Nonlinear Whistlerons

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Introduction

We show that nonlinear interactions between whistlers and finite amplitude density perturbations are governed by a a nonlinear Schrödinger equation for the modulated whistlers (whistlerons), and a set of equations for arbitrary large amplitude density perturbations in the presence of the whistler ponderomotive force. The governing equations are solved numerically to show the existence of large scale density perturbations that are self-consistently created by localized modulated whistler wavepackets. Our numerical results are found to be in good agreement with experimental results.

Theory

The dynamics of modulated whistler wavepacket in the presence of electron density perturbations associated with low-frequency ion-acoustic fluctuations and nonlinear frequency-shift caused by the magnetic field-aligned free streaming of electrons (with the flow speed v_{ez}) is governed by a nonlinear Schrödinger equation

$$i(\partial_t + v_g\partial_z)E + (v_g'/2)\partial_{zz}^2 E + (\omega_0 - \omega)E = 0, \qquad (1)$$

where $\omega = k_0^2c^2\omega_{ce}/(\omega_{pe}^2 + k_0^2c^2) + k_0 v_{ez}$, and $\omega_{pe}^2 = \omega_{pe,0}^2 n_e/n_0$ is the local plasma frequency including the electron density n_e of the plasma slow motion. The group velocity and the group dispersion of whistlers are $v_g = \partial\omega_0/\partial k_0 = 2(1-\omega_0/\omega_{ce})\omega_0/k_0$ and $v_g' = \partial^2\omega_0/\partial k_0^2 = 2(1-\omega_0/\omega_{ce})(1-4\omega_0/\omega_{ce})\omega_0/k_0^2$, respectively.

The equations for the ion motion involved in the low-frequency (in comparison with the whistler wave frequency) ion-acoustic perturbations are

$$\partial_t n_i + \partial_z(n_i v_{iz}) = 0 \qquad (2)$$

and

$$\partial_t v_{iz} + (1/2)\partial_z v_{iz}^2 = -(e/m_i)\partial_z\phi - (\partial_z p_i)/m_i n_i, \qquad (3)$$

where the ion pressure is given by $p_i = p_{i,0}(n_i/n_0)^3$.

The electron dynamics in the plasma slow motion is governed by the continuity and momentum equations, viz.

$$\partial_t n_e + \partial_z(n_e v_{ez}) = 0 \qquad (4)$$

and

$$0 = (e/T_e)\partial_z\phi - \partial_z \ln(n_e/n_0)) + F, \qquad (5)$$

where T_e is the electron temperature, ϕ is the ambipolar potential, and the low-frequency ponderomotive force of electron whistlers is

$$F = [\omega_{pe,0}^2/\omega_0(\omega_{ce}-\omega_0)](\partial_z + \frac{2}{v_g}\partial_t)|E|^2/4\pi n_0 T_e. \qquad (6)$$

The system of equations is closed by the quasi–neutrality $n_i = n_e \equiv n$, giving $v_{iz} = v_{ez} \equiv v_z$ by the continuity equations, so that $\partial_t n + \partial_z(nv_z) = 0$. Eliminating $\partial_z\phi$ from the governing equations for low- frequency density perturbations, we have

$$\partial_t v_z + (1/2)\partial_z v_z^2 = -(T_e/m_i)[\partial_z \ln(n/n_0) - F] - (\partial_z p_i)/m_i n \quad (7)$$

together with

$$\partial_t n + \partial_z(nv_z) = 0. \qquad (8)$$

The nonlinear Schrödinger equation for the whistler electric field together with the low-frequency equations form a closed set for our purposes.

The normalized system of equations are [1]

$$\partial_\tau N = -\partial_\xi(Nu), \qquad (9)$$

$$\partial_\tau\left(u - \frac{2\alpha}{V_g}|\mathcal{E}|^2\right) = \partial_\xi\left[-\frac{u^2}{2} - \frac{\ln N + 1.5\eta N^2}{1+3\eta} + \alpha|\mathcal{E}|^2\right], \qquad (10)$$

and

$$\partial_\tau\mathcal{E} = -V_g\partial_\xi\mathcal{E} + i\left[P\partial_{\xi\xi}^2\mathcal{E} + \left(\frac{1}{1+\kappa^2} - \frac{1}{N+\kappa^2} - \frac{2}{(1+\kappa^2)^2}\frac{u}{V_g}\right)\Omega_c\kappa^2\mathcal{E}\right], \qquad (11)$$

where the constants are $\alpha = (1+\kappa^2)^2\omega_{pe,0}^2/\omega_{ce}^2\kappa^2$ and $P = (1+\kappa^2)(1-3\kappa^2)V_g^2/4\kappa^2\Omega_c$ where $\kappa = ck_0/\omega_{pe,0}$. The sign of the coefficient P, multiplying the dispersive term in Eq. (11), depends on κ: When $\kappa < 1/\sqrt{3}$, P is positive and for $\kappa < 1/\sqrt{3}$ we see that P is negative.

