Stability of linear oscillations in dusty plasma crystals in the presence of ion flow

Padma Kant Shukla [1] and Ioannis Kourakis [2]

Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr–Universität Bochum, D–44780 Bochum, Germany

email: [1] ps@tp4.rub.de , [1] ioannis@tp4.rub.de

Introduction

We present a comprehensive investigation of inter-grain interaction in the presence of ion flow in dusty plasmas (DP) [1]. Specifically, we exploit the results of Ref. [2] to provide detailed information regarding the role of ion flow on short range Debye-Hückel and long range attractive potentials around dust grains that are levitated at the same height above a negative electrode. It is shown that both short and long range potentials are modified by the ion flow. We then carry out a study of the stability of dust oscillations in a horizontal arrangement of dust grains. It is found that dust lattice modes may become unstable due to ion flow towards the electrode.

Figure 1. Horizontal dust layer (heuristic picture).

Longitudinal dust-lattice (LDL) modes: prerequisites

Let us consider dust lattice oscillations in a plasma whose constituents are electrons, streaming ions, and an ensemble of negatively charged dust grains. The *longitudinal* motion of charged dust grains (mass M and charge Q, both assumed constant for simplicity) in a DP crystal (lattice constant r_0) obeys [1, 3, 4, 5, 6]:

$$M\left(\frac{d^2x_n}{dt^2}+\nu\frac{dx_n}{dt}\right) = -\sum_n \frac{U_{nm}(r_{nm})}{\partial r} \equiv Q\,E(x_n), \qquad (1)$$

- $U_{nm}(r_{nm}) \equiv Q\,\phi(x)$ is a binary interaction potential function related to the electrostatic potential $\phi(x)$ around the $m-$th grain;
- $E(x) = -\partial\phi(x)/\partial x$ is the electric field;
- $r_{nm} = x_n - x_m$: distance between $n-$th and $m-$th grains.
- ν is the damping rate due to dust-neutral collisions.

A 1d DP layer is considered here, yet generalization to a 2d grid is straightforward. Retaining only first neighbor interactions [7] and considering small (linear) displacements $\delta x_n = x_n - x_n^{(0)}$:

$$\frac{d^2\delta x_n}{dt^2}+\nu\frac{\delta x_n}{dt} = \omega_{0,L}^2\,(\delta x_{n-1}+\delta x_{n+1} - 2\,\delta x_n) \qquad (2)$$

[1, 3, 4, 8]. The LDL oscillation 'eigenfrequency' $\omega_{0,L}$ is given by:

$$\omega_{0,L}^2 = \frac{Q}{M}\frac{\partial^2\phi(r)}{\partial r^2}\bigg|_{r=r_0}. \qquad (3)$$

$\phi(r)$ is *often* assumed to be the Debye-Hückel potential

$$\phi^{(D)}(r) = \frac{Q}{r}\,\exp(-k_D r), \qquad (4)$$

where $k_D = \lambda_D^{-1}$ is the inverse Debye radius [9], so $\omega_{0,L}$ is [1, 3]

$$\omega_{0,L}^{(D)^2} = \frac{2Q^2}{Mr_0^3}\left(1+\frac{r_0}{\lambda_D}+\frac{r_0^2}{2\lambda_D^2}\right)\exp(-k_D\,r_0)\,.$$

The associated linear dispersion relation reads:

$$\omega\,(\omega+i\,\nu) = 4\,\omega_{0,L}^2\,\sin^2\left(\frac{kr_0}{2}\right), \qquad (5)$$

which reduces to

$$\omega\,(\omega+i\,\nu) \approx \omega_{0,L}^2 r_0^2\,k^2 \equiv c_L^2 k^2 \qquad (6)$$

in the continuum (long-wavelength) limit, i.e. for $\lambda \gg r_0$. We see that *stability of the LDLW is ensured only if the RHS is positive*, i.e. if (and only if) $\omega_{0,L}$ is real (viz. $\omega_{0,L}^2 > 0$). Therefore, *stability depends on the interaction potential* $\phi(r)$ via Eq. (3).

