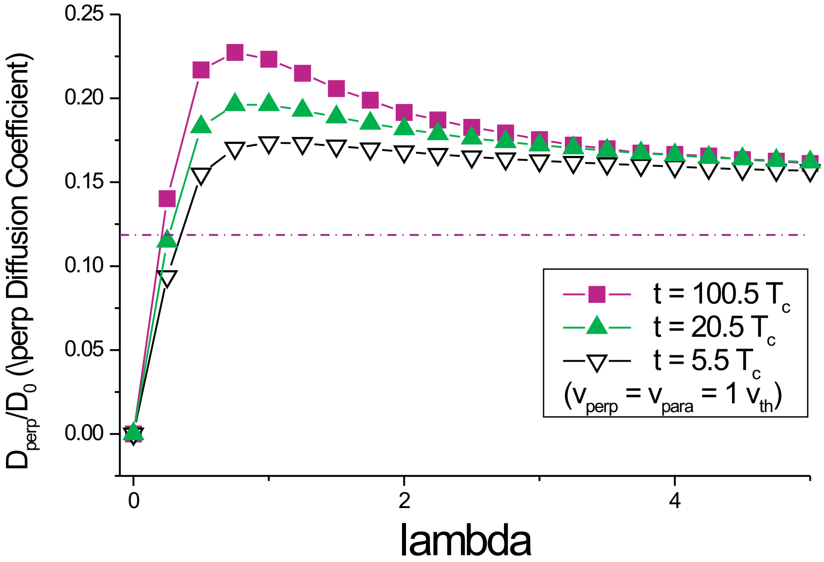


Figure 2: The diffusion coefficient $D_\perp$ and the corresponding friction vector $\mathcal{F}_\perp$, plotted against velocity $v_\perp$, for $B = 1T$ (solid line) and $B = 0$ (dashed line).



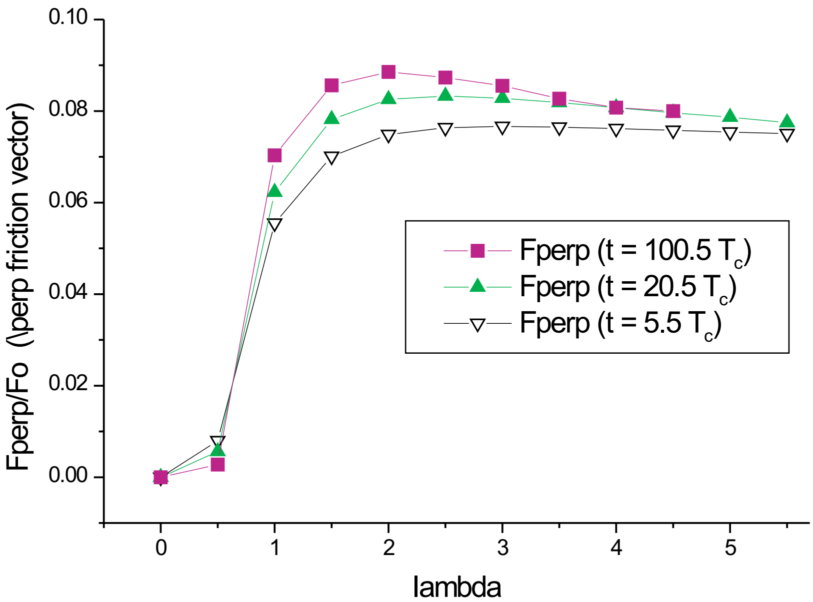
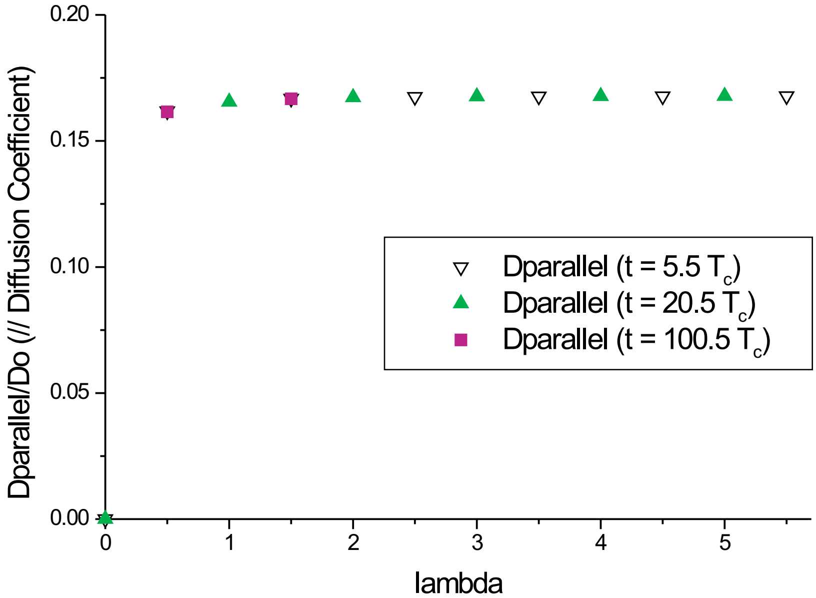


Figure 3: The perpendicular diffusion coefficient $D_\perp$ and the friction vector (norm) $\mathcal{F}_\perp$ (top), and their $\|$ −counterparts (bottom) plotted against λ ($\sim 1/B$), at different instants of t. $D_\perp$ slightly increases in time, yet only around $\lambda \approx 1$ (i.e. $\rho_L \approx r_D$), above which it practically remains constant. The field-dependence is smoothed out, as $D_\perp$ approaches the asymptotic value for $\Omega \to 0$ (dash-dot line).



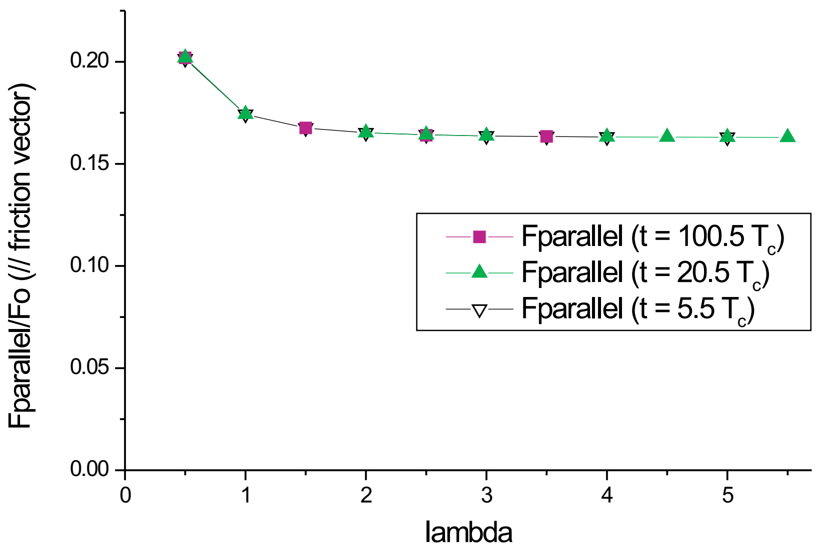


Figure 4: $D_{||}$ comes out to be independent of the field and so does $\mathcal{F}_{||}$.