Solitary waves in the small-amplitude limit

In the small-amplitude limit, viz. $N = 1 + N_1$, $u = u_1$, where N_1, $u_1 \ll 1$, Eqs. (9)–(11) yield

$$\partial_\tau N_1 = -\partial_\xi u_1, \qquad (12)$$

$$\partial_\tau\left(u_1 - \frac{2\alpha}{V_g}|\mathcal{E}|^2\right) = \partial_\xi\left(-N_1 + \alpha|\mathcal{E}|^2\right), \qquad (13)$$

and

$$\partial_\tau\mathcal{E} = -V_g\partial_\xi\mathcal{E} + i\left[P\partial_{\xi\xi}^2\mathcal{E} + \left(N_1 - \frac{u}{V_g}\right)\frac{\Omega_c\kappa^2}{(1+\kappa^2)^2}\mathcal{E}\right]. \qquad (14)$$

Here, we look for whistler envelope solitary waves moving with the group speed V_g, so that N_1 and u_1 depends only on $\chi = \xi - V_g\tau$, while the electric field envelope is assumed to be of the form $\mathcal{E} = W(\chi)\exp(i\Omega\tau - ik\xi)$, where W is a real-valued function of one argument. Using the boundary conditions $N_1 = 0$, $u_1 = 0$ and $W = 0$ at $|\xi| = \infty$, we have $k = 0$, $N_1 = -W^2\alpha/(1-V_g^2)$ and $u_1 = V_g N_1$. We here note that subsonic ($V_g < 1$) solitary waves are characterized by a density cavity while supersonic ($V_g > 1$) envelope solitary waves are characterized by a density hump. The system of equations (12)–(14) is then reduced to the cubic Schrödinger equation

$$P\partial_{\chi\chi}^2 W + QW^3 - \Omega W = 0, \qquad (15)$$

where $Q = \alpha\Omega_c\kappa^2/(1+\kappa^2)(1-V_g^2)$. Localized solutions of Eq. (7) only exist if the product PQ is positive. We note that $P > 0$ ($P < 0$) when the whistler frequency $\omega_0 < \omega_{ce}/4$ ($\omega_0 > \omega_{ce}/4$), and that $Q < 0$ ($Q > 0$) when $|V_g| < 1$ ($|V_g| > 1$), so in the frequency band where $\omega_0 < \omega_{ce}/4$, only subsonic solitary waves, characterized by a localized density cavity can exist, while in the frequency band $\omega_0 > \omega_{ce}/4$, only supersonic solitary waves characterized by a localized density hump exist. Equation (7) has exact solitary wave solutions of the form

$$W = (2\Omega/Q)^{1/2}\mathrm{sech}[(\Omega/P)^{1/2}(\xi - V_g\tau - \xi_0)], \qquad (16)$$

where V_g and Ω and the displacement ξ_0 are the three free parameters for a given set of physical plasma parameters. Finally, we recall that the dispersion relation for the electron whistlers used here is valid if $\omega_0 > \sqrt{\omega_{ce}\omega_{ci}}$. For subsonic whistlers having the group speed $v_g = C_s V_g$ (where $V_g < 1$), where $v_g \approx 2\omega_0/k_0$ and $\omega_0 \approx k_0^2c^2\omega_{ce}/\omega_{pe,0}^2$, we have $ck_0/\omega_{pe,0} = (C_s/c)(\omega_{pe,0}/\omega_{ce})V_g/2 > (m_e/m_i)^{1/4}$.

