MPIPKS – Max-Planck-Institut für Physik komplexer Systeme

Dresden 24.11.2004

Dusty Plasmas: a new paradigm in Nonlinear Science

Focus issue: Localized excitations in dust crystals

Ioannis Kourakis

R.U.B. Ruhr-Universität Bochum, Institut für Theoretische Physik IV

Fakultät für Physik und Astronomie, D–44780 Bochum, Germany

email: `ioannis@tp4.rub.de`

in collaboration with Padma Kant Shukla & Bengt Eliasson

`www.tp4.rub.de/∼ioannis/conf/2004-MPIPKS-oral.pdf`

Outline

1. *Dusty Plasma (DP)*: an overview of notions and ideas.

2. *Dust Crystals*: occurrence, structure and dynamics.
   (i) Focus: 1d dust crystals in lab; *prerequisites*.
   (ii) *Nonlinearity* in 1d DP crystals: Origin and modeling.

3. Nonlinear effects on *transverse* dust-lattice waves (*TDLW* s): amplitude modulation, transverse envelope structures.

4. Nonlinear effects on *longitudinal* dust-lattice waves (*LDLW* s): amplitude modulation, envelope structures, solitons.

5. 1d *Discrete Breathers* (*Intrinsic Localized Modes*) ?

6. Conclusions.

1. Plasma: definition and characteristics

- *Ordinary* $e - i$ (*electron - ion*) plasmas are defined as large ensembles of interacting charged particles.

- Ingredients:
  - – *electrons* e^- (charge $-e$, mass m_e),
  - – *ions* i^+ (charge $+Z_i e$, mass m_i).

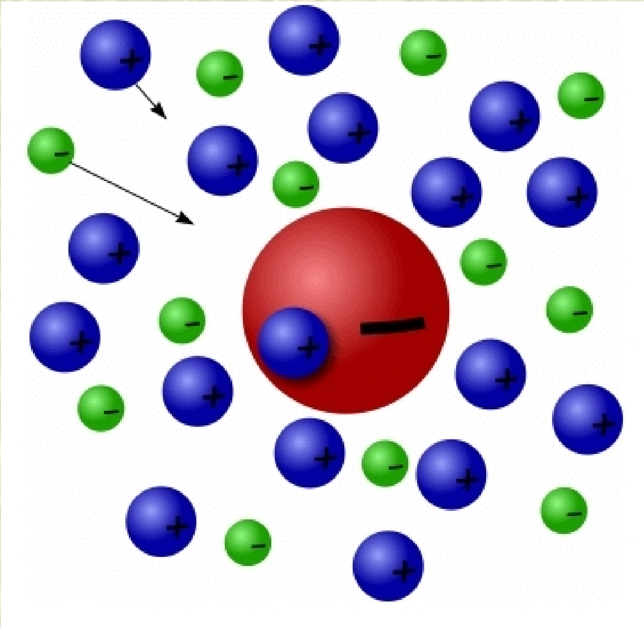
  The *overall charge neutrality* condition reads:

$$Q_{total} = e(Z_i n_{i,0} - n_{e,0}) = 0 \qquad \Rightarrow \qquad Z_i n_{i,0}/n_{e,0} = 1$$

- *Physics of Plasmas*: relevance with *Astrophysics*, *Space Physics*, *Atmospheric Physics*, *EM wave propagation*, *Fusion Theory*, *Material technology*, *Plasma Processing industry*, ...

- Plasma is (long known as) a very *complex* form of matter !

1. DP – Dusty Plasmas (or *Complex Plasmas*): definition and characteristics



❑ Ingredients:

– *electrons* e^- (charge $-e$, mass m_e),

– *ions* i^+ (charge $+Z_i e$, mass m_i), *and*

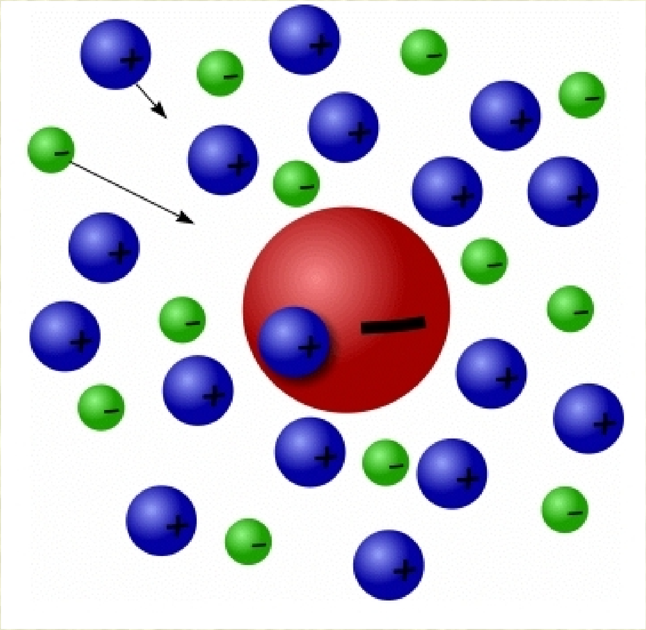
– charged particulates $\equiv$ *dust grains* d (most often d^-):

charge $Q = sZ_d e \sim \pm(10^3 - 10^4)\, e, \quad (s = \pm 1)$

mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,

radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu$m.

1. DP – Dusty Plasmas (or *Complex Plasmas*): definition and characteristics



- Ingredients:
  - *electrons* e^- (charge $-e$, mass m_e),
  - *ions* i^+ (charge $+Z_i e$, mass m_i), *and*
  - charged micro-particles $\equiv$ *dust grains* d (most often d^-):

    charge $Q = sZ_d e \sim \pm(10^3 - 10^4)\, e$, $\quad$ ($s = \pm 1$).

Overall neutrality condition: $Z_i n_{i,0} - n_{e,0} + sZ_d n_{d,0} = 0$

Occurrence and origin: Where does the dust come from?

(a) *Space: cosmic debris* (silicates, graphite, amorphous carbon), comet tails, near - Earth man-made pollution (Shuttle exhaust, satellite remnants), ...

e.g. *interstellar dust*:

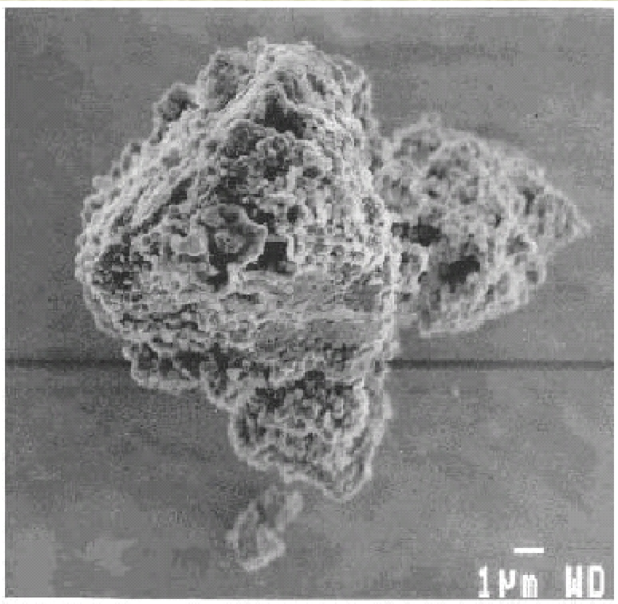


Figure 1.1. The appearance of interplanetary dust particles (courtesy of Dr Scott Messenger, Washington University).

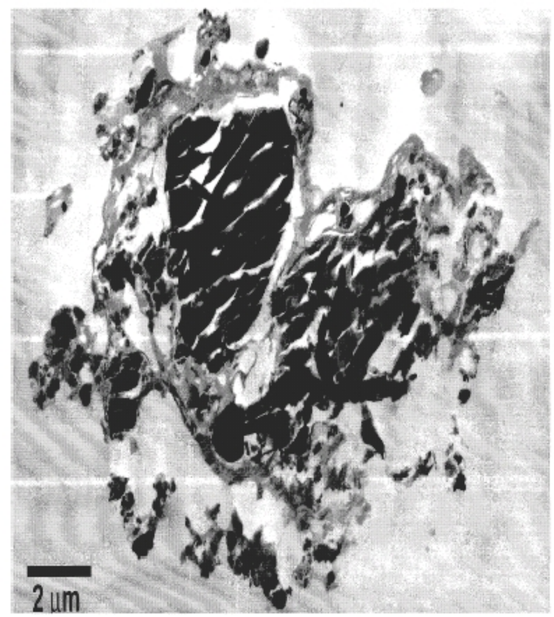
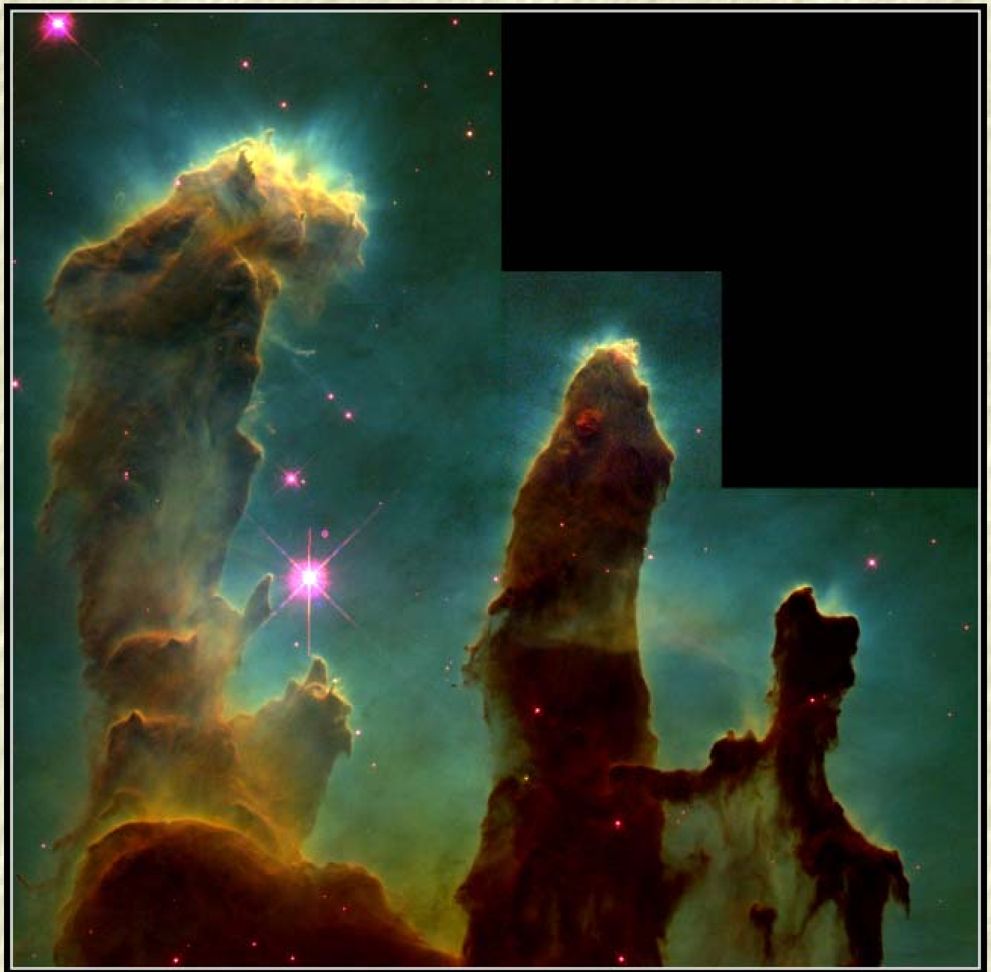


Figure 1.2. Typical interior view of anhydrous interplanetary dust viewed by transmission electron microscopy (courtesy of Dr Lindsay Keller, Johnson Space Center).

Source: [P. K. Shukla & A. Mamun 2002]

e.g. *interstellar dust*: Evaporating gaseous globules (EGGs); Hubble 1995 data, from the Eagle nebula (Serpens constellation), show columns of puffing dust (→ star formation):



Gaseous Pillars · M16 HST · WFPC2
PRC95-44a · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA

e.g. *Comet tails* (*Hale-Bopp*, *Hyakutake* comets):

Review of 1996 data from the *Ulysses* spacecraft revealed: *Hyakutake*'s tail was as long as 3.5 times the Earth-Sun distance (!), creating an enormous stream of charged particles (ions); Nature **157** (15), 228 (2000).

e.g. Planetary rings: *Jupiter, Saturn (here: B ring), Uranus, Neptune*, ... (data from space missions Voyager 1 and 2)

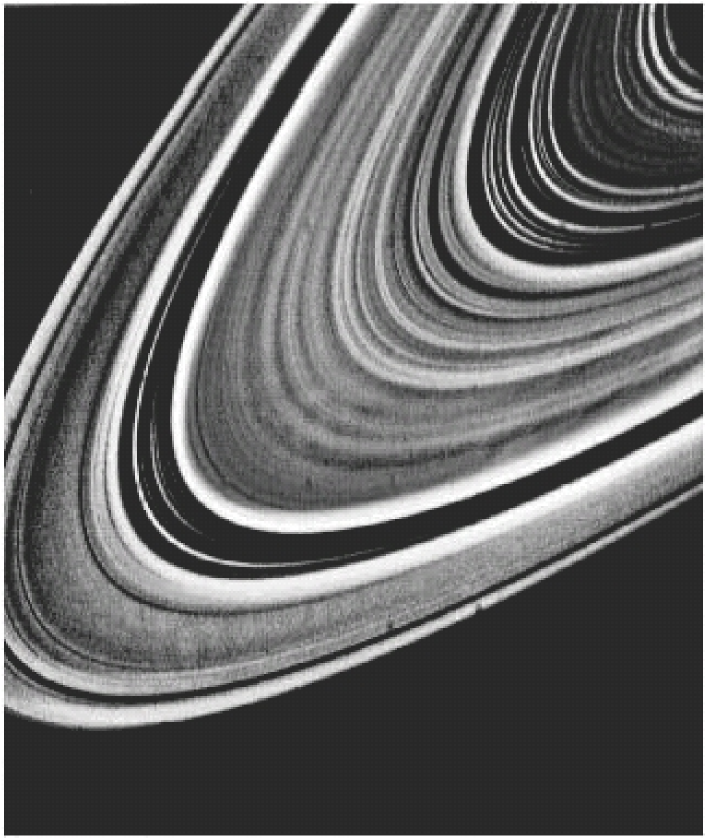


Figure 1.4. A view of the nearly radial spokes in Saturn's B ring. The azimuthal width of a spoke is typically a few thousand kilometres, which in the electrostatic levitation model corresponds to the size of the meteoric-impact-produced plasma cloud (courtesy of Jet Propulsion Laboratory (JPL)).

Source: [P. K. Shukla & A. Mamun 2002]

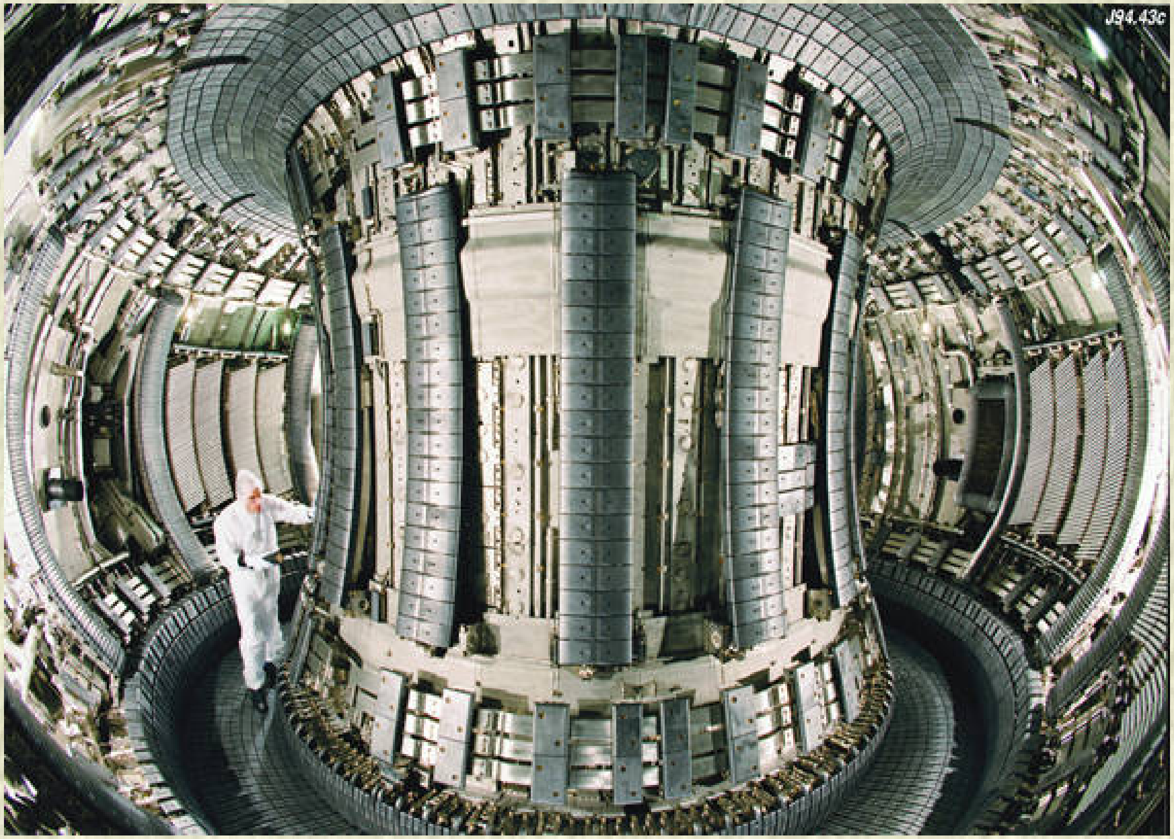
Occurrence and origin: Where from? *(cont.)*

(b) *Earth's atmosphere:* volcanic eruptions (e.g. Krakatoa, 1883), *extraterrestrial* origin (*meteorites*): $\geq 2 \cdot 10^4$ tons a year (!)(*), atmospheric *pollution*, chemical *aerosols*, ...; phenomena observed in the polar summer mesopause, e.g. *Noctilucent clouds (NLCs)* (**), Polar mesospheric summer echoes (PMSEs), polar mesopause temperature profile, and Radar backscatter $\rightarrow$ manifestation of dust (e.g. ice cubicles).

Sources: [P. K. Shukla & A. Mamun 2002], (*) [DeAngelis 1992], (**) G. E. Thomas, Rev. Geophys. **29**, 553 (1991), T. W. Backhouse, Meteorol. Mag. **20**, 133 (1885).