A more sophisticated interaction potential

In full rigor, one should take into account the wake potential generated by ion flow towards the electrode [2, 10]. It has been recently shown from first physical principles [2] that the electrostatic interaction potential $\phi(r) = Q\,k_D\,W(r)$ around a charged dust grain in the vicinity of a conducting wall penetrated by in-flowing ions may strongly deviate from the simple Debye-Hückel picture. In specific, the (normalized) potential $W(r;z_0) = W(\rho;a)$ was found in Ref. [2] to be:

$$W(\rho) = 2\int_0^\infty dq_\perp\,q_\perp\,J_0(q_\perp\rho)\,g(k), \qquad (7)$$

where $g(k) = (k_D/4\pi)\,G_{\mathbf{k}_\perp}(z_0)$ is the (dimensionless) function:

$$g(k) = \frac{e^{-\kappa a}}{\kappa^2+q^2}\left[\kappa\,\sinh(\kappa a) + q\,\sin(qa)\right] \qquad (8)$$

and $J_0(x)$ is a Bessel function of the first kind; also,
- $r \equiv \rho\,\lambda_D$: inter – grain distance;
- $z_0 \equiv a\,\lambda_D$: levitation height (above electrode);
- $u \equiv M\omega_{p,i}\lambda_D$: velocity of ion flow ($M$: *Mach number*);
- $q_\perp = k\lambda_D$: wavenumber *(normalized)*.

Finally, q and κ, related to the poles of the dielectric function, are:

$$q^2 = \frac{1}{2}\left[\sqrt{(q_\perp^2+1-M^{-2})^2+4\,M^{-2}\,q_\perp^2} - q_\perp^2 + M^{-2} - 1\right],$$
$$\kappa^2 = \frac{1}{2}\left[\sqrt{(q_\perp^2+1-M^{-2})^2+4\,M^{-2}\,q_\perp^2} + q_\perp^2 - M^{-2} + 1\right] \qquad (9)$$

The 1st term in *RHS* (8) is related to the *Debye-Hückel potential* (distorted by the ion flow), while the 2nd term accounts for the *wake potential* generated downstream by grains. For vanishing ion velocity ($u_i \to 0$) and infinite electrode-to-grain distance ($a \to \infty$), the Fourier transform of the Debye potential is recovered.

Numerical study of the interaction potential

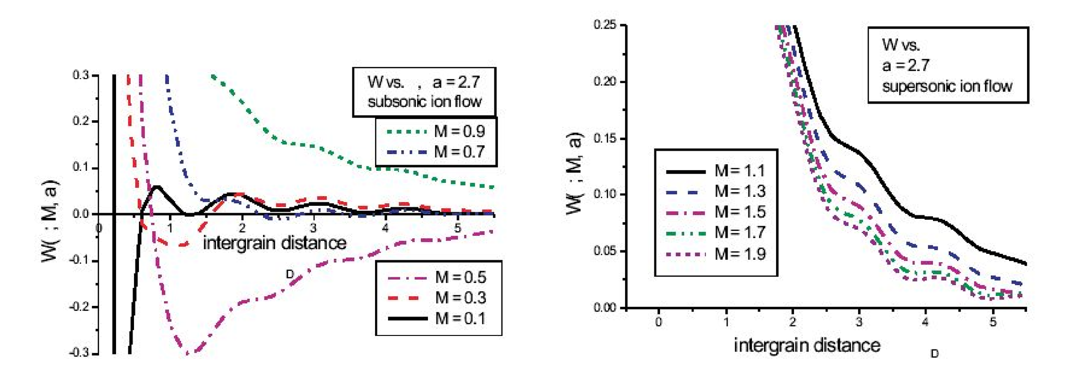

Figure 2. The normalized interaction potential W is depicted against the (normalized) intergrain distance $k_D r$, for $a = 2.7$. (a) Subsonic ion flow ($M < 1$). (b) Supersonic ion flow ($M > 1$).