Numerical results

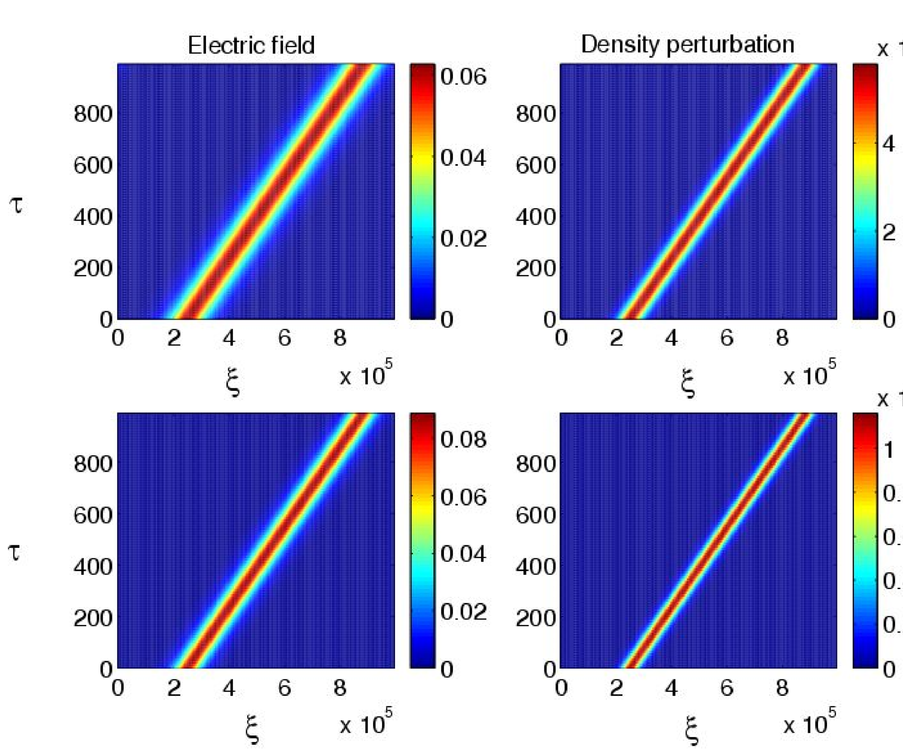

Figure 1: A supersonic whistler wave

We have investigated the properties of modulated whistlers wave packets by solving numerically Eqs. (9)–(11). We have here chosen parameters from a recent experiment, where the formation of localized whistler envelopes have been observed [3]. In the experiment, one has $n_0 = 1.2\times10^{12}\,\mathrm{cm}^{-3}$ and $B_0 = 100$ G, so that $\omega_{pe,0} = 6.7\times10^{10}\,\mathrm{s}^{-1}$ and $\omega_{ce} = 1.76\times10^{9}\,\mathrm{s}^{-1}$, respectively. Hence, $\omega_{ce}/\omega_{pe,0} = 0.026$. The frequency of the whistler wave is $\omega_0 = 2\pi\times160\times10^6\,\mathrm{s}^{-1} = 1.0\times10^9\,s^{-1}$, so that $\omega_0/\omega_{ce} \approx 0.57 > 0.25$. Thus, the whistlers have negative group dispersion. From the dispersion relation of whistlers, we have $\kappa \approx 1.15$, which gives $k_0 \approx 257\,\mathrm{m}^{-1}$. The latter corresponds to whistlers with a wavelength of 2.4 cm. Furthermore, the whistler group velocity is $v_g = 3.36\times10^6$ m/s. The argon ion-electron plasma ($m_i/m_e = 73400$) had the temperatures of $T_e = 10$ eV and $T_i = 0.5$ eV, giving the sound speed $5.25\times10^3\,$m/s, and the normalized group velocity $V_g = v_g/C_s = 640$. In Fig. 1, we have illustrated the existence of localized whistler envelope solitons, in which the electric field envelope (left panels) is accompanied with a density hump (right panels). We notice that the density hump is relatively small, due to the large group velocity of the whistler waves.

In Fig. 2, we have presented the development of a large-amplitude whistler pulse, which was launched in a plasma perturbed by ion-acoustic waves, with a density modulation of one percent. This simulates, to some extent, the experiment by Kostrov *et al.*, where the density and magnetic field were perturbed by a low-frequency conical refraction wave, giving rise to a modulation of the electron whistlers.