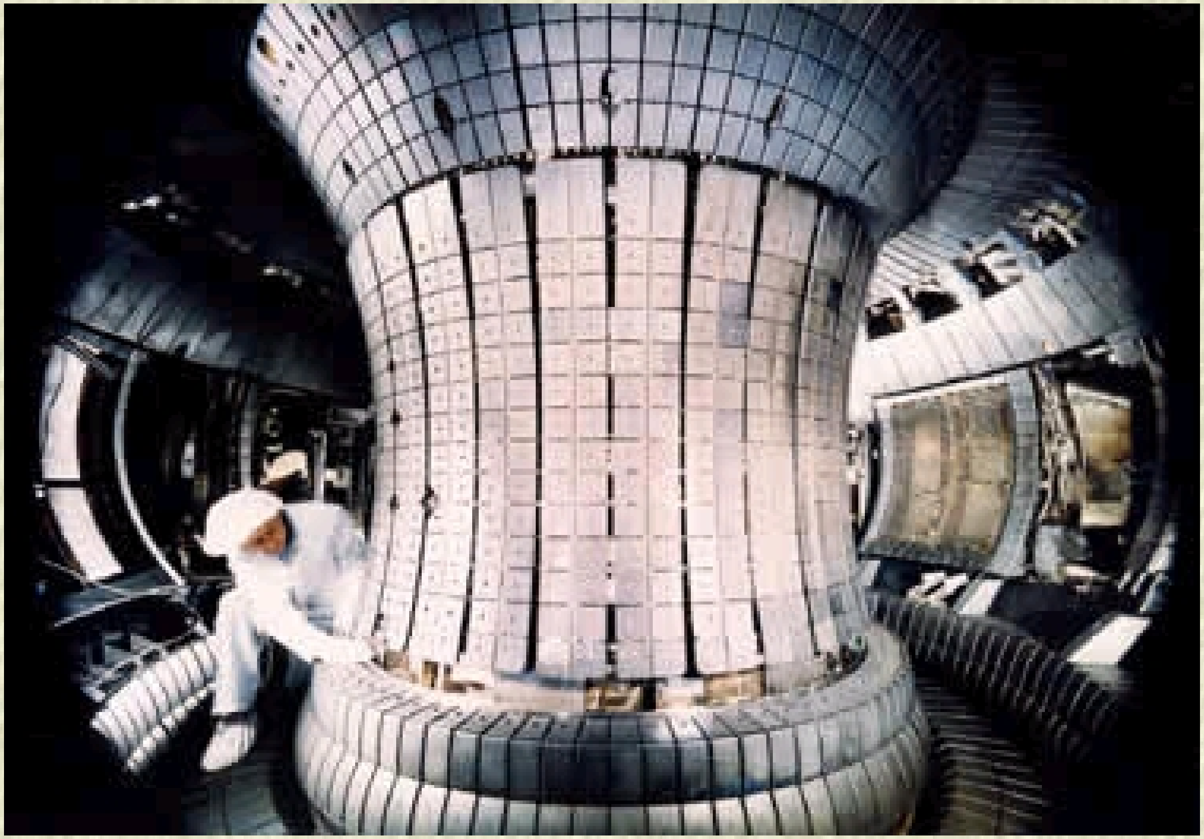
Occurrence and origin: Where from? *(cont.)*

(c) *Fusion devices:* plasma-surface interaction, carbonaceous particulates resulting from *Tokamak* wall erosion-created debris (graphite, CFCs: Carbon Fiber Composites, ...); e.g. JET fusion reactor (Culham, UK):



Occurrence and origin: Where from? *(cont.)*

(c) *Fusion devices:* plasma-surface interaction, carbonaceous particulates resulting from *Tokamak* wall erosion-created debris (graphite, CFCs: Carbon Fiber Composites, ...); e.g. ASDEX Upgrade (Garching, Germany):

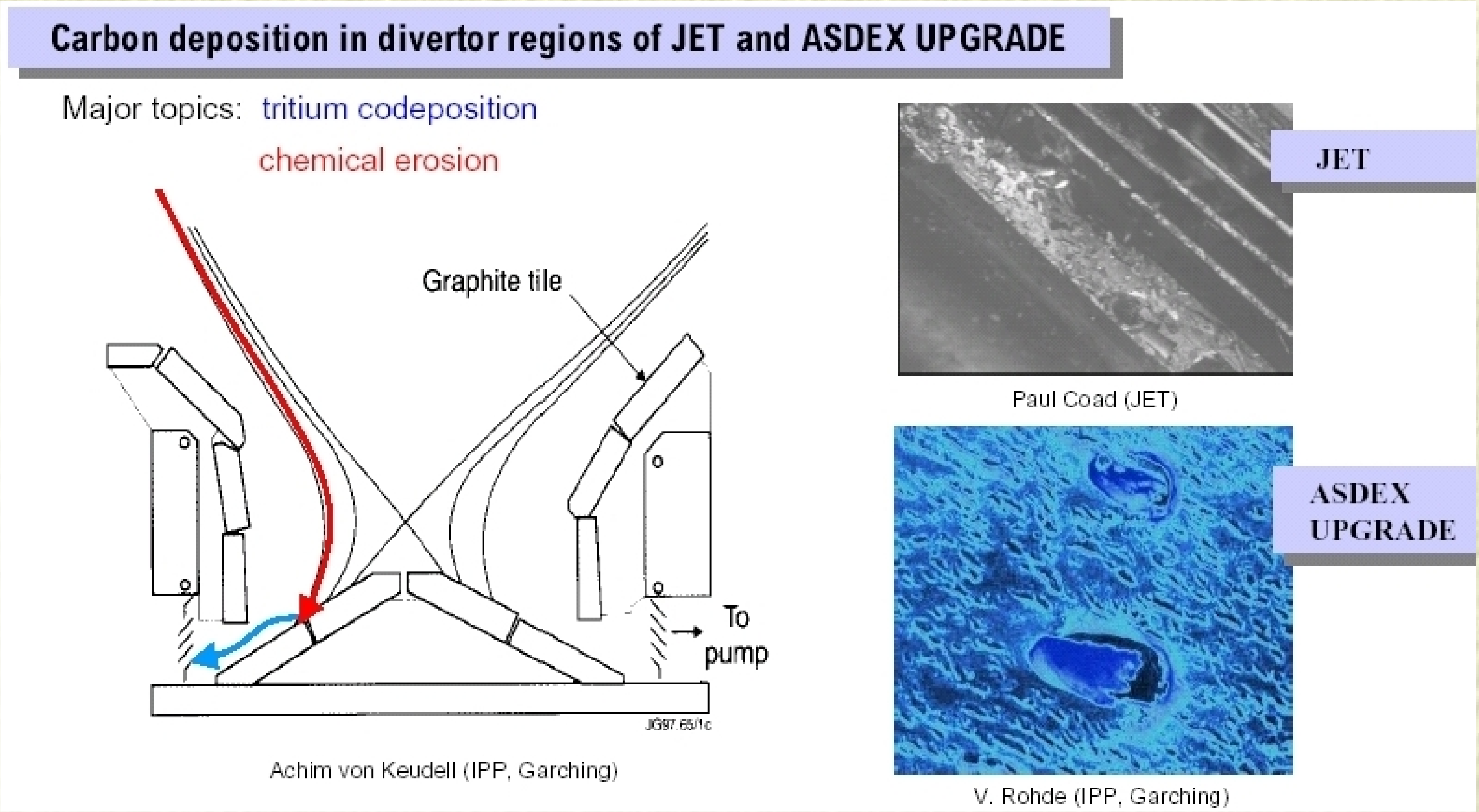


Occurrence and origin: Where from? *(cont.)*

(c) *Fusion devices:* plasma-surface interaction, carbonaceous particulates resulting from *Tokamak* wall erosion-created debris (graphite, CFCs: Carbon Fiber Composites, ...); e.g. TEXTOR (Jülich, Germany):

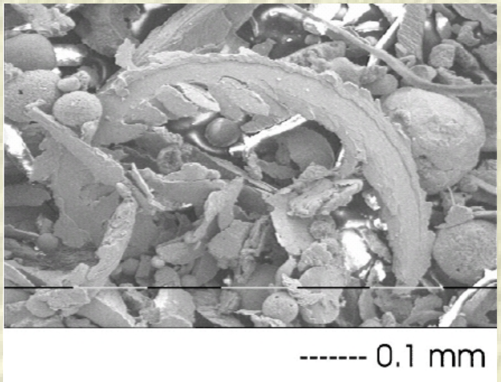
Dust in fusion machines (Tokamaks) *(cont.)*

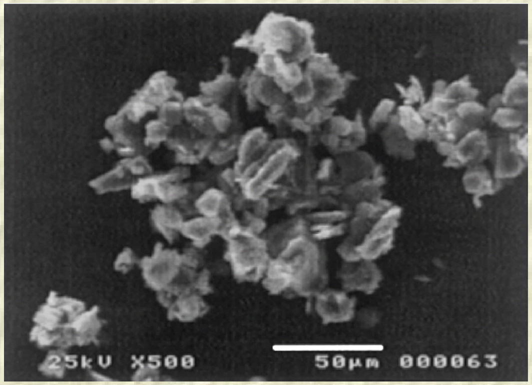
Dust observation near the tokamak walls (bottom, *divertor* area):



(after K. MATYASH, Max Planck Institüt - Univ. Greifswald).

- Dust reported in: JIPPT-IIU 1997, TEXTOR 1999, JET 1999, TORE-SUPRA 2001, TFTR 2001, ASDEX-UPGRADE 2003 ...





- Material: Carbon (graphite composites), tungsten, berellium, ...

  [Rubel *et al.* Nucl.Fus. 41, 1087 (2001)]

- Relevance with disruption (JIPPT-IIU 1997 study).

- Dust can *survive high* T and *migrate* towards the core plasma

  [Martin *et al.* 2004, Krashenninikov 2004].

- *Safety* issue invoked in ITER design.

Occurrence and origin: Where from? *(cont.)*

(d) *Technology:* Semiconductor industry, (Si) microchip fabrication, deposition and implant, dust contamination, solar cell stabilization ...;

(e) *Laboratory:* (man-injected) melamine–formaldehyde particulates (**) injected in *rf* or *dc* discharges; 3d ($=$ multiple 2d layers) or 1d crystals (via appropriate experimental setting).

($\rightarrow$ *See details in later slides*).

Sources: [P. K. Shukla & A. Mamun 2002], (**) [G. E. Morfill *et al.* 1998]

Some unique features of *the Physics of* Dusty Plasmas:

- Complex plasmas are *overall charge neutral*; most (sometimes *all*!) of the negative charge resides on the microparticles;
- The microparticles can be *dynamically dominant*: mass density $\approx 10^2$ times higher than the neutral gas density and $\approx 10^6$ times higher than the ion density !
- Studies in *slow motion* are possible due to high M i.e. *low Q/M ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!) $\rightarrow$ video;
- Dust charge ($Q \neq \mathrm{const.}$) is now a dynamical variable, associated to a new *collisionless damping* mechanism;

(...continued) More "heretical" features are:

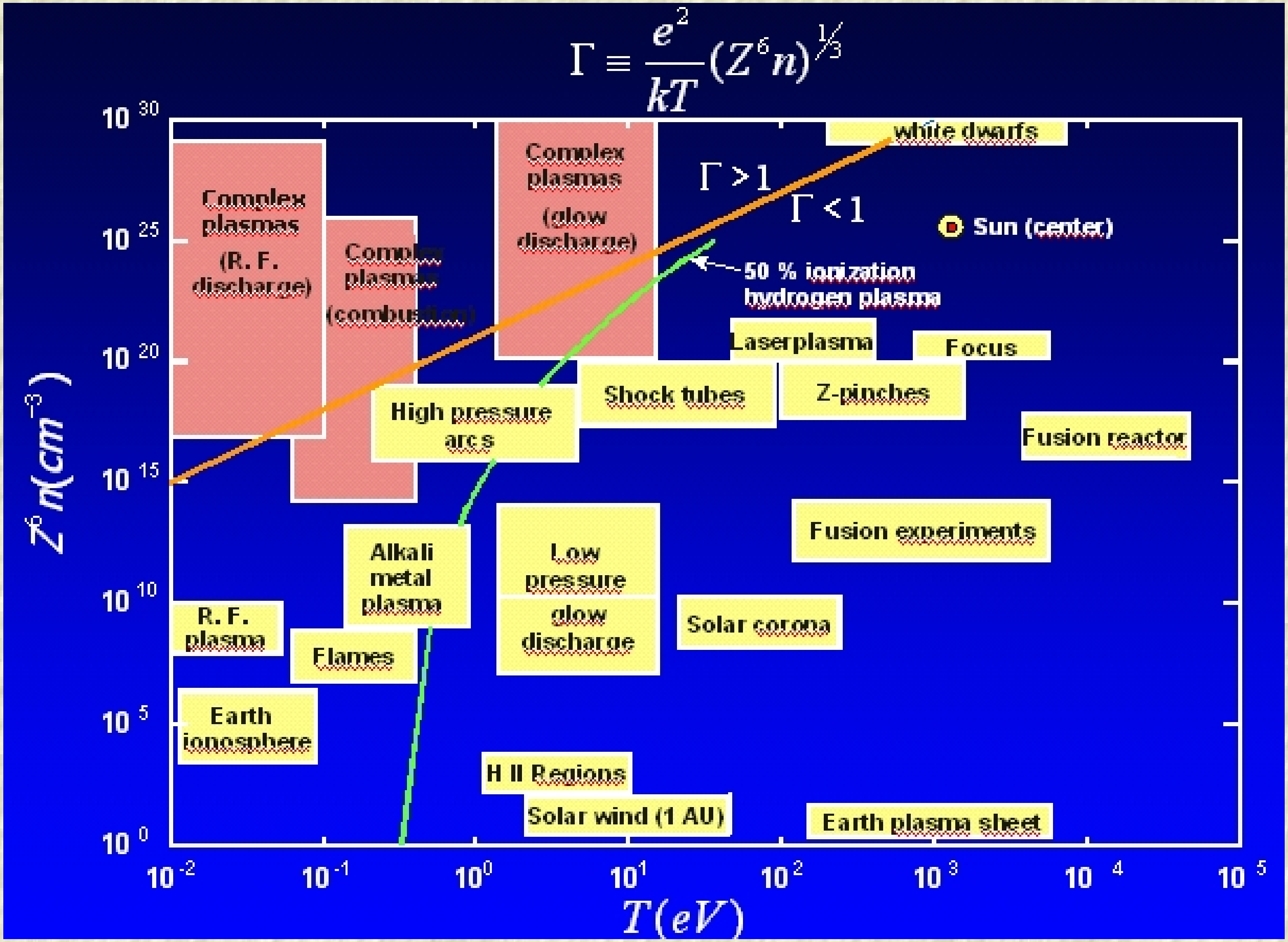
- Important *gravitational* (compared to the *electrostatic*) interaction effects; gravito-plasma physics; gravito-electrodynamics; Jeans-type (gravitational) plasma instabilities etc. [Verheest PPCF 41 A445, 1999]

- Contrary to *weakly-coupled* $e - i$ plasmas ($\Gamma \ll 1$), Complex Plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [IKEZI 1986]) states, depending on the value of the *effective coupling (plasma) parameter* Γ;

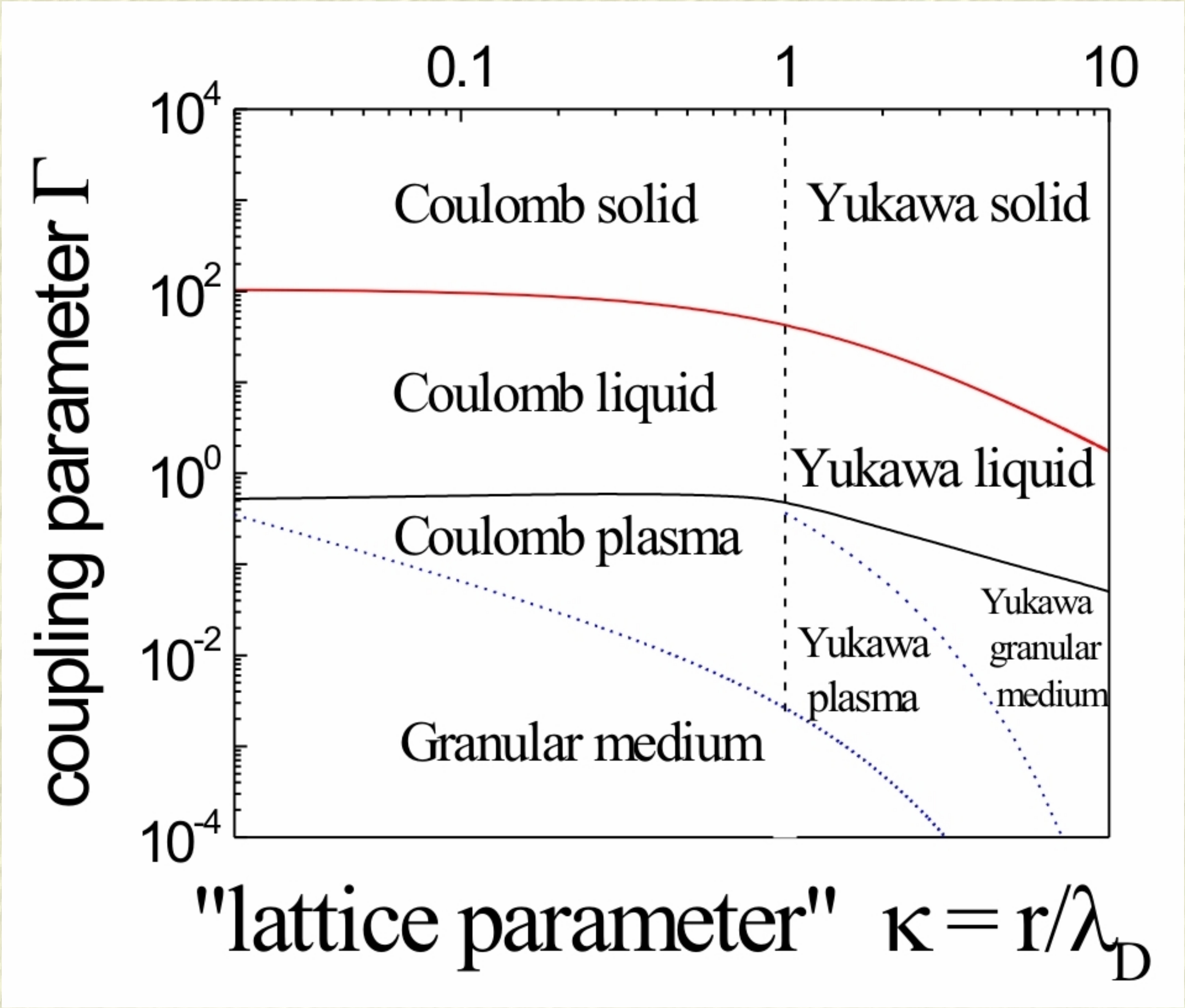
$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \sim \frac{Q^2}{r\,T} e^{-r/\lambda_D}$$

(r: inter-particle distance, T: temperature, λ_D: Debye length).