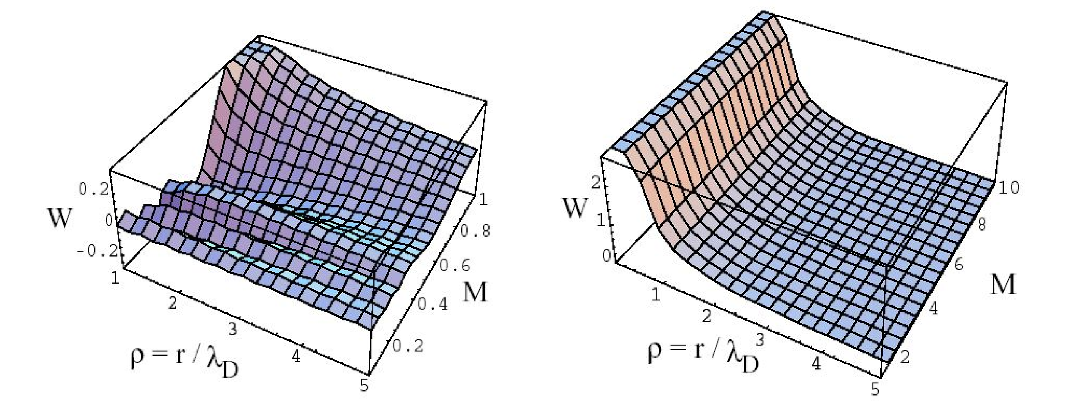

Figure 3. The interaction potential W is depicted against the (normalized) intergrain distance $\rho = k_D\,r$ and the Mach number M, for $a = 2.7$. (a) Subsonic ion flow ($0 < M < 1$). (b) Supersonic ion flow ($1 < M < 10$). We see that the character (attractive/repulsive) of the potential w at a given distance $\rho = k_D\,r$ may depend on the subsonic ion flow.

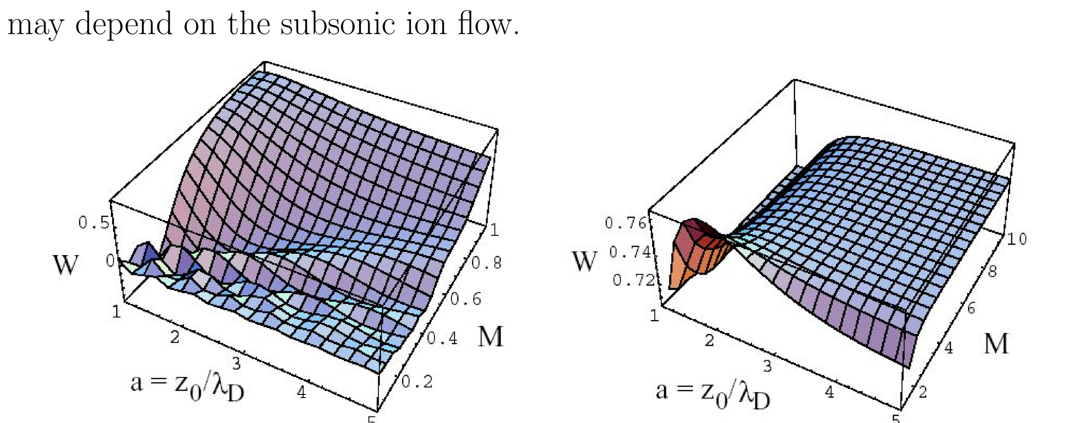

Figure 4. The interaction potential W is depicted against the (normalized) levitation height a ($1 < a < 5$ here) and the Mach number M, for an intergrain distance value of $\rho = k_D\,r = 1$. (a) Subsonic ion flow ($0 < M < 1$). (b) Supersonic ion flow ($1 < M < 10$). The character (attractive/repulsive) of the potential w at a given height may depend on the (subsonic) ion velocity, and vice versa.