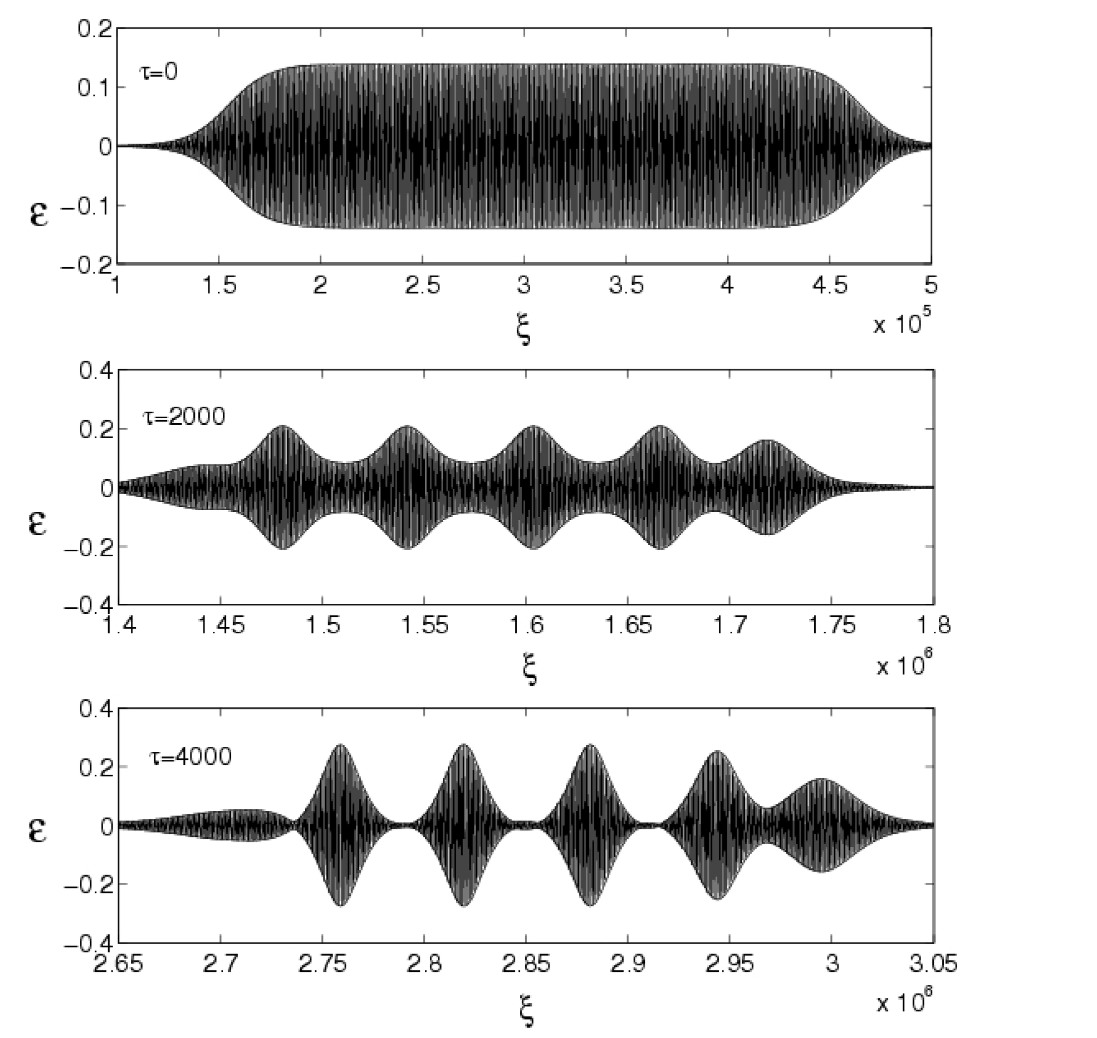

Figure 2: Formation of solitary whistler waves

Here, as in the experiment, we observe that a modulated electron whistler pulse (middle panel of Fig. 2) develops into isolated solitary electron whistlers (lower panel). We note that the wavelength of the whistlers is ≈ 2.5 cm, while the typical width of a solitary pulse is $\Delta\xi \approx 3\times10^4$ in the scaled length units, corresponding to ≈ 64 cm, so that each solitary wave train contains 25 wavelengths of the high-frequency whistlers. In one experiment, illustrated in the lower panel of Fig 4 in Ref. [3], one finds that the width of the solitary whistler pulse in time is $0.2\,\mu$s, which with the group speed $v_g = 3.36\times10^6$ m/s gives the width ~ 60 cm in space of the solitary wave packets, in good agreement with our numerical results. From the relation $N_1 = -W^2\alpha/(1-V_g^2)$ valid for solitary whistlers in the small-amplitude limit, and with the amplitude of $W = |\mathcal{E}|$ approximately 0.3 seen in the lower panel of Fig. 2, we can estimate the relative amplitude of the density hump associated with the solitary waves to be of the order 10^{-3}, i.e. much smaller than the modulation $\sim 10^{-2}$ due to the ion-acoustic waves excited in the initial condition.