State portrait of charged matter: density vs. temperature

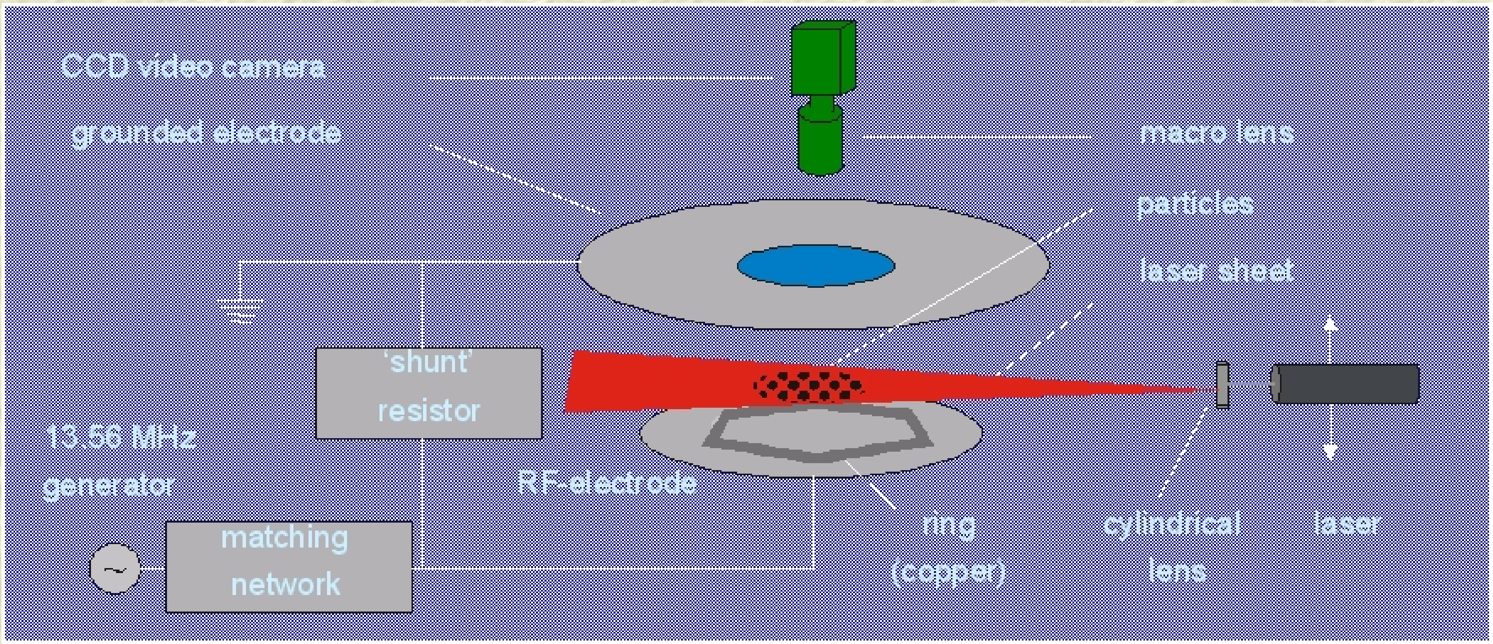


Phase diagram of (complex) charged matter

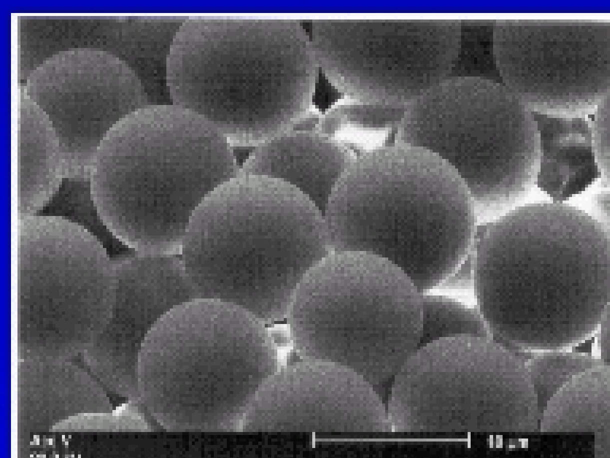


(after G. E. MORFILL, Max Planck Institüt - CIPS).

2. Dust Crystal experiments on Earth:



Particles:
Melamine-Formaldehyde
diameter: few µm



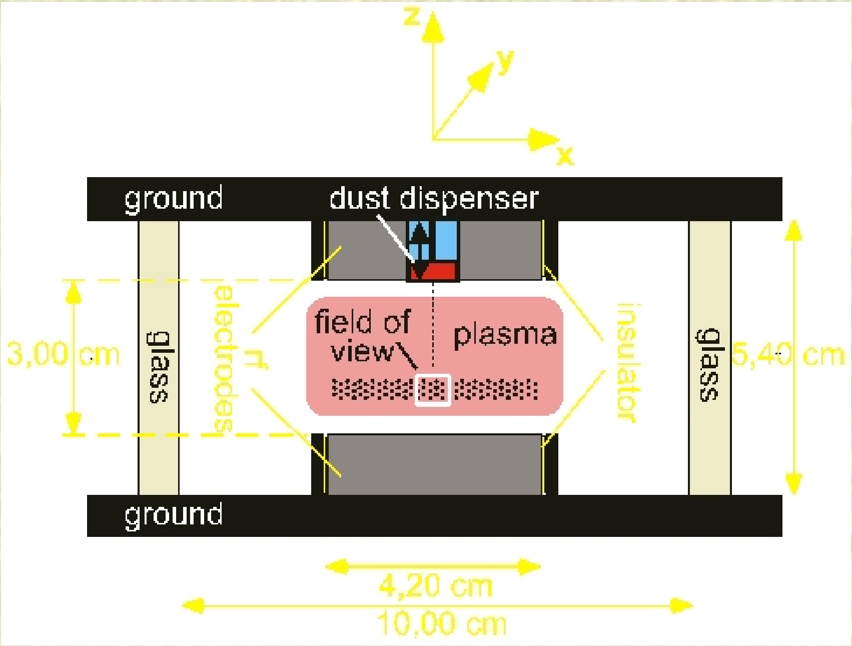
Gas:
noble gas (argon, krypton)
pressure: few Pa ... 100 Pa (=1 mbar)

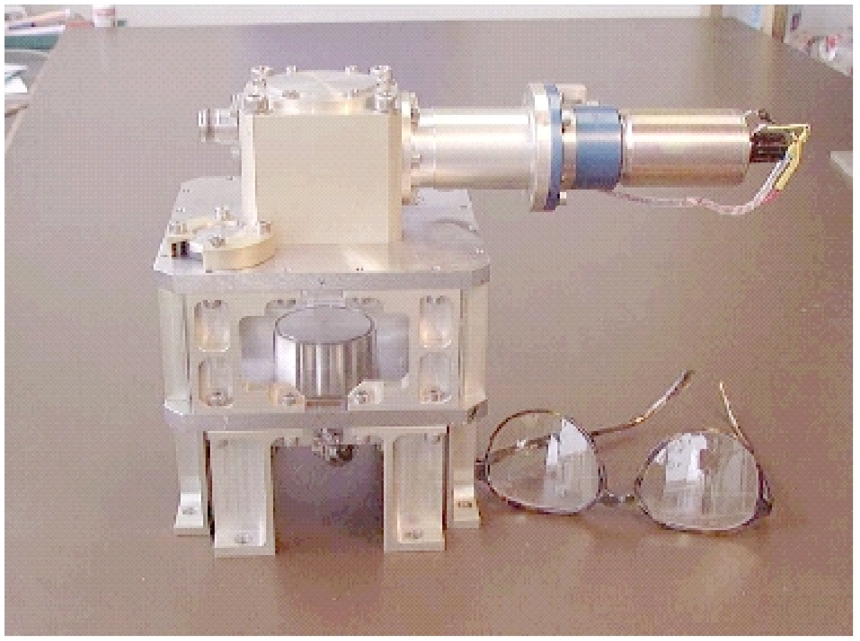
Ionisation fraction: 10^{-6} - 10^{-7}

Temperatures:
kT_e ~2-4 eV (electrons)
kT_i ~0.03 eV (ions)
kT_d ~0.025 eV (microparticles) in crystalline state

[Thomas *et al.*, PRL 1994].

2. Dust Crystal experiments on Earth: *(cont.)*

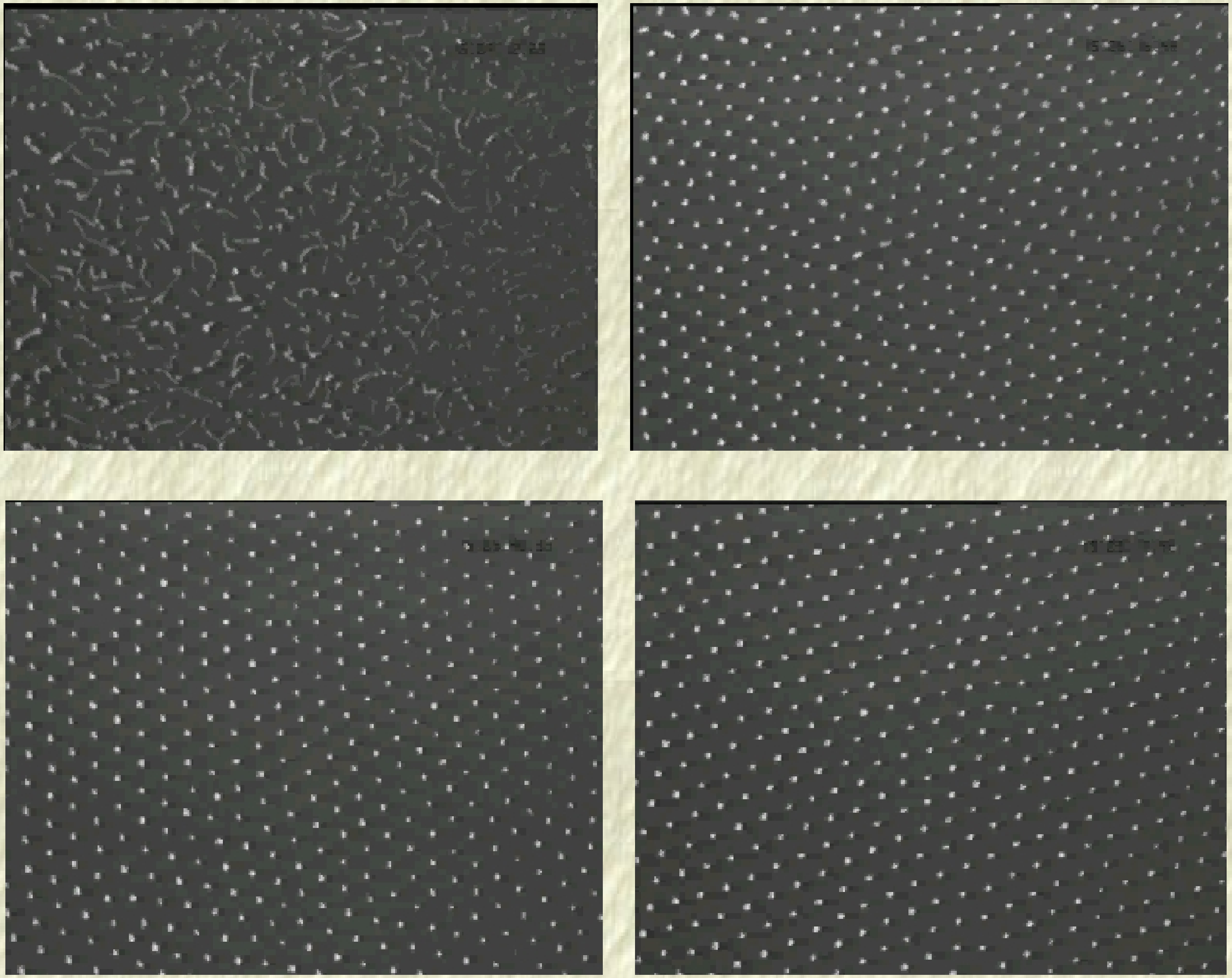




Plasma crystals:

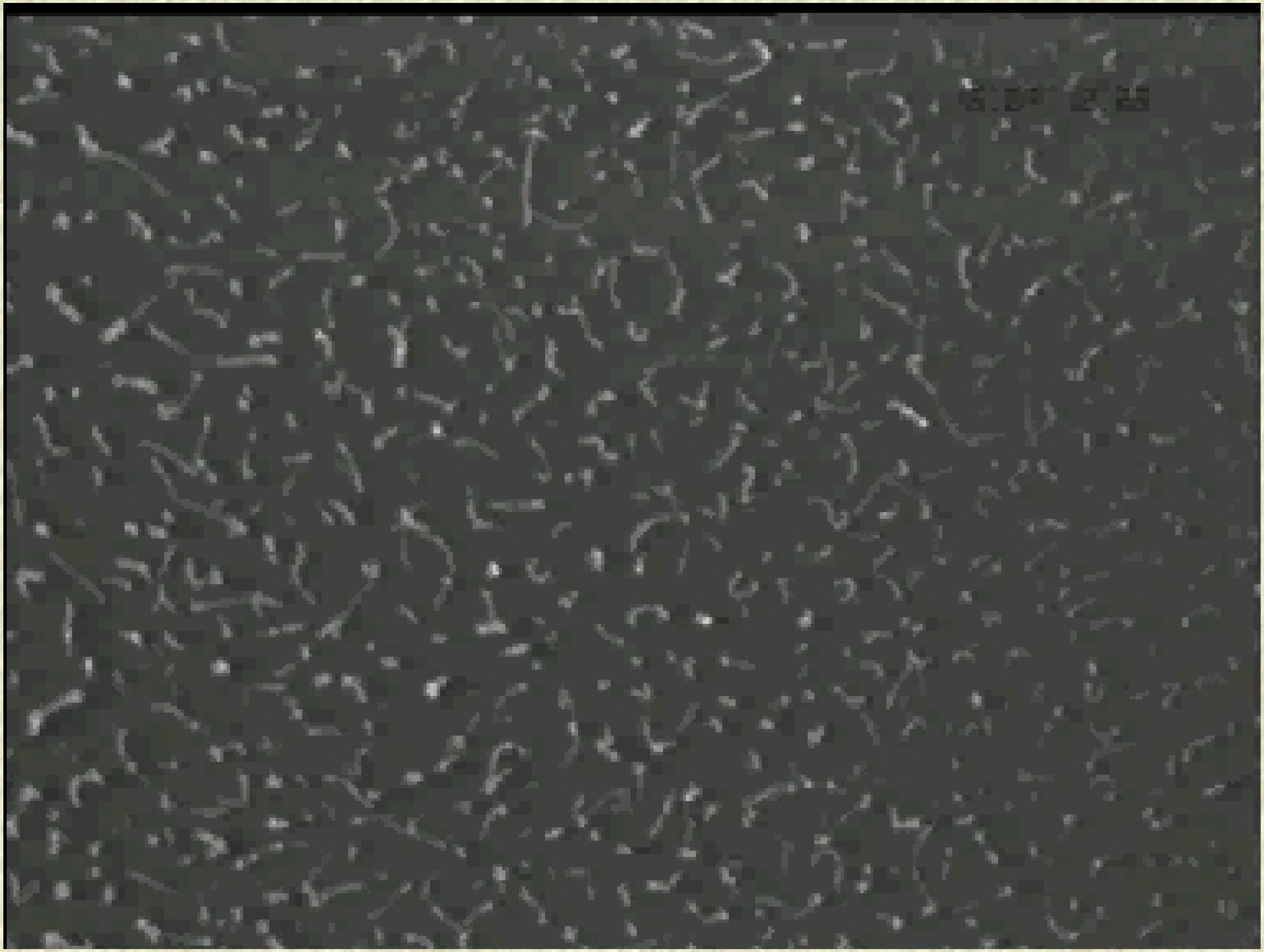
- Theoretical prediction: Ikezi 1986;
- Experimental realization: Thomas *et al.* 1994, Chu & Lin. I 1994, Hayashi & Tachibana 1994.