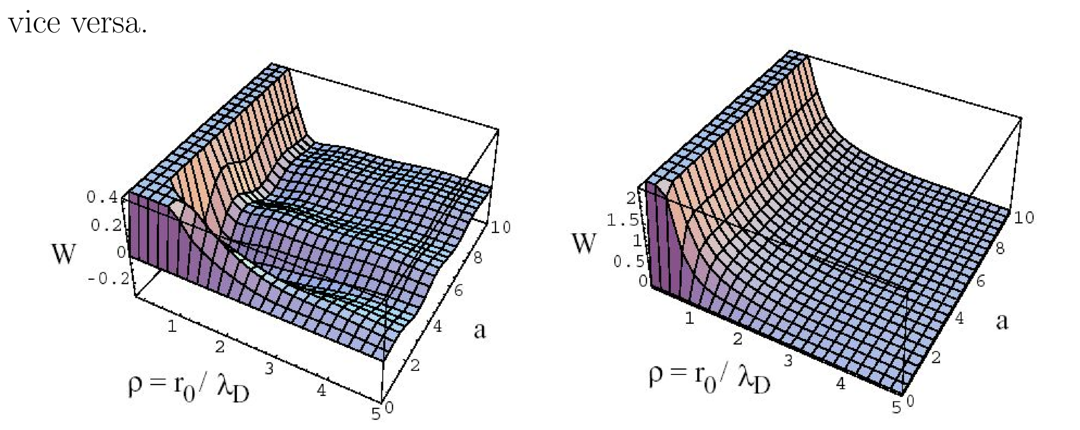

Figure 5. The potential W is represented against the (normalized) levitation height a ($0 < a < 10$) and the intergrain distance $\rho = k_D\,r$ ($0 < \rho < 5$) for a fixed Mach number M, equal to (a) $M = 0.5$, (b) $M = 1.5$. Notice that a given distance may correspond to stable or unstable oscillations, depending on the levitation height.

Dispersion relation for a 2d grid: stability analysis

Considering a simple two-dimensional dust gas (grid), it was shown [2] that the oscillation frequency ω is:

$$\omega^2 \approx \frac{Q^2}{M\lambda_D^3}g(k)\,k^2\,. \qquad (10)$$

Therefore, the sign of ω^2 may be investigated by a numerical analysis of the (sign of the) *RHS* (10). One finds that dust grain oscillations subject to a *subsonic* ion flow may become unstable for certain (combinations of) M and a.

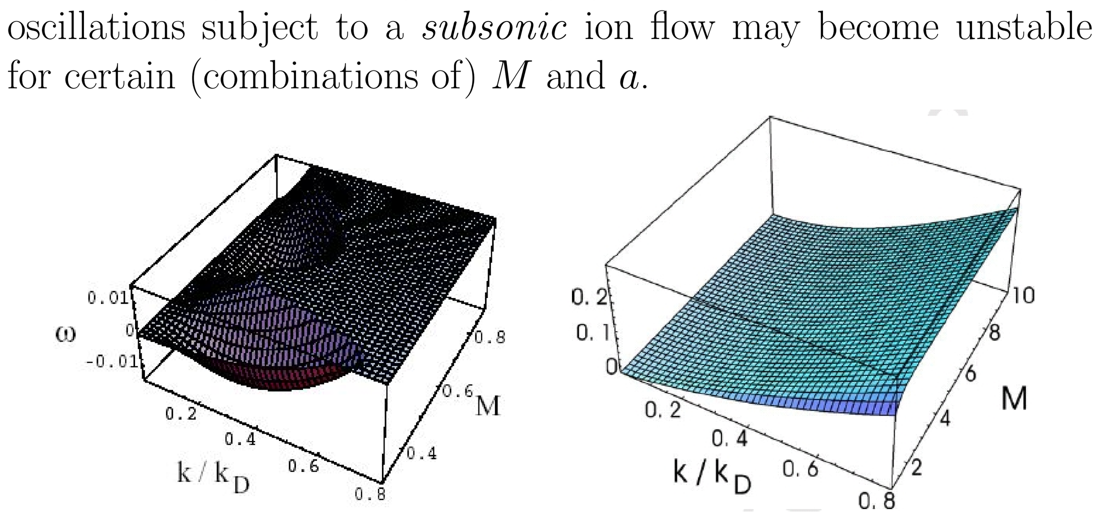

Figure 6. (a) The dust grid dispersion curve, i.e. the oscillation frequency squared ω^2, as given by (8) and (10), is represented against the wavenumber k/k_D and the Mach number M ($0 < M < 1$: subsonic case). (b) As in (a), for the supersonic case ($M > 1$). The levitation height was chosen equal to $a = 2.7$. Two distinct unstable regions ($\omega^2 < 0$) appear in (a), as the ion velocity increases. The supersonic case is globally stable.