Next, we study the properties of subsonic whistler envelope solitary pulses which have the normalized group speed $V_g = 0.5$. Here, the restrictive condition $ck_0/\omega_{pe,0} = (C_s/c)(\omega_{pe,0}/\omega_{ce})V_g/2 > (m_e/m_i)^{1/4}$ requires somewhat higher values of the plasma temperature and $\omega_{pe,0}/\omega_{ce}$ for their existence. With $m_i/m_e = 30000$, we have $(m_e/m_e)^{1/4} \approx 0.1$. We take $\kappa = ck_0/\omega_{pe,0} = 0.2$, $C_s = 10^5$ m/s (corresponding to $T_e \sim 1400$ eV) $\eta = 0.1$, and $\omega_{pe,0}/\omega_{ce} = 2400$. Thus, $\Omega_c = 0.072$ and $\omega_0/\omega_{ce} \approx 0.039$. For these values of the parameters, there exist solitary whistler pulse solutions, which we have displayed in Fig. 3.

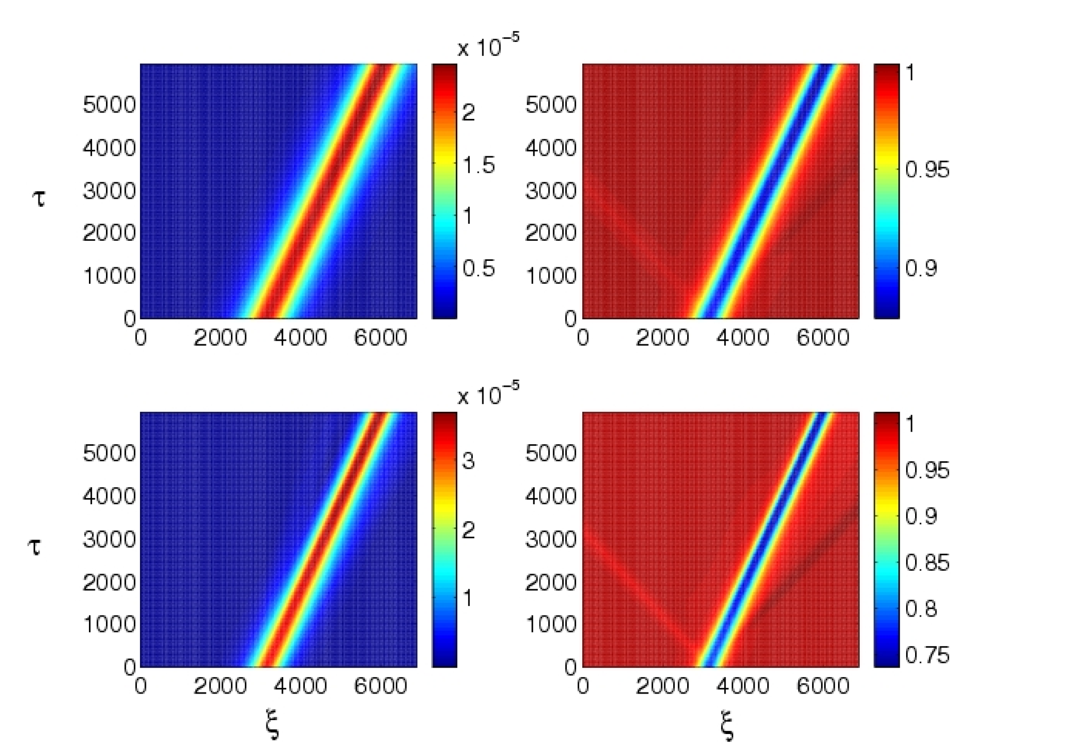

Figure 3: Subsonic whistler waves

Here, we have used the exact solution in the small-amplitude limit as an initial condition for the simulation of the full system of equations (1)–(3). The bell-shaped whistler electric field envelope is accompanied with a large-amplitude plasma density cavity.

References

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