Different states (*phases*) are observed (top view):

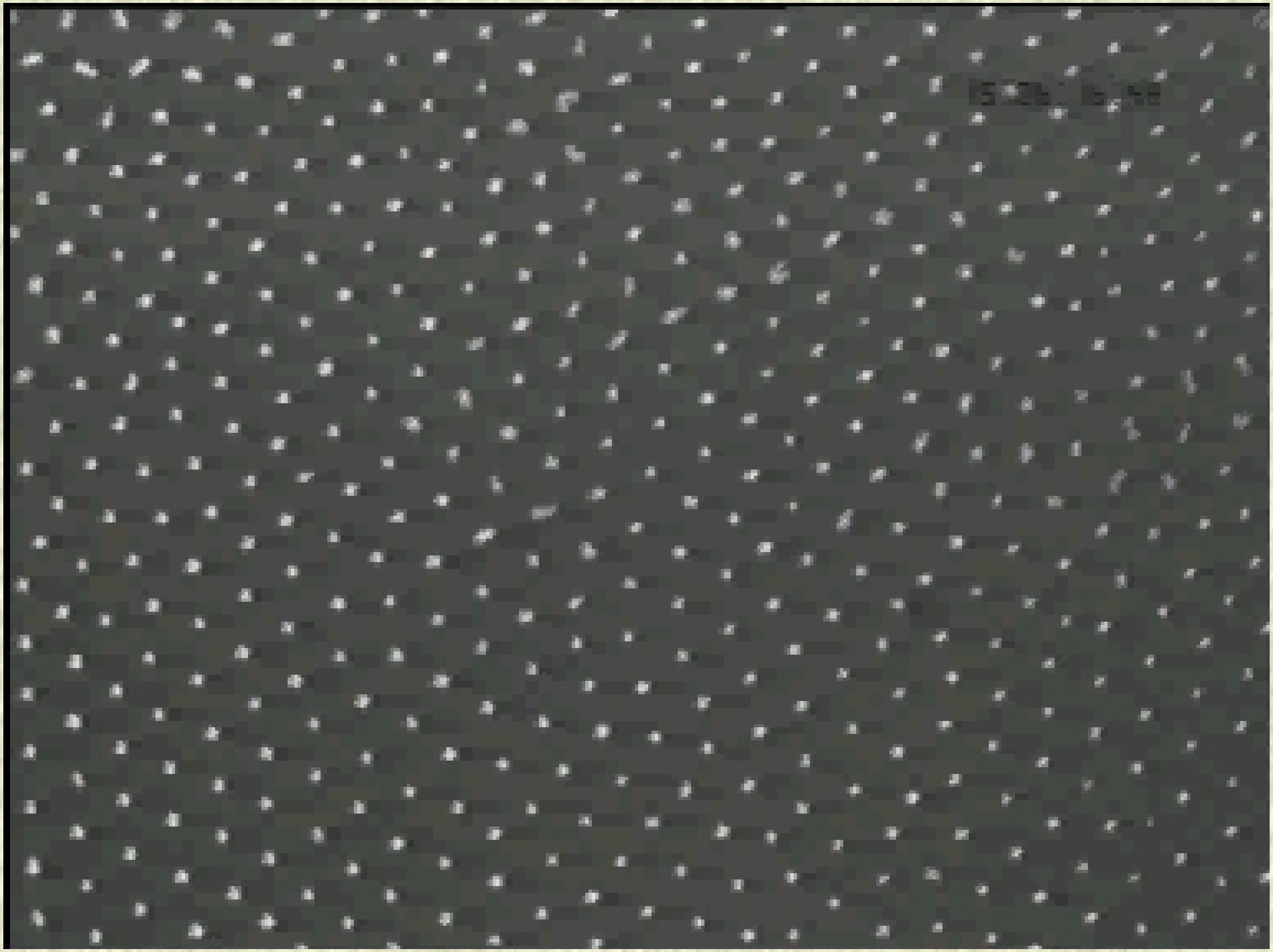


Material courtesy of G. Morfill, MPI Garching.

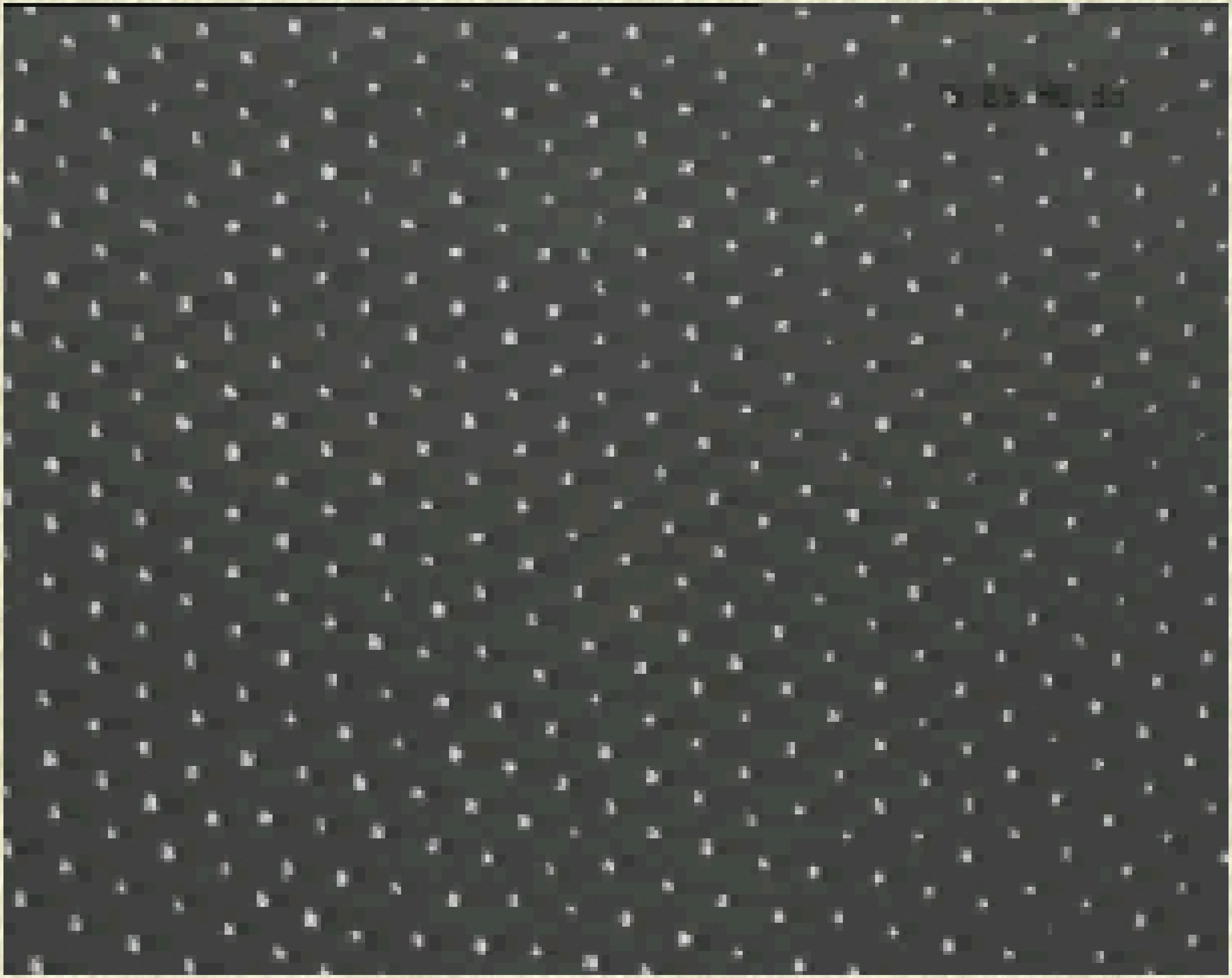
Disordered (“*gas*”) phase (top view):



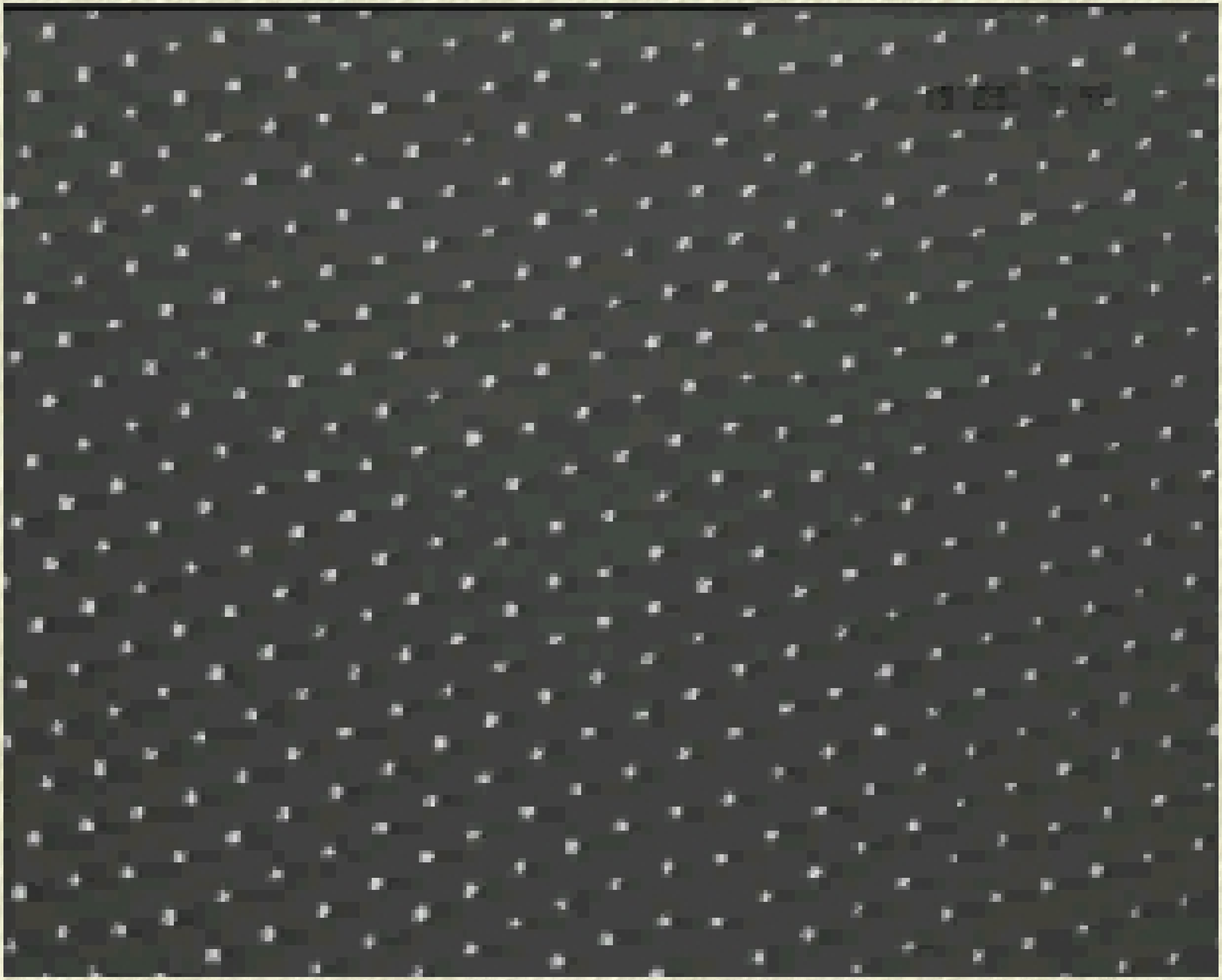
Vibrational (*"viscous"*) phase (top view):



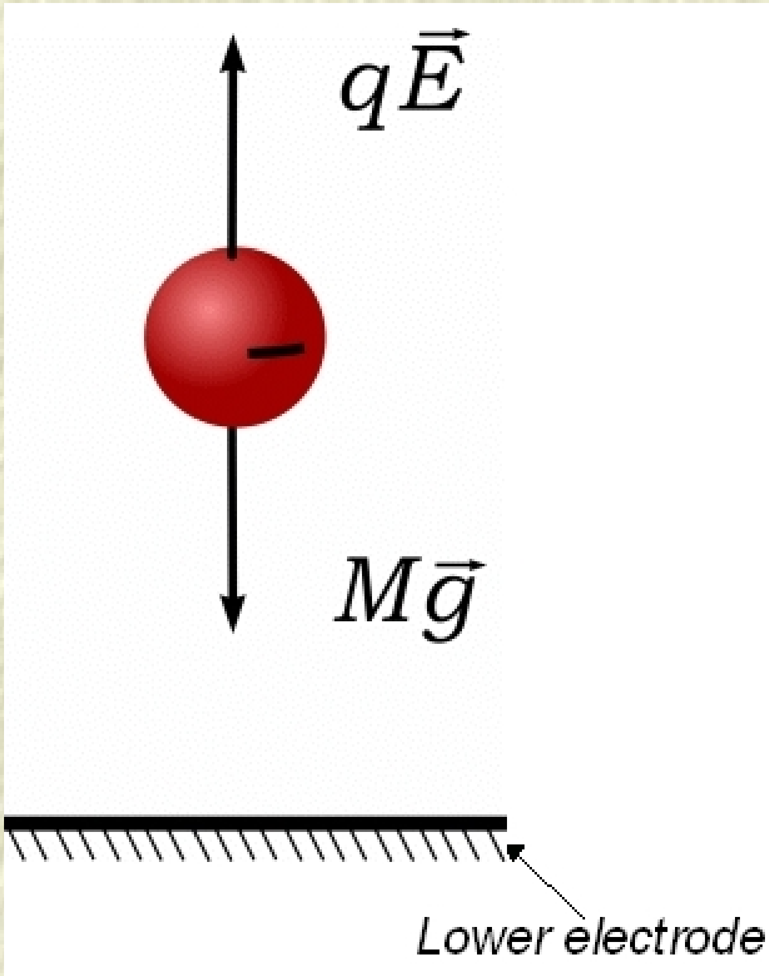
(*“Liquid - crystal”*) phase (top view):



Crystalline (“*solid*”) phase (top view):



Earth experiments are subject to gravity, which balances electric forces at the (*levitated*) equilibrium position:

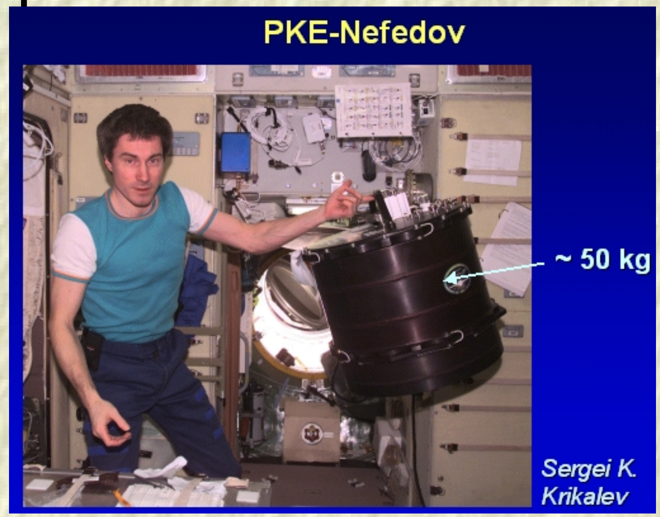


levitation in strong sheath electric field

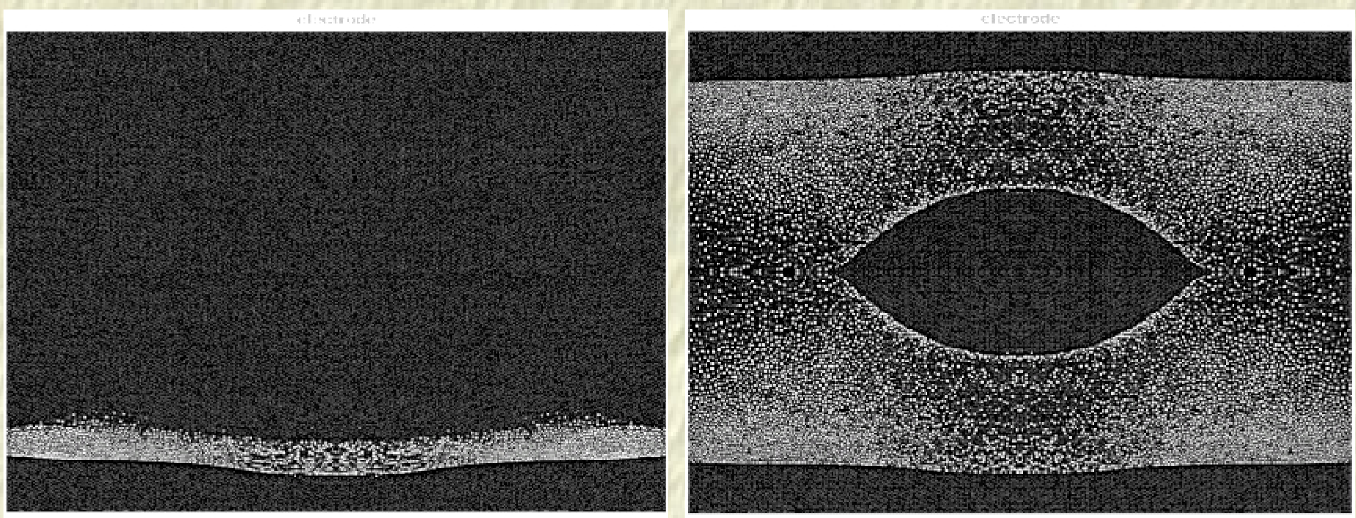
thus ...

Dust experiments in ISS (International Space Station):

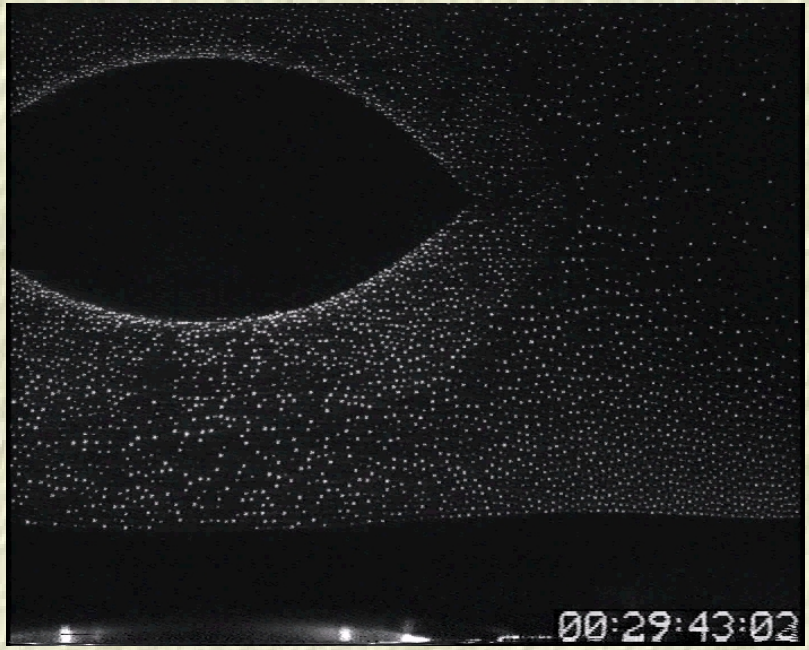
μg conditions: less stress on the plasma.