LDL mode in a 1d crystal: stability analysis

In a similar manner, one may derive the characteristic frequency of LDL oscillations $\omega_{0,L}^2$ in Eqs. (2), (5), from (3).

One finds that $\omega_{0,L}^2$ is related to the potential (Fourier) transform $G_{\mathbf{k}_\perp} = (4\pi/k_D)g(k)$ via the expression

$$\omega_{0,L}^2 = -\frac{Q^2}{M}\frac{1}{(2\pi)^\delta}\int d^\delta\mathbf{k}_\perp\,k^2\,G_{\mathbf{k}_\perp}(z_0)\,\exp(i\mathbf{k}_\perp\mathbf{r_0})\,, \qquad (11)$$

[11] i.e.

$$\omega_{0,L}^2 = -\frac{2Q^2k_D^3}{M}\int_{-\infty}^\infty dq_\perp\,q_\perp^2\,g(q_\perp)\,\cos(q_\perp\rho_0) \qquad (12)$$

for a one-dimensional lattice ($\delta = 1$). See that $\omega_{0,L} = \omega_{0,L}(r_0,z_0) \equiv \omega_{0,L}(\rho_0,a)$ i.e. depends on the levitation height $a = k_D z_0$ and the lattice constant $\rho_0 = r_0/\lambda_D$ (normalized).

We may now evaluate the *RHS*(12) numerically.

As intuitively expected, the stability of the chain for a given value of ρ depends on the velocity u_i of the ion flow, so that *an increase in* u_i *may result in unstable oscillations and melting* of the chain. Inversely, stability for given u_i may be ensured for a certain value of ρ and excluded for another; see Fig. 7.

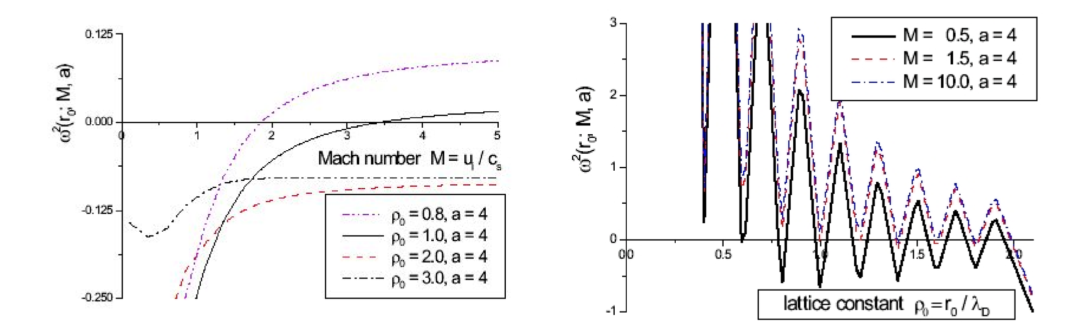

Figure 7. (a) The characteristic (square) lattice frequency $\omega_{0,L}^2$, given by (12), is depicted against the Mach number M for fixed (levitation height) a and different values of the (normalized) lattice constant $\rho_0 = r_0/\lambda_D$. (b) $\omega_{0,L}^2$ vs. lattice constant $\rho_0 = r_0/\lambda_D$ for fixed (levitation height) a and different values of the Mach number M.

Stable configurations correspond to lattice constants of the order of a Debye length λ_D, in agreement with known experimental results. Note the dependence on the levitation height a (for given u_i, ρ) and the lattice constant ρ. In fact, different values of ρ -yet possibly *close* ones- may present a completely different stability profile; cf. Figs. 8a, b.