Side view on Earth (left) vs. μg (right); $\rightarrow$ *see video*:

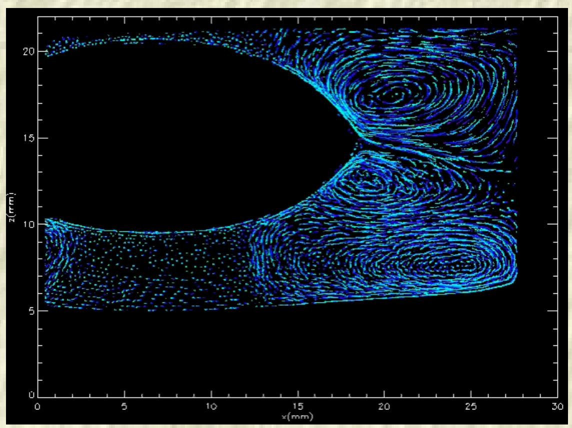


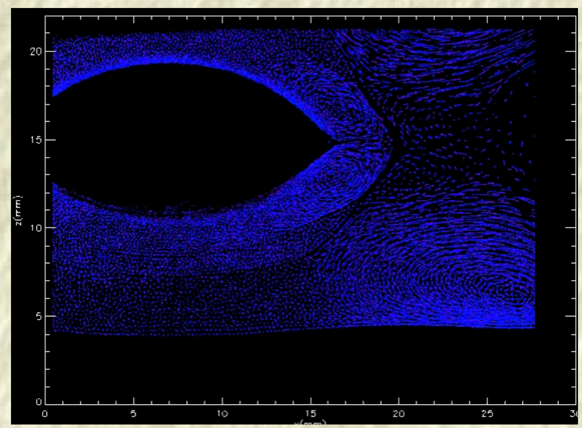
Side view in μg conditions:



– void:

see movie: dustvoid.avi

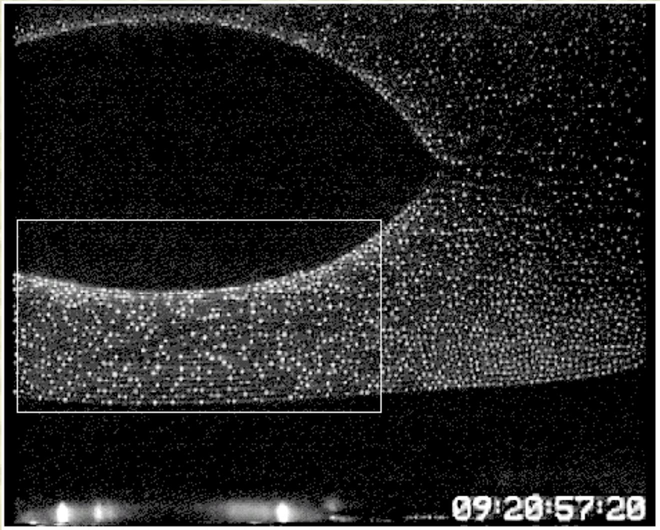


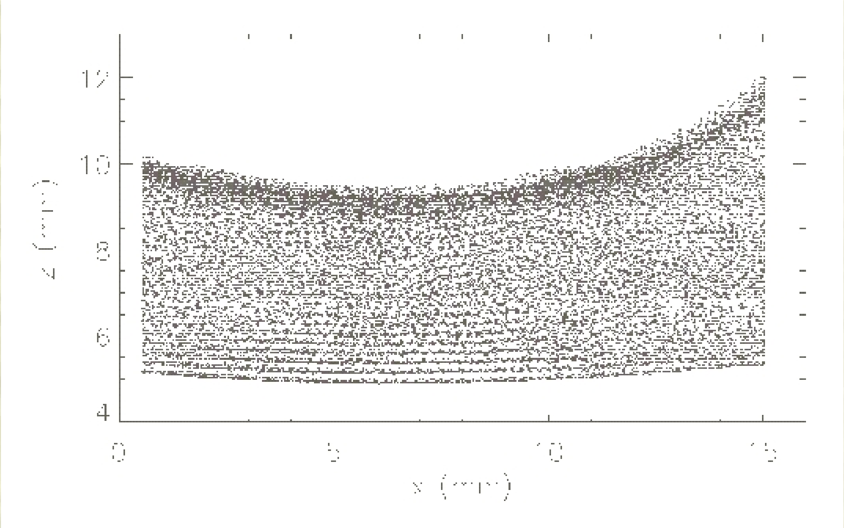


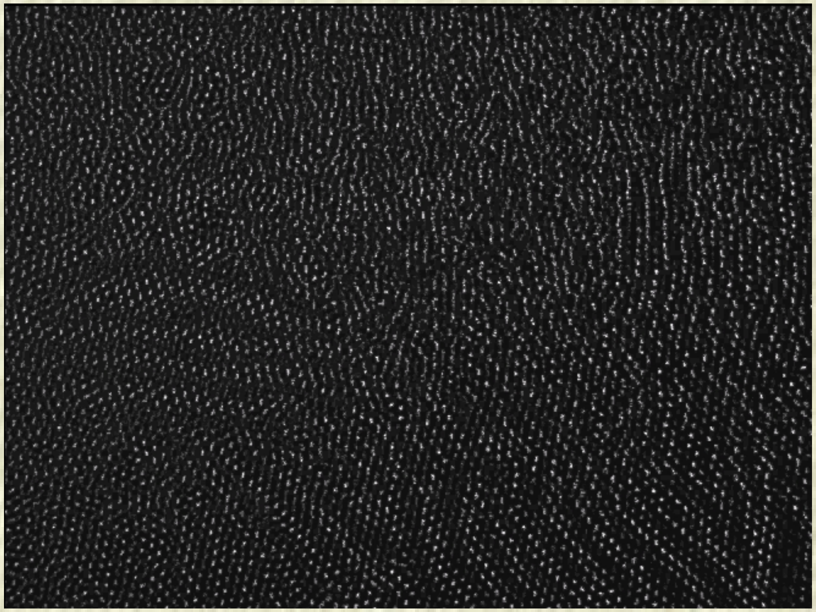
– vortices:

see movies: voidvortex1.avi, voidvortex2.avi

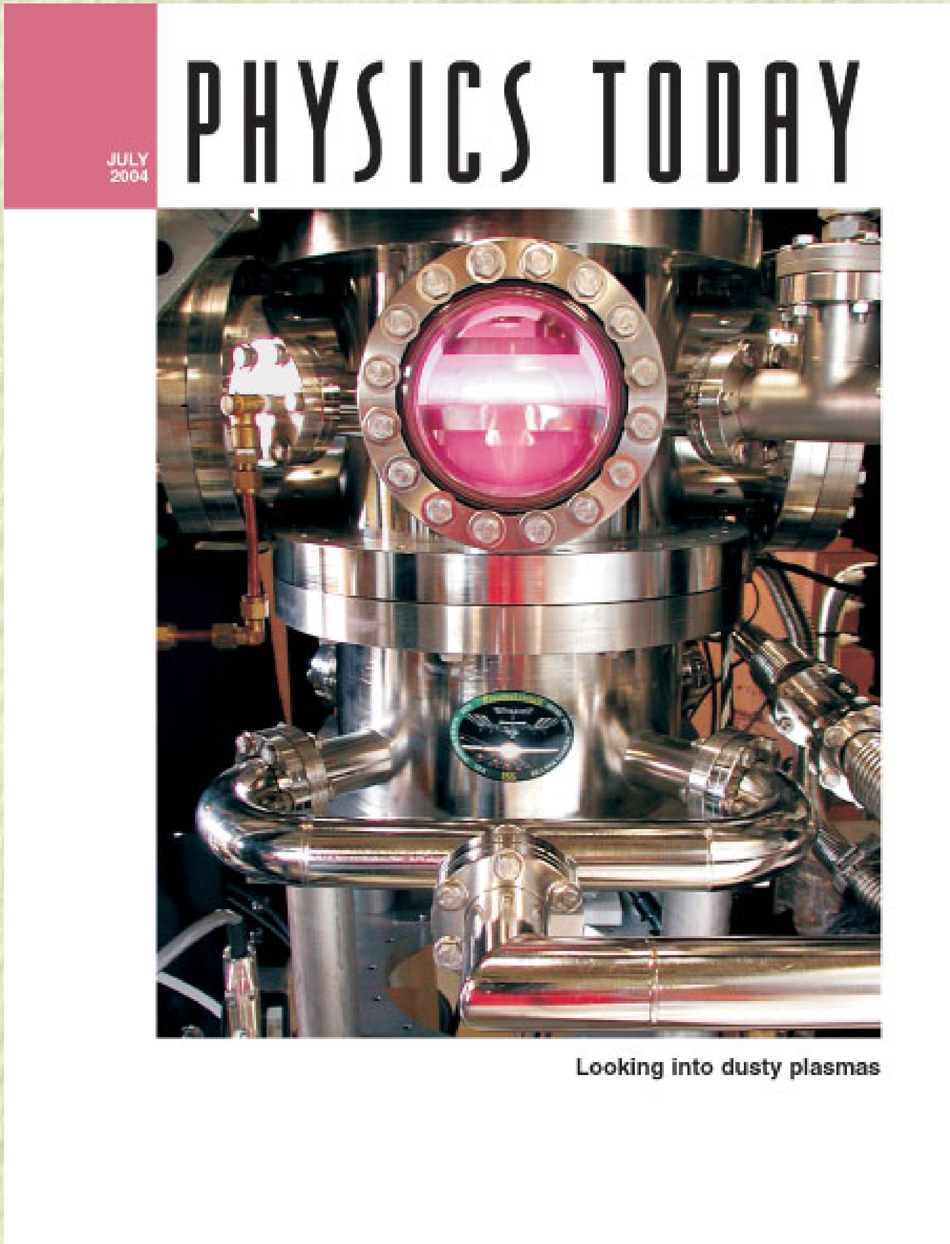
ISS data: Crystallization process







→ movie



(end of Part 1).

2. (i) Focusing on 1d DP crystals: known linear modes.

- *Longitudinal Dust Lattice (LDL)* mode:
  – *Horizontal* oscillations ($\sim \hat{x}$): cf. *phonons* in atomic chains;
  – *Acoustic* mode: $\omega(k = 0) = 0$;
  – *Restoring force* provided by electrostatic interactions.

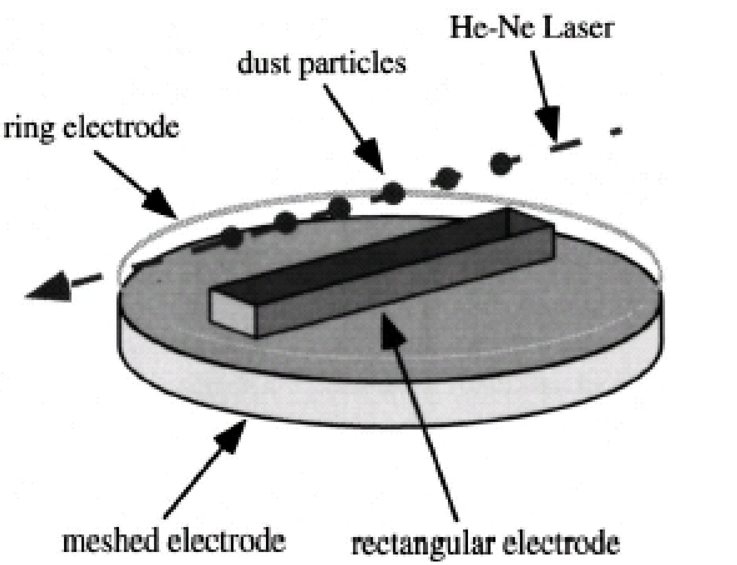


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

* Figure from: S. Takamura *et al.*, Phys. Plasmas **8**, 1886 (2001).

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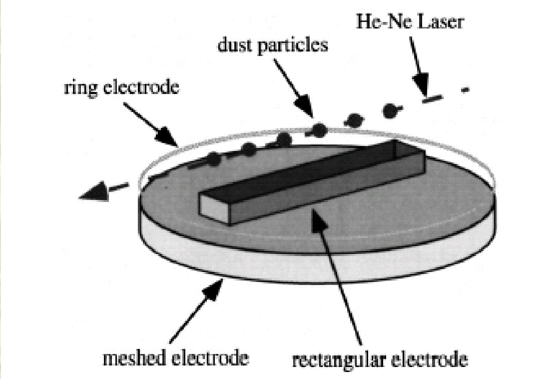
  - – *Restoring force* provided by electrostatic interactions.
- *Transverse Dust Lattice (TDL)* mode:
  - – *Vertical* oscillations ($\sim \hat{z}$);
  - – *Optical* mode:
    $\omega(k = 0) = \omega_g \neq 0$
    (center of mass motion);
  - – *Single* grain vibrations (propagating $\sim \hat{x}$ for $k \neq 0$):
    Restoring force provided by the *sheath electric potential* (*and* interactions).



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2. (i) Focusing on 1d DP crystals: known linear modes.

❑ *Longitudinal Dust Lattice (LDL)* mode:
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$\omega(k=0)=\omega_g \neq 0$
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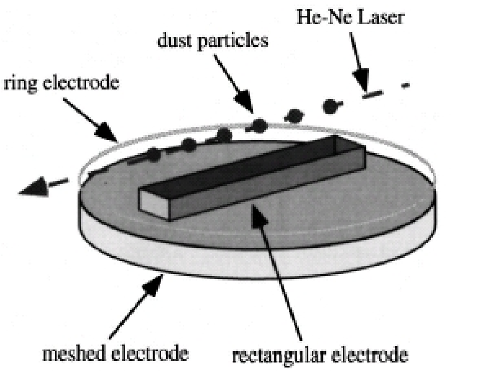


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

– *Single* grain vibrations (propagating $\sim \hat{x}$ for $k \neq 0$):
Restoring force provided by the *sheath electric potential* (*and* interactions).

❑ Transverse ($\sim \hat{y}$, in-plane, optical) d.o.f. *suppressed*.

* Figure from: S. Takamura *et al.*, Phys. Plasmas **8**, 1886 (2001).

Model *Hamiltonian for 1 dust grain*:

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt} \right)^2 + \Phi_{ext}(\mathbf{r}_n)$$

where:

– *Kinetic Energy* (1st term);

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *‘external’ force fields*:
may account for *confinement potentials* and/or *sheath electric forces*, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.

Model *Hamiltonian for N dust grains:*

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

where:

– *Kinetic Energy* (1st term);

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *'external' force fields*:
may account for *confinement potentials* and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.
+ coupling:
– $U_{int}(r_{nm})$ is the (binary) *interaction potential energy*;

Q.: Nonlinearity: *Origin: where from ?*
Effect: which consequence(s) ?

2. (ii) Nonlinearity: Where does it come from?

(i) *Sheath* environment (*anharmonic* vertical potential):

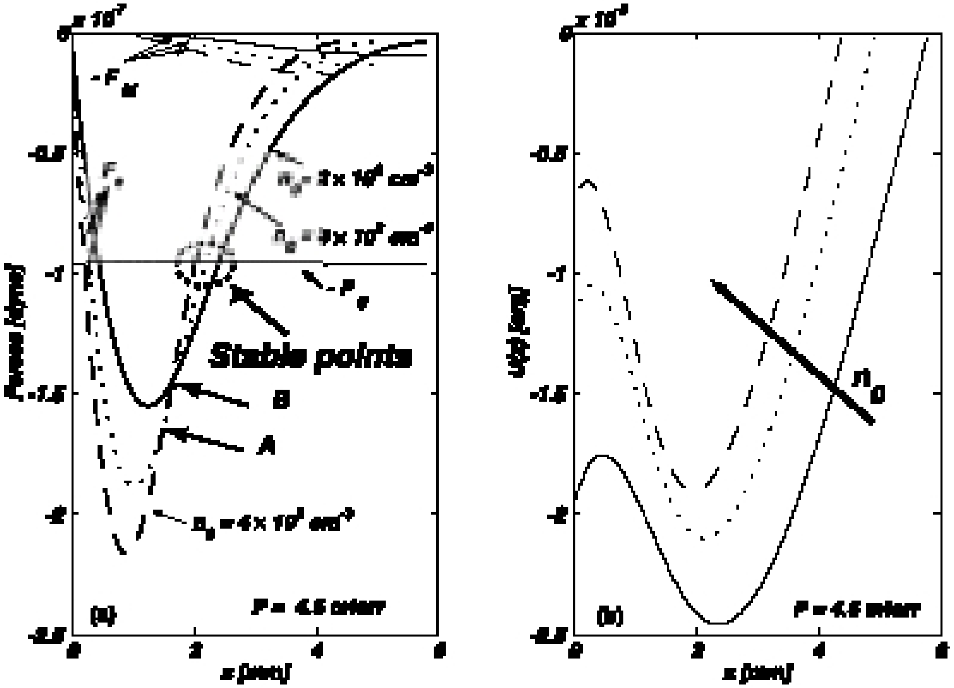


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2\times10^8 cm^{-3}$ (solid line), $n_0 = 3\times10^8 cm^{-3}$ (dashed line), $n_0 = 4\times10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$.

Source: Sorasio *et al.* (2002).

2. (ii) Nonlinearity: Where does it come from?

(i) *Sheath* environment (*anharmonic* vertical potential):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2} M \omega_g^2 (\delta z_n)^2 + \frac{1}{3} M \alpha \, (\delta z_n)^3 + \frac{1}{4} M \beta \, (\delta z_n)^4 + ...$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z_{(0)}$; α, β, ω_g are defined via $E(z)$, $[B(z)]^\dagger$ and $Q(z)$;

(in fact, functions of n and P) [† V. Yaroshenko *et al.*, NJP 2003; PRE 2004]

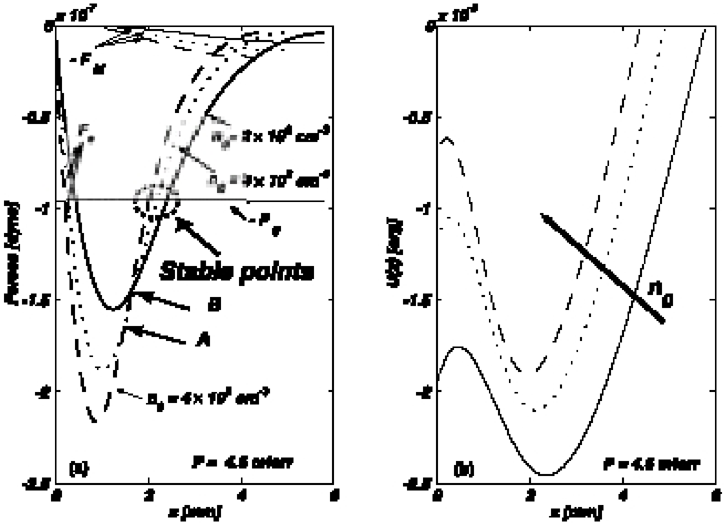


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2 \times 10^8 cm^{-3}$ (solid line), $n_0 = 3 \times 10^8 cm^{-3}$ (dashed line), $n_0 = 4 \times 10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$.

Source: Sorasio *et al.* (2002).

Nonlinearity: Where from? *(continued ...)*

(ii) *Interactions* between grains: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened: $r_0/\lambda_D \approx 1$), *anharmonic*; typically:

$$U_{Debye}(r) = \frac{q^2}{r} \exp\left(-r/\lambda_D\right)$$

Nonlinearity: Where from? *(continued ...)*

(ii) *Interactions* between grains: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened: $r_0/\lambda_D \approx 1$), *anharmonic*; typically: $U_{Debye}(r) = \frac{q^2}{r}\exp\left(-r/\lambda_D\right)$.

Expanding $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$ near equilibrium:

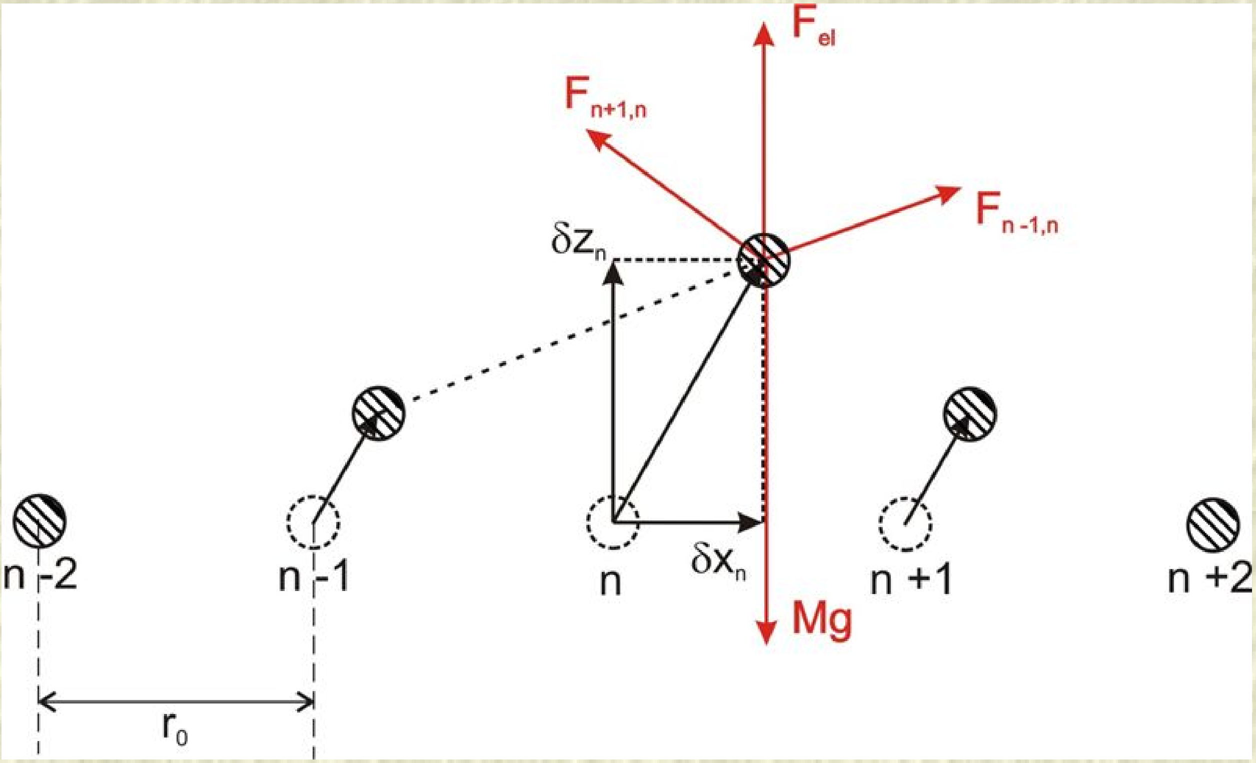
$\Delta x_{nm} = x_n - x_{n-m} = m r_0,\qquad \Delta z_{nm} = z_n - z_{n-m} = 0,$

one obtains:

$$
\begin{aligned}
U_{nm}(r) \;\approx\;& \frac{1}{2} M\omega_{L,0}^2 (\Delta x_{nm})^2 + \frac{1}{2} M\omega_{T,0}^2 (\Delta z_{nm})^2 \\
&+ \frac{1}{3} u_{30} (\Delta x_{nm})^3 + \frac{1}{4} u_{40} (\Delta x_{nm})^4 + \ldots + \frac{1}{4} u_{04} (\Delta z_{nm})^4 + \\
&+ \frac{1}{2} u_{12} (\Delta x_{nm}) (\Delta z_{nm})^2 + \frac{1}{4} u_{22} (\Delta x_{nm})^2 (\Delta z_{nm})^2 + \ldots
\end{aligned}
$$

Nonlinearity: Where from? *(continued ...)*

(iii) Mode *coupling* also induces non linearity: anisotropic motion, *not* confined along one of the main axes $(\sim \hat{x}, \hat{z})$.



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

3. *Transverse* oscillations

The vertical $n-$th grain displacement $\delta z_n = z_n - z_{(0)}$ obeys

$$\frac{d^2(\delta z_n)}{dt^2} + \nu \frac{d(\delta z_n)}{dt} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n \right) + \omega_g^2\,\delta z_n = 0$$

Part 1: *Transverse* oscillations

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* $\omega_{T,0} \; = \left[-qU'(r_0)/(M r_0)\right]^{1/2} = \omega_{DL}^2 \, \exp(-\kappa)\,(1+\kappa)/\kappa^3 \quad ^{(\dagger)}$

$^{(\dagger)}$ (*for Debye interactions*); $\kappa = r_0/\lambda_D$ is the *lattice parameter*;

* $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$; λ_D is the *Debye length*;

Part 1: *Transverse* oscillations

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* Set $\nu = 0$ in the following;

* *Continuum* analogue: $\delta z_n(t) \to u(x,t)$

$$\frac{\partial^2 u}{\partial t^2} + c_T^2 \frac{\partial^2 u}{\partial x^2} + \omega_g^2 \, u = 0$$

where $c_T = \omega_{T,0}\, r_0$ is the *transverse "sound" velocity*.

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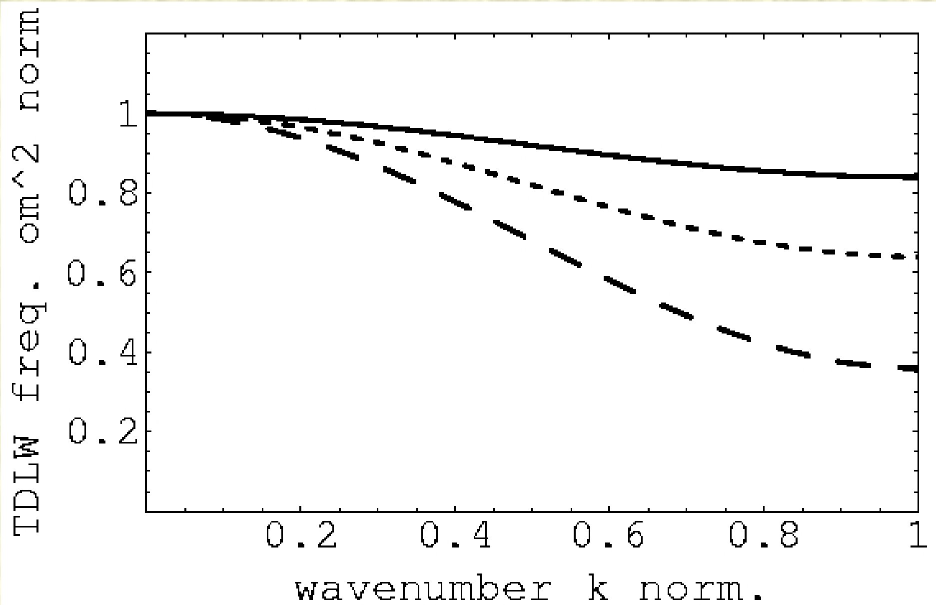
* Set $\nu = 0$ in the following;

* *Optical* dispersion relation (*backward* wave, $v_g < 0$) †:

$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(kr_0/2)$$



† Cf. experiments: T. Misawa *et al.*, *PRL* **86**, 1219 (2001); B. Liu *et al.*, *PRL* **91**, 255003 (2003).

What if *nonlinearity* is taken into account?

$$\frac{d^2\delta z_n}{dt^2} + \nu\,\frac{d(\delta z_n)}{dt} + \omega_{T,0}^2\,(\,\delta z_{n+1} + \,\delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n$$
$$+\alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,.$$

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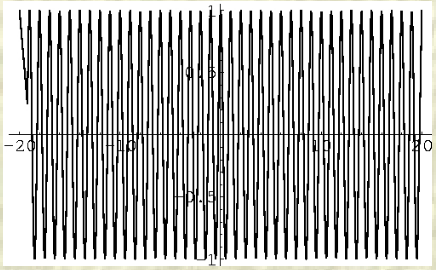
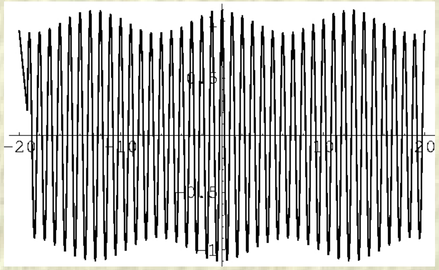
$$+\alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0$$

* *Intermezzo:* The mechanism of *wave amplitude modulation*:

The *amplitude* of a harmonic wave may vary in space and time:



$\longrightarrow$



What if *nonlinearity* is taken into account?

$$\frac{d^2\delta z_n}{dt^2} + \nu\,\frac{d(\delta z_n)}{dt} + \omega_{T,0}^2\,(\,\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n$$
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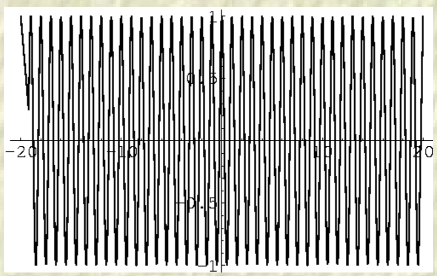
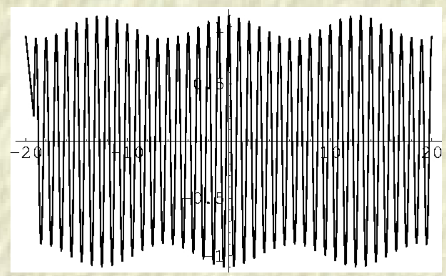
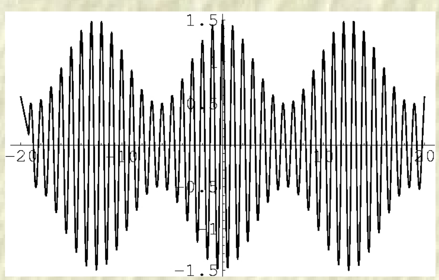


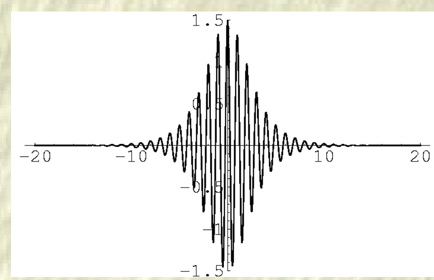
$\longrightarrow$



This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* or formation of *envelope solitons*:



$\longrightarrow$?



Large amplitude oscillations - envelope structures

A reductive perturbation (multiple scale) technique, viz.

$$t \to \{t_0, t_1 = \epsilon t, t_2 = \epsilon^2 t, ...\}, \;\; x \to \{x_0, x_1 = \epsilon x, x_2 = \epsilon^2 x, ...\}$$

yields ($\epsilon \ll 1$; damping omitted):

$$\delta z_n \approx \epsilon\,(A\,e^{i\phi_n} + \text{c.c.}) + \epsilon^2\,\alpha\left[-\frac{2|A|^2}{\omega_g^2} + \left(\frac{A^2}{3\omega_g^2}\,e^{2i\phi_n} + \text{c.c.}\right)\right] + ...$$

($\phi_n = nkr_0 - \omega t$);

Large amplitude oscillations - envelope structures

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$t \to \{t_0, t_1 = \epsilon t, t_2 = \epsilon^2 t, \dots\}, \; x \to \{x_0, x_1 = \epsilon x, x_2 = \epsilon^2 x, \dots\}$

yields ($\epsilon \ll 1$; damping omitted):

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($\phi_n = nkr_0 - \omega t$); the harmonic amplitude $A(X, T)$:

– depends on the *slow* variables $\{X, T\} = \{\epsilon(x - v_g t), \epsilon^2 t\}$;

– obeys the *nonlinear Schrödinger equation* (NLSE):

$$i \, \frac{\partial A}{\partial T} + P \, \frac{\partial^2 A}{\partial X^2} + Q \, |A|^2 \, A = 0 \, , \qquad (1)$$

– *Dispersion coefficient*: $P = \omega''(k)/2$ $\to$ see dispersion relation;

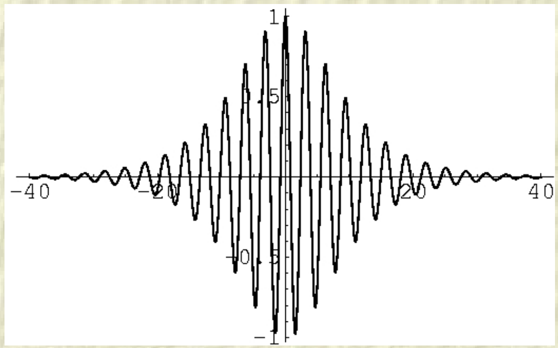
– *Nonlinearity coefficient*: $Q = \left[10\alpha^2/(3\omega_g^2) - 3\,\beta\right]/2\omega$.

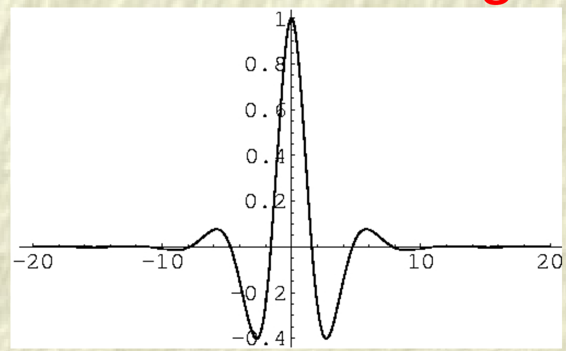
Known properties of the NLS Eq.

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); **11**, 3665 (2004).]

Modulational stability analysis & envelope structures

- $PQ > 0$: *Modulational instability* of the carrier, *bright solitons*:



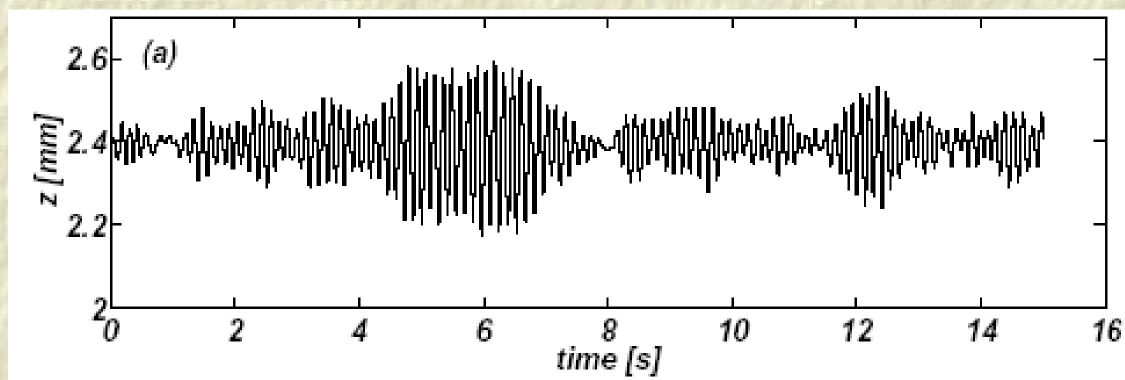


$\rightarrow$ *TDLW*s: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

Rem.: $Q > 0$ for *all* known experimental values of α, β.

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)]

Figure 9: Dust grain oscillations induced by a 1% fluctuation in plasma density. The simulation parameters are: $P = 0.9\ mtorr$, $n_0 = 0.8 \times 10^8\ cm^{-3}$, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$, $\varsigma_t = 0.06$, $\varsigma_p = 1\% n_0$



Source: G. Sorasio *et al.* (2002).

Modulational stability analysis & envelope structures

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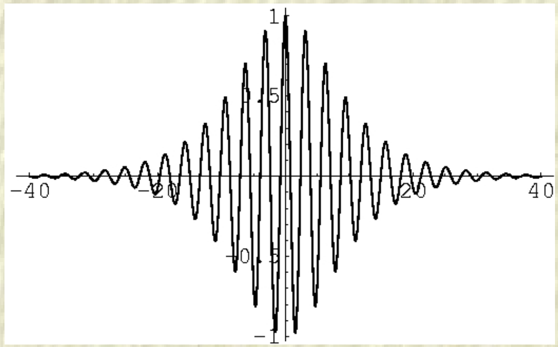
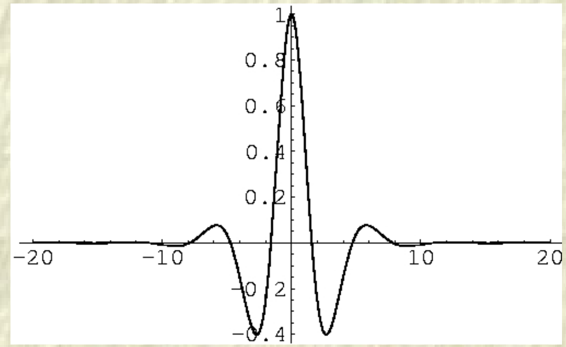
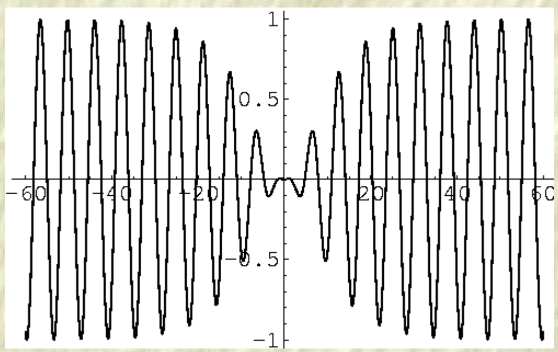
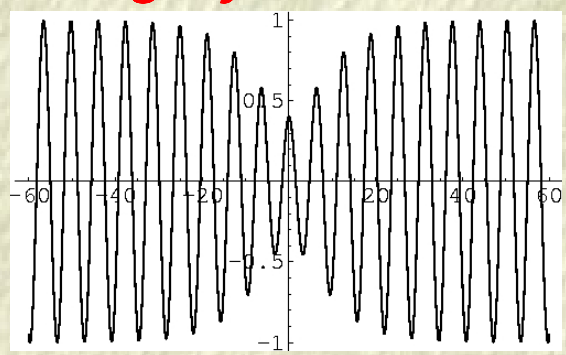




$\rightarrow$ *TDLW*s: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

- $PQ < 0$: Carrier wave is *stable*, *dark/grey solitons*:





$\rightarrow$ *TDLW*s: possible for *long* wavelengths i.e. $k < k_{cr}$.