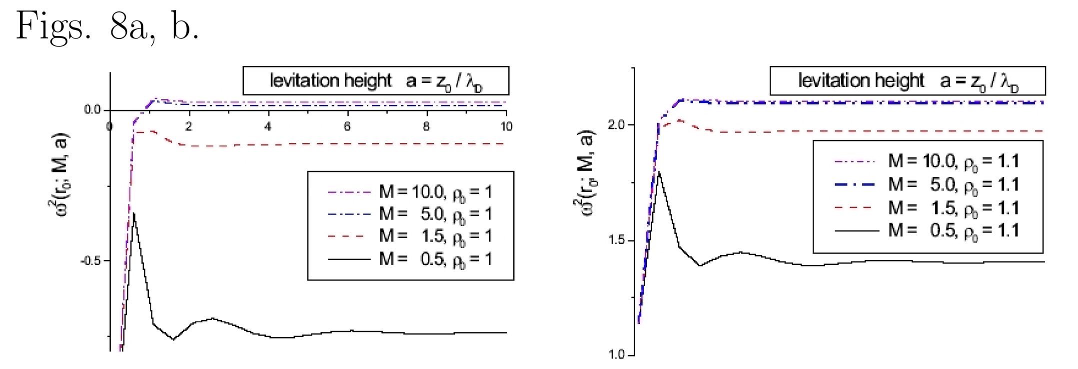

Figure 8. $\omega_{0,L}^2$ is depicted against the (normalized) levitation height) a for different values of the the Mach number M and lattice constant: (a) $\rho_0 = 1.0$; (b) $\rho_0 = 1.1$. We see that the latter is always stable, while the former is only stable at specific height values a.

In conclusion, both Debye-Hückel repulsive and ion wake attractive potentials are significantly modified by the ion flow.

Melting may occur
- by modifying the ion flow and/or
- when strong intergrain distance variations occur (possibly related to wide amplitude longitudinal oscillations due to nonlinear effects), or
- if the equilibrium position (related to the linear term in the sheath field) is modified.

These remarks remain valid in both sub- ($M < 1$) and supersonic ($M > 1$) ion flow cases (in contrast with the Ignatov grid model - see above - [2], where stability was prescribed for $M > 1$ only).

These results may help to elucidate the melting of dust crystals in the sheath region, observed in experiments.

References

[1] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics Publishing Ltd., Bristol, 2002).

[2] A. M. Ignatov, Plasma Physics Reports **29**, 296 (2003).

[3] F. Melandsø, Phys. Plasmas **3**, 3890 (1996).

[4] B. Farokhi *et al.*, Phys. Lett. A **264**, 318 (1999); *idem*, Phys. Plasmas **7**, 814 (2000).

[5] O. Ishihara *et al*, Physica Scripta T **75**, 79 (1998).

[6] D. Tskhakaya and P. K. Shukla, Phys. Lett. A **286**, 277 (2001).

[7] This approximation is valid as long as $r_d \ll a \ll \lambda_D$, where r_d is the dust radius, a is the intergrain separation distance, and λ_D is the effective Debye radius of unmagnetized dusty plasma. On the other hand, when $a \sim \lambda_D$, the approximation of nearest neighbor dust grain interaction has to be relaxed in order to obtain the modified spectrum [4, 6] of longitudinal dust lattice waves in an infinite chain of charged dust.

[8] I.Kourakis and P. K. Shukla, *"Study of the intergrain interaction potential and associated instability of dust-lattice plasma oscillations in the presence of ion flow"*, Physics Letters **A 316**, in press (2003).

[9] In a dusty plasma, a dust grain is typically shielded by ions when electrons and ions are in thermodynamical equilibrium. However, in a plasma with flowing ions, the Debye shielding is due to electrons, and hence $\lambda_D = \lambda_{De}$.

[10] M. Nambu *et al.*, Physics Letters A **203**, 40 (1995).

[11] $\delta = 1/2$ for a 1d / 2d lattice.