Rem.: $Q > 0$ for *all* known experimental values of α, β

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)] *(end of Part 3)*.

4a. *Longitudinal* excitations (linear).

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} + \nu \frac{d(\delta x_n)}{dt} = \omega_{0,L}^2 \left(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n\right)$$

– $\delta x_n = x_n - n r_0$: longitudinal dust grain displacements

– *Acoustic* dispersion relation (for $\nu = 0$):

$$\omega^2 = 4\omega_{L,0}^2 \sin^2\left(k r_0/2\right) \equiv \omega_L^2(k)$$

– $\omega_{0,L}^2 = U''(r_0)/M) = 2\,\omega_{DL}^2 \exp(-\kappa)\,(1 + \kappa + \kappa^2/2)/\kappa^3 \quad ^{(*)}$

$^{(*)}$ *for Debye interactions*;   Rem.: $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$.

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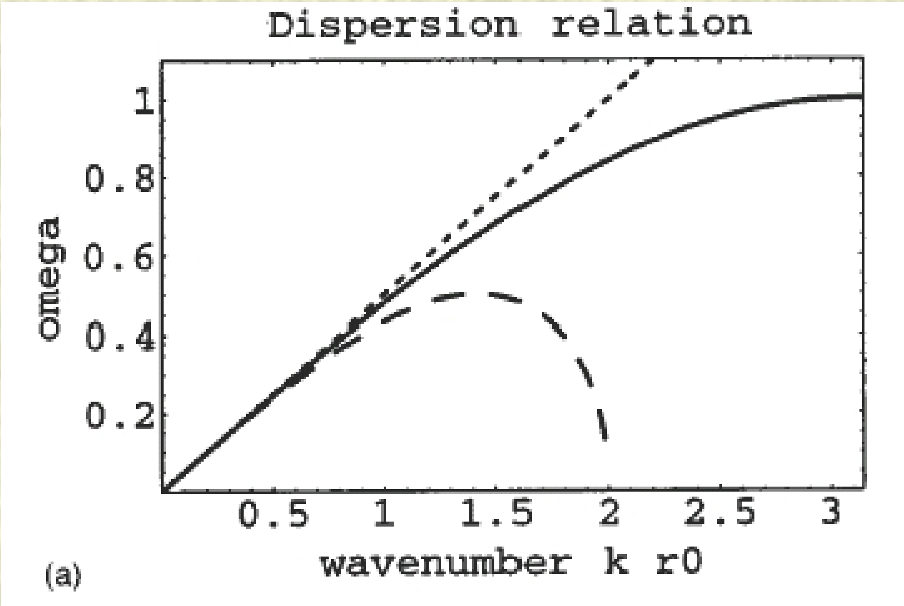

4a. *Longitudinal* excitations (*nonlinear*).

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} + \nu \frac{d(\delta x_n)}{dt} = \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$- a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right]$$
$$+ a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right]$$

– $\delta x_n = x_n - n r_0$: longitudinal dust grain displacements

– Cf. *Fermi-Pasta-Ulam (FPU) problem*:
anharmonic spring chain model.

Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$$\delta x_n \approx \epsilon \left[u_0^{(1)} + \, (u_1^{(1)} \, e^{i\phi_n} + \text{c.c.})\right] + \epsilon^2 \, (u_2^{(2)} \, e^{2i\phi_n} + \text{c.c.}) \, + \ldots ,$$

[Harmonic generation; Cf. experiments: K. Avinash PoP 2004].

Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$$\delta x_n \approx \epsilon \left[u_0^{(1)} + \left(u_1^{(1)}\, e^{i\phi_n} + \text{c.c.}\right)\right] + \epsilon^2 \left(u_2^{(2)}\, e^{2i\phi_n} + \text{c.c.}\right) + \dots,$$

[Harmonic generation; Cf. experiments: K. Avinash PoP 2004].

where the amplitudes now obey the coupled equations:

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L\, \frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0\, |u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L}\, u_1^{(1)} \frac{\partial u_0^{(1)}}{\partial X} = 0\,,$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - c_L^2}\, \frac{\partial}{\partial X}\, |u_1^{(1)}|^2 \equiv R(k) \frac{\partial}{\partial X}\, |u_1^{(1)}|^2$$

– $Q_0 = -\frac{k^2}{2\omega} \left(q_0\, k^2 + \frac{2p_0^2}{c_L^2 r_0^2} \right)$;

– $v_{g,L} = \omega_L{}'(k)$; $\{X, T\}$ are *slow* variables (as above);

– $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$, $q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$.

– $R(k) > 0$, since $\forall\, k$ $\quad v_{g,L} < \omega_{L,0}\, r_0 \equiv c_L$ *(sound velocity)*.

Asymmetric *longitudinal* envelope structures.

– The system of Eqs. for $u_1^{(1)}$, $u_0^{(1)}$ may be combined into a closed (*NLSE*) equation (for $A = u_1^{(1)}$, here);

$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2\,A = 0$$

– $P = P_L = \omega_L''(k)/2 < 0$;

– $Q > 0$ (< 0) prescribes *stability* (instability) at *low* (high) k.

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– $P = P_L = \omega_L''(k)/2 < 0$;
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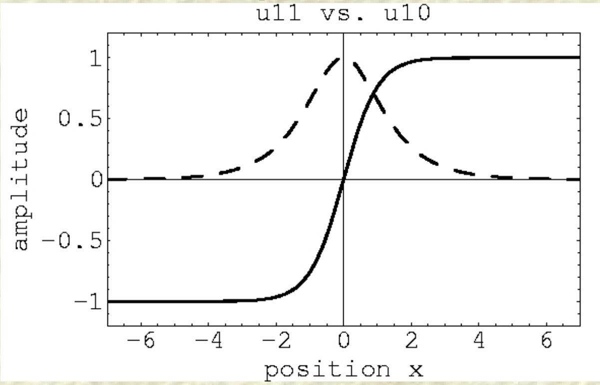
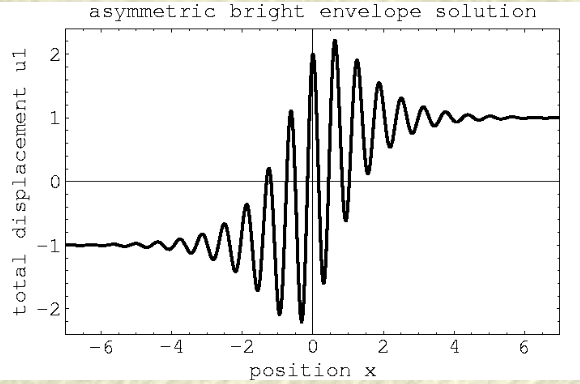
– Envelope excitations are now *asymmetric*:





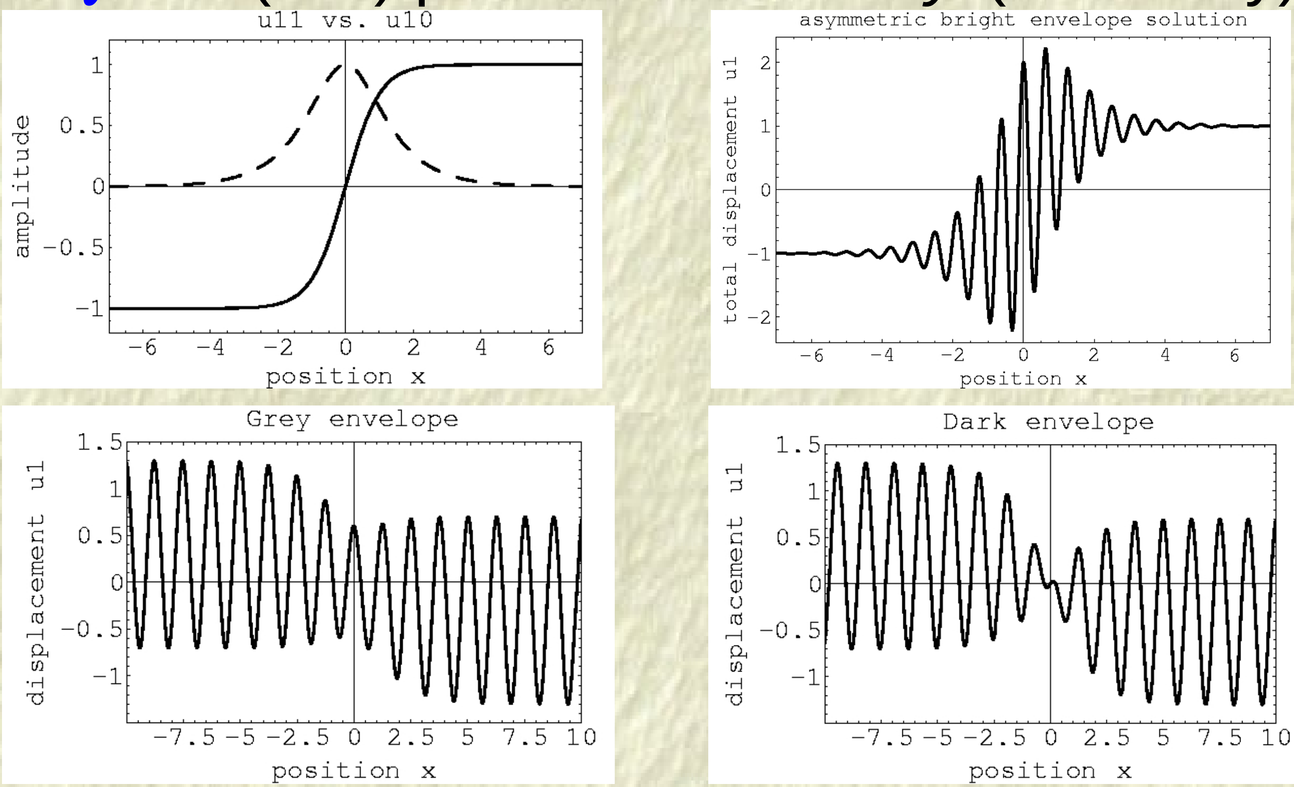
(at high k)

Asymmetric *longitudinal* envelope structures.

– The system of Eqs. for $u_1^{(1)}$, $u_0^{(1)}$ may be combined into a closed (NLSE) equation, which yields *asymmetric* envelope solutions.

– $P = P_L = \omega_L''(k)/2 < 0$;

– $Q > 0$ (< 0) prescribes *stability* (instability) at *low* (high) k.



(at high k)

(at low k)

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).]

(end of Part 4a).

4b. *Longitudinal* soliton formalism.

Q.: *A link to soliton theories: the Korteweg-deVries Equation.*

– *Continuum approximation*, viz. $\delta x_n(t) \to u(x,t)$.

– *"Standard"* description: keeping *lowest order* nonlinearity,

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$ and p_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), slow profile evolution in time τ and defining the *relative displacement* $w = u_\zeta$, one obtains

$$w_\tau - a\,w\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0$$

(for $\nu = 0$); $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$.

– This KdV Equation yields soliton solutions, ... ($\to$ *next page*)

The KdV description

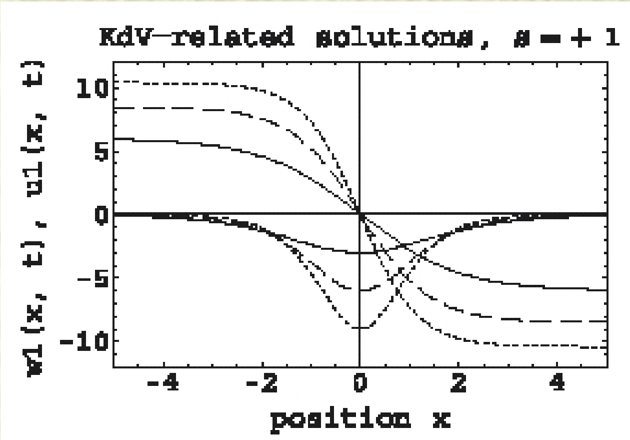
The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, here) solutions, in the form (here):

$$w_1(\zeta, \tau) = -w_{1,m} sech^2 \left[(\zeta - v\tau - \zeta_0)/L_0 \right]$$

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$.



The KdV description

The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, here) solutions, in the form (here):

$$w_1(\zeta, \tau) = -w_{1,m} sech^2 \left[(\zeta - v\tau - \zeta_0)/L_0 \right]$$

– Pulse amplitude: $w_{1,m} = 3v/a = 6vv_0/|p_0|$;

– Pulse width: $L_0 = (4b/v)^{1/2} = [2v_1^2 r_0^2/(vv_0)]^{1/2}$;

– Note that: $w_{1,m} L_0^2 = \text{constant}$ (cf. experiments)†.

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$.

– This is the standard treatment of dust-lattice solitons today ... †

† F. Melandsø 1996; S. Zhdanov *et al.* 2002; K. Avinash *et al.* 2003; V. Fortov *et al.* 2004.

Experiments on DL solitons

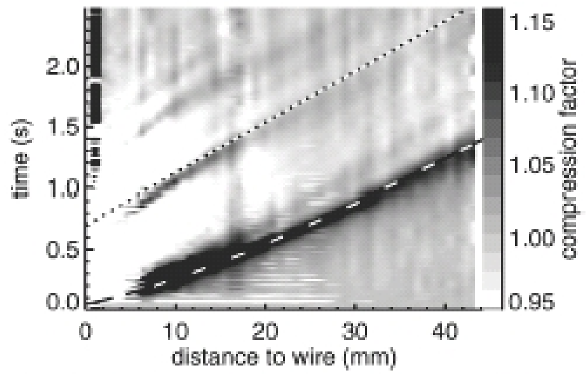


FIG. 2. Compression factor n/n_0 as a function of time and distance to the wire. Darker regions correspond to higher compression. The lower dashed curve is a fit to the trajectory of the soliton. The upper dotted line was drawn above a weak secondary pulse with Mach number $M \simeq 1$. Its slope is determined by the dust lattice wave speed $C_{DL} = 23$ mm/s in the middle of the lattice.

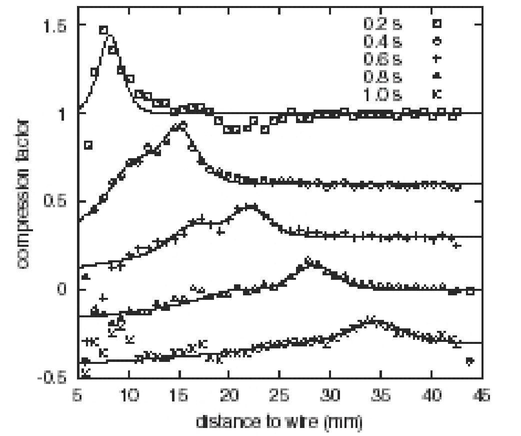


FIG. 3. Compression factor n/n_0 versus distance to the wire at different times. The solid lines show the theoretical fits to the experimental data. Two solitons are present. The fits and experimental points at later times are offset down (by 0.4, 0.7, 1.0, 1.3, respectively).

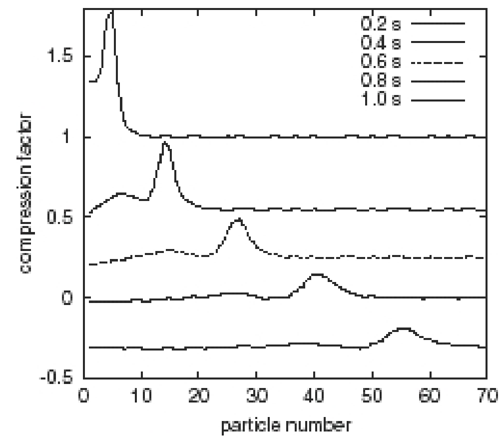


FIG. 5. Compression factor versus particle number for a simulated linear chain model. It describes the formation of two (or more) solitons by a single excitation pulse and qualitatively agrees with the experiment. The curves at later times are offset down (by 0.45, 0.75, 1.0, and 1.3, respectively).

[Samsonov *et al.*, PRL 2002].

Characteristics of the KdV theory

The *Korteweg - deVries* theory, as applied in DP crystals:

– provides a *correct qualitative description* of *compressive* excitations observed in experiments;

– draws benefit from the KdV *"artillery"* of analytical *know-how* obtained in the past: *integrability*, *multi-soliton* solutions, *conservation laws*, ... ;

Characteristics of the KdV theory

The *Korteweg - deVries* theory presented above:

– provides a *correct qualitative description* of *compressive* excitations observed in experiments;

– benefits from the KdV *"artillery"* of analytical know-how obtained throughout the years: *integrability*, *multi-soliton* solutions, *conservation laws*, ... ;

but possesses a few drawbacks:

– *approximate* derivation: (i) propagation velocity v near (longitudinal) sound velocity c_L, (ii) time evolution terms omitted *'by hand'*, (iii) higher order nonlinear contributions omitted;

– *only* accounts for *compressive* solitary excitations (for Debye interactions); nevertheless, the existence of *rarefactive* dust lattice excitations is, *in principle*, *not excluded*.

Longitudinal **soliton formalism** ***(continued)***

Q.: *What if we also kept the next order in nonlinearity ?*

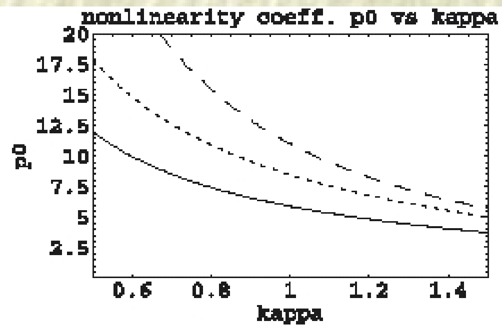
Longitudinal **soliton formalism *(continued)***

Q.: *What if we also kept the next order in nonlinearity ?*

– *"Extended"* description: :

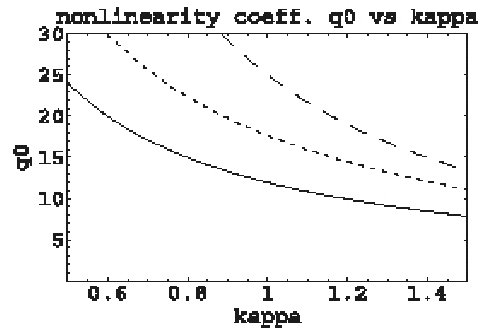
$$\ddot{u} + \nu\, \dot{u} - c_L^2\, u_{xx} - \frac{c_L^2}{12}\, r_0^2\, u_{xxxx} = -\, p_0\, u_x\, u_{xx} + q_0\, (u_x)^2\, u_{xx}$$

$c_L = \omega_{L,0}\, r_0$; $\quad \omega_{L,0},\; p_0 \sim -U'''(r)$ and $q_0 \sim U''''(r)$ (cf. above).



(b)

Fig. 4. (a) The nonlinearity coefficient p_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.



(b)

Fig. 5. (a) The nonlinearity coefficient q_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.

Rq.: q_0 is *not* negligible, compared to p_0! (instead, $q_0 \approx 2p_0$ practically!)

Longitudinal **soliton formalism *(continued)***

Q.: *What if we also kept the next order in nonlinearity ?*

– *"Extended"* description: :

$$\ddot{u} + \nu \dot{u} - c_L^2 u_{xx} - \frac{c_L^2}{12} r_0^2 u_{xxxx} = -p_0 u_x u_{xx} + q_0 (u_x)^2 u_{xx}$$

$c_L = \omega_{L,0} r_0$; $\omega_{L,0}$, p_0 and q_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), and defining the *relative displacement* $w = u_\zeta$, one has

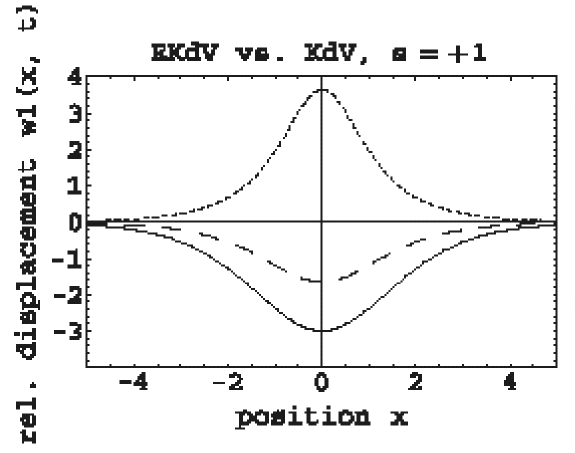
$$w_\tau - a w w_\zeta + \hat{a} w^2 w_\zeta + b w_{\zeta\zeta\zeta} = 0 \quad (2)$$

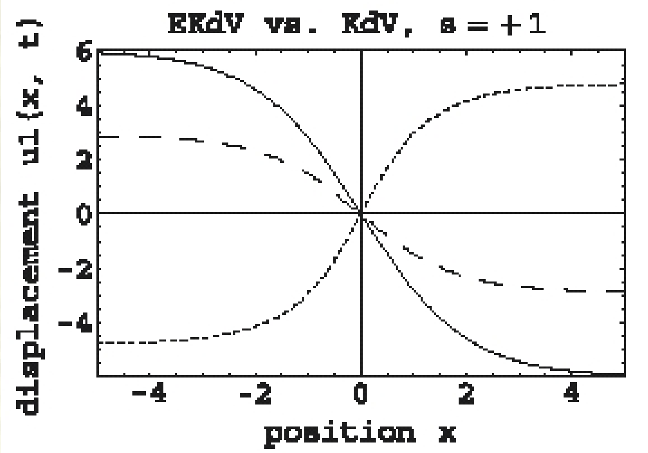
(for $\nu = 0$); $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$; $\hat{a} = q_0/(2c_L) > 0$.

Characteristics of the EKdV theory

The *extended Korteweg - deVries* Equation:

– accounts for *both compressive and rarefactive* excitations;





(*horizontal grain displacement* $u(x,t)$)

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– is previously widely studied, in literature;

Still, ...

– It was derived under the *assumption*: $v \approx c_L$.

One more alternative: the Boussinesq theory

The *Generalized Boussinesq* (Bq) Equation (for $w = u_x$):

$$\ddot{w} - c_L^2 \, w_{xx} = \frac{c_L^2 r_0^2}{12} \, w_{xxxx} - \frac{p_0}{2} \, (w^2)_{xx} + \frac{q_0}{2} \, (w^3)_{xx}$$

– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– has been widely studied in literature;

and, ...

One more alternative: the Boussinesq theory

The *Generalized* *Boussinesq* (Bq) Equation (for $w = u_x$):

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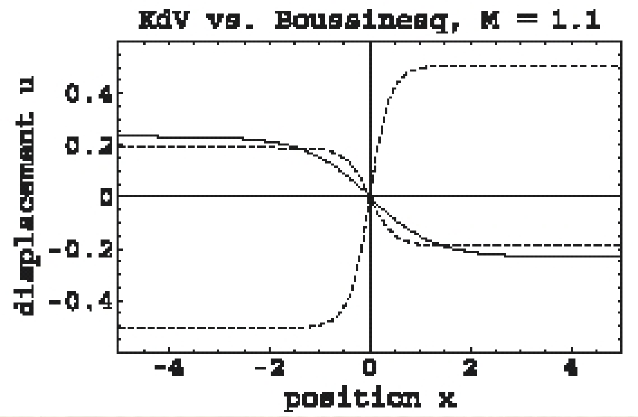
– predicts *both compressive and rarefactive* excitations;

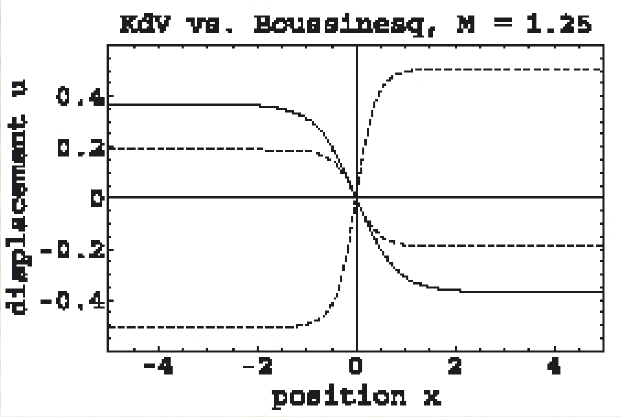
– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– has been widely studied in literature;

and, ...

– relaxes the velocity assumption, i.e. is valid $\forall \quad v > c_L$.





(end of Part 4b)

5. Transverse Discrete Breathers - DB

- Recall the eq. of motion in the *transverse* direction:

$$\frac{d^2u_n}{dt^2} + \omega_{T,0}^2\,(\,u_{n+1} + \,u_{n-1} - 2\,u_n) + \omega_g^2\,u_n + \alpha\,u_n^2 + \beta\,u_n^3 = 0$$

- 1d DP crystals are intrinsically *highly discrete* lattice configurations: $\epsilon = \omega_0^2/\omega_g^2 \simeq 0.016$ ([Misawa et al., PRL 2001]); $\epsilon \simeq 0.181$ ([Liu et al., PRL 2003]).

- Damping may be neglected (for low plasma density and/or pressure): $\nu/\omega_g \simeq 0.00154$ ([Misawa et al., PRL 2001]).

- One may seek *discrete breather* solutions (localized modes):

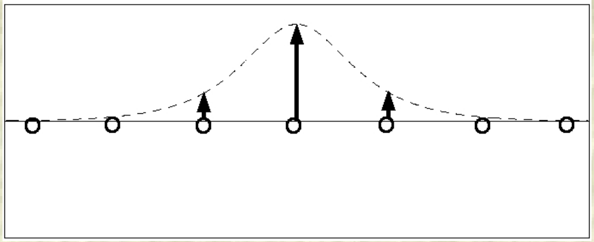
$$u_n(t) = \sum_{m=-\infty}^{\infty} A_n(m)\exp(im\omega t)$$

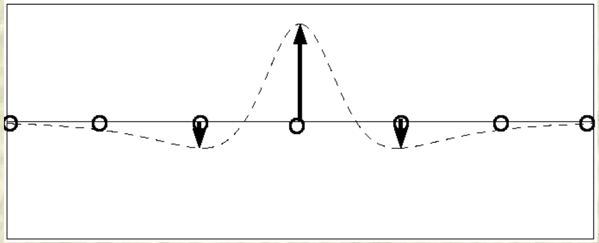
5. Transverse Discrete Breathers - DB *(cont.)*

– The well-known *non - resonance condition* is recovered:

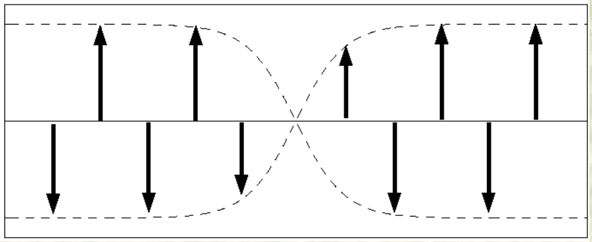
$$m\omega_B \neq \omega_T(k) \qquad \forall k,\, m = 1, 2, \ldots$$

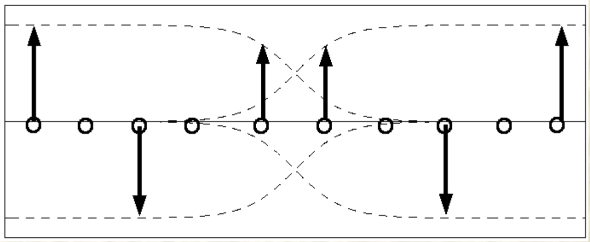
– One obtains the *bright-type* DB solutions (localized pulses):





– Also: *dark-type* excitations (holes; *Kivshar dark* modes):





– Existence and stability criteria still need to be examined: current work with V. Basios (ULB) and V. Koukouloyannis (AUTh).

– Similar modes may be sought in the *longitudinal* direction.

Conclusions

We have seen that:

– *Dusty (Complex) plasmas* provide an excellent test-bed for physical theories;

– *Observations are possible at the kinetic level*: Unique possibility for data processing in view of visualization /quantification of physical quantities e.g. mean values (velocity moments), phase space profile, entropy, phase transitions, Fourier analysis, ...

– *Technology for experiment*: *cheap* and *readily available*;

– Link to *Statistical Mechanics, Solid State Physics, Nonlinear Science* ...

– Challenging physical problems in Space and on Earth.

Conclusions (*continued*): focusing upon DP crystals

– *Energy localization* via *modulational instability* leading to the formation of *envelope excitations* is possible in both *transverse* and *longitudinal* directions ;

– Solitary waves can be efficiently modeled by existing soliton theories (e.g. KdV, EKdV, MKdV; more accurately: Bq, EBq) ;

– Compressive *and* rarefactive excitations are predicted ;

– *Discrete Breather*-type localized modes exist (study further);

– *Urge for experimental confirmation*;

– Future directions: include *dissipation* (dust-neutral friction, ion drag); *particle-wake effects*; *mode coupling* effects; ... *(Realism!)*

– Fertile soil for future studies: still *a lot to be done!...*

Thank You !

Ioannis Kourakis

Acknowledgments: Sergej Flach (MPIPKS, Dresden).

Collaboration: Padma Kant Shukla & Bengt Eliasson (R.U.B.), V. Basios (U.L.B., Brussels), V. Koukouloyannis (AUTh, Greece), T. Bountis (Patras, Greece).

Material from:

I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004);
idem, *Phys. Plasmas*, **11**, 3665 (2004).
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idem, *European Phys. J. D*, **29**, 247 (2004).

Available at: `www.tp4.rub.de/∼ioannis`

ioannis@tp4.rub.de

Appendix I: Solutions of the NLSE
Localized envelope excitations 1: *bright solitons*

- The NLSE accepts various soliton solutions: $\psi = \rho\, e^{i\Theta}$; the *total* wavepacket is then: $u \approx \epsilon\, \rho\, \cos(kx - \omega t + \Theta)$ where the *amplitude* ρ and *phase correction* Θ depend on ζ, τ.

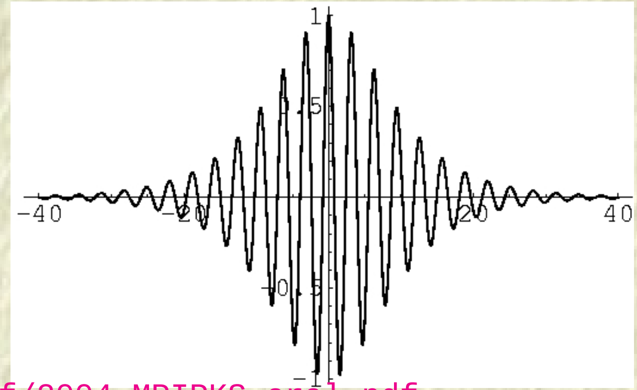
- Bright–type envelope soliton (pulse):

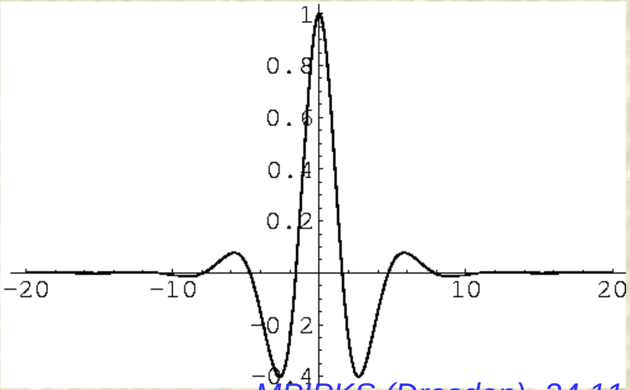
$$\rho = \rho_0\, sech\left(\frac{X - u_e\, T}{L}\right), \quad \Theta = \frac{1}{2P}\left[u_e\, X + (\Omega - \frac{1}{2}u_e^2)T\right]. \quad (3)$$

$$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$$

$(X_0 = 0)$

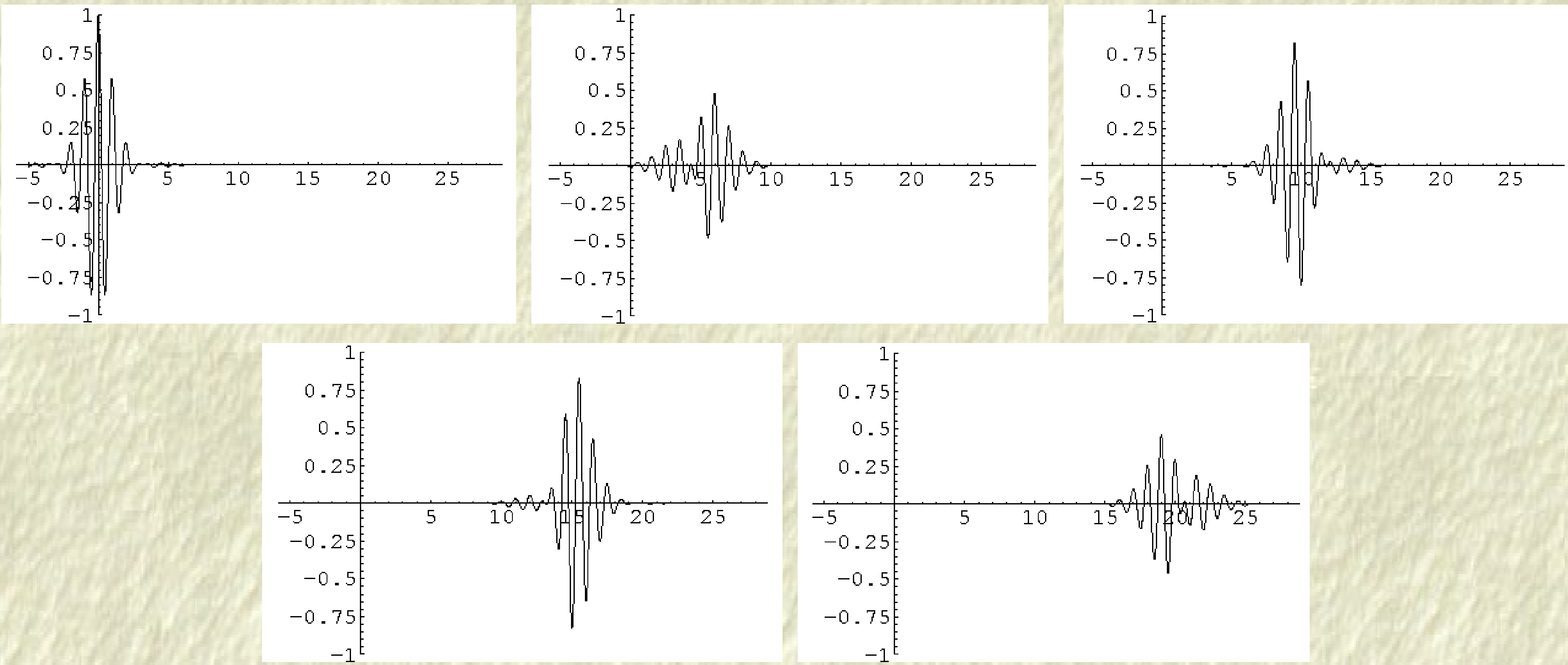
$(\Theta_0 = 0)$





Propagation of a bright envelope soliton (pulse)

This *envelope modulated wavepacket* is essentially *a propagating (and oscillating) localized pulse*, confining the *carrier wave*:



Localized envelope excitations 2: ***dark/grey solitons***

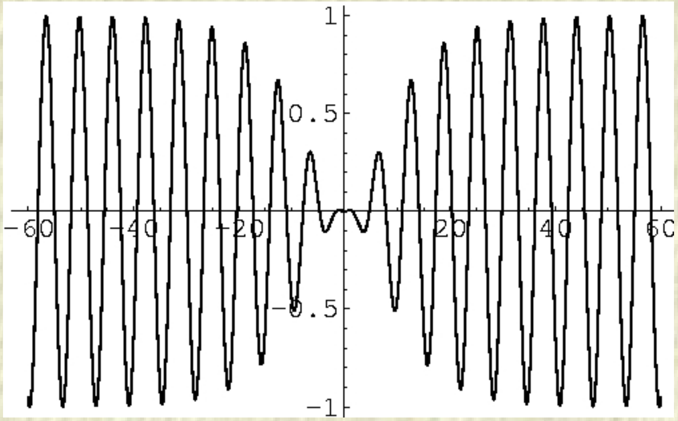
- Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{X - u_e T}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{X - u_e T}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[u_e X - \left(\frac{1}{2}u_e^2 - 2PQ\rho_1^2\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{\rho_1}} \qquad (X_0 = 0) \qquad (4)$$

This is a *propagating localized* *hole* (zero density void):



dark/grey solitons (continued...)

❑ Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - d^2\, sech^2\left(\frac{X - u_e T}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

$$L'' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{d\rho_2}} \qquad (5)$$

This is a propagating *(non zero-density)* void: