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Electrostatic wave propagation in dusty plasmas

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www.tp4.rub.de/∼ioannis/conf/2004-oral-VolosGR.pdf

Outline

A. Introduction

(i) *Dusty Plasma (DP)*: a rapid overview of notions and ideas

(ii) Occurrence of DP: Space, laboratory and fusion plasmas

B. A physical paradigm: Dust ion–acoustic waves (DIAWs)

C. Focus issue: Dust acoustic waves (DAWs)

(i) Linear features

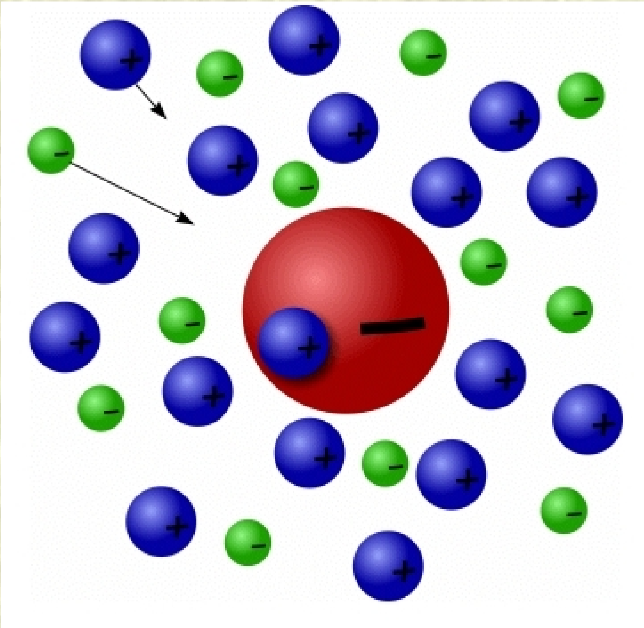
(ii) The mechanism of *wave amplitude modulation (AM)*

(iii) *Envelope excitations*

D. Dust-lattice waves (DLW): $\longrightarrow$ poster

E. Conclusions

A. Intro. (i) DP – Dusty Plasmas (or *Complex Plasmas*): definition and characteristics



❑ Ingredients:

– *electrons* e^- (charge $-e$, mass m_e),

– *ions* i^+ (charge $+Z_i e$, mass m_i), *and*

– charged micro-particles ≡ *dust grains* d (most often d^-):

charge $Q = \pm Z_d e \sim \pm(10^3 - 10^4)\, e$,

mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,

radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu$m.

Origin: Where does the dust come from?

- *Space:* cosmic debris (silicates, graphite, amorphous carbon), comet dust, man-made pollution (Shuttle exhaust, satellite remnants), ...

- *Atmosphere:* extraterrestrial dust (meteorites): $\geq 2 \cdot 10^4$ tons a year (!)(*), atmospheric pollution, chemical aerosols, ...

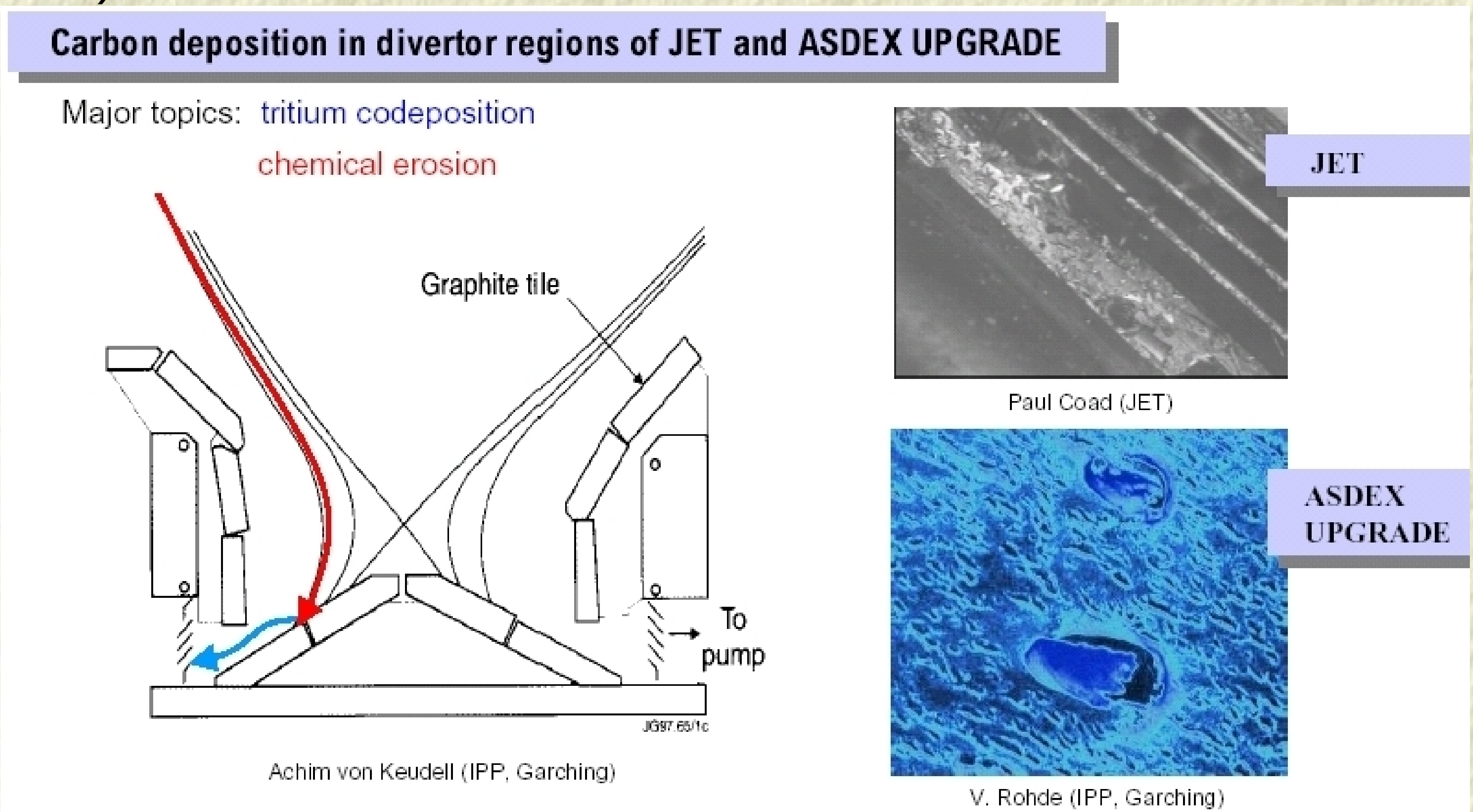
- *Laboratory:* (man-injected) melamine–formaldehyde particulates (**), ...

- *Fusion reactors:* plasma-surface interaction, carbonaceous particulates resulting from wall erosion-created debris (graphite, Carbon Fiber Composites, ...)

Sources: [P. K. Shukla & A. Mamun 2002], (*) [DeAngelis 1992], (**) [G. E. Morfill *et al.* 1998]

Dust in fusion machines (e.g. Tokamaks)

Observation of dust near the tokamak walls (bottom, *divertor* area):

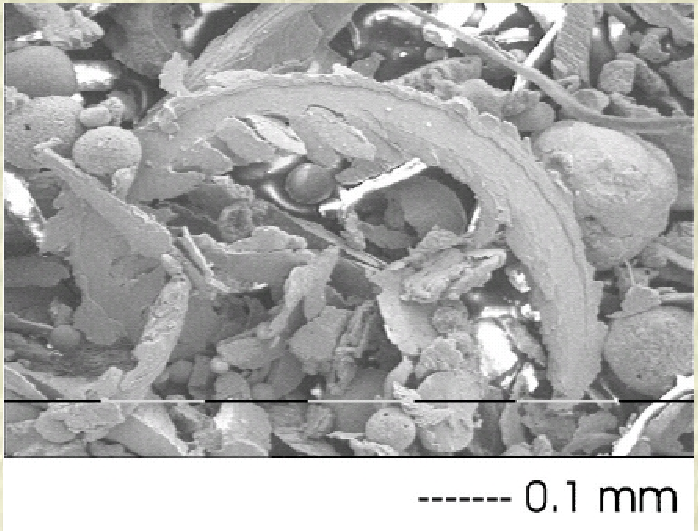


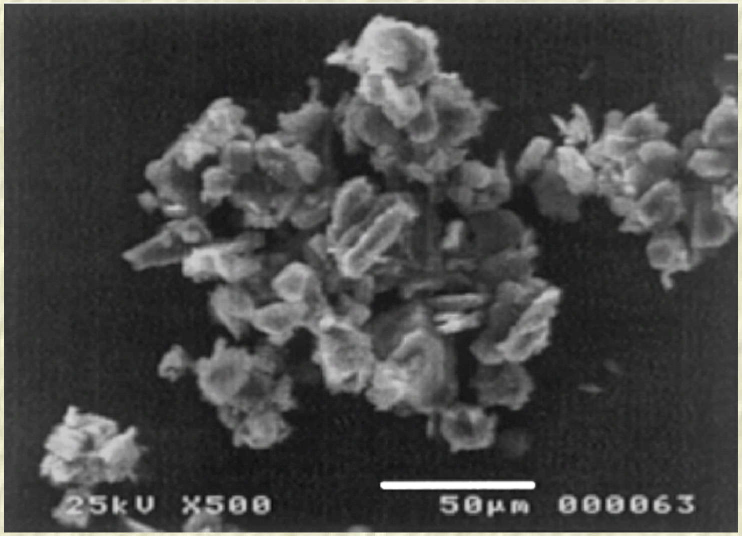
(after K. MATYASH, Max Planck Institüt - Univ. Greifswald).

Occurrence of dust (+ DP) in fusion devices

- Post-operation collection of DP near the reactor walls (*divertor* area);
- Dust reported in: JIPPT-IIU 1997, TEXTOR 1999, JET 1999, TORE-SUPRA 2001, TFTR 2001, ASDEX-UPGRADE 2003 …
- Material: Carbon (graphite composites), tungsten, berellium, ...

  [Rubel *et al.* Nucl.Fus. 41, 1087 (2001)]





Issues of importance, regarding dust in fusion devices :

- *Fact*: Dust grains in Tokamaks may be *charged* and *ferromagnetic*; interact with E/M fields;

- *Fact*: Impact on reactor operation $\longrightarrow$ *disruptions*, ... $\longrightarrow$ experimental study at JIPPT-IIU [Narihara *et al.*, 1997]; *also* ...

- *Fact*: Possible impact on *safety*: carbon dust absorbs radioactive tritium $^{1}_{3}\mathrm{H}$ [Rubel 2001], [Federici *et al.* 2000, 2001]; *thus* ...

- *Fact*: Impurity production is related to *Tritium inventory*;

- *Phenomenology*: possible interaction with wave modes e.g. *ELMs* [Tsytovich-Winter 1998, Rubel *et al.* 2001, Federici *et al.* 2001]; impurity inwards drift reported in JET SOL [J. P. Coad et al. 1999];

- *Aim*: Understanding (and control of) the dynamics of dust may be a major issue involved in ITER design [Federici *et al.* 2000, 2001].

Some unique features of *the Physics of* Dusty Plasmas:

- Complex plasmas are *overall charge neutral*; most (sometimes *all*!) of the negative charge resides on the microparticles;

- The microparticles can be *dynamically dominant*: mass density $\approx 10^2$ times higher than the neutral gas density and $\approx 10^6$ times higher than the ion density !

- Studies in *slow motion* are possible due to high M i.e. *low Q/M ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);

- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);

- Dust charge ($Q \neq \mathrm{const.}$) is now a dynamical variable, associated to a new *collisionless damping* mechanism;

(...continued) More "heretical" features are:

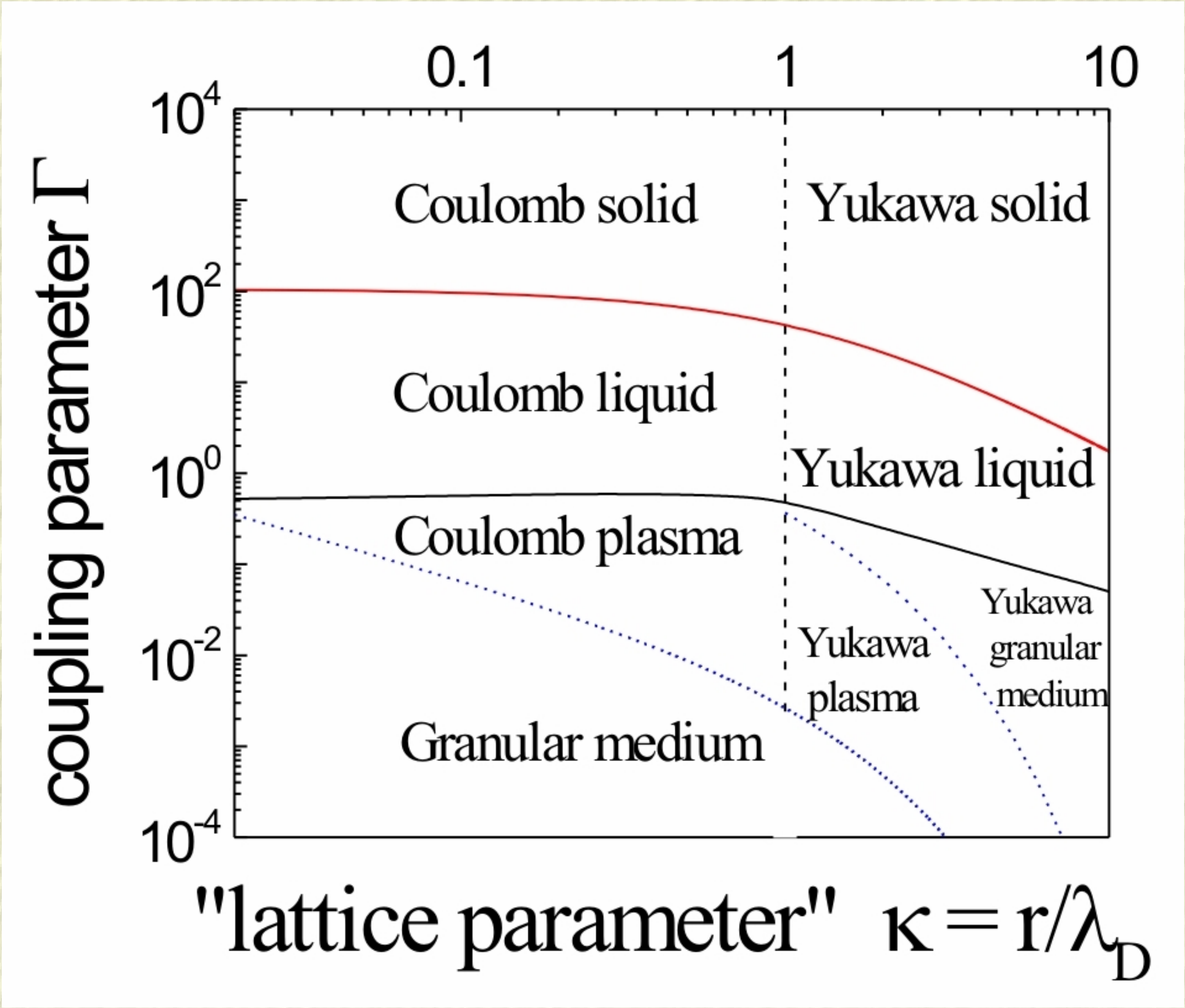
- Important *gravitational* (compared to the *electrostatic*) interaction effects; gravito-plasma physics; gravito-electrodynamics; Jeans-type (gravitational) plasma instabilities etc. [Verheest PPCF 41 A445, 1999]

- Complex plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [Ikezi 1986]) states, depending on the value of the *effective coupling (plasma) parameter* Γ;

$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \sim \frac{Q^2}{r\,T} e^{-r/\lambda_D}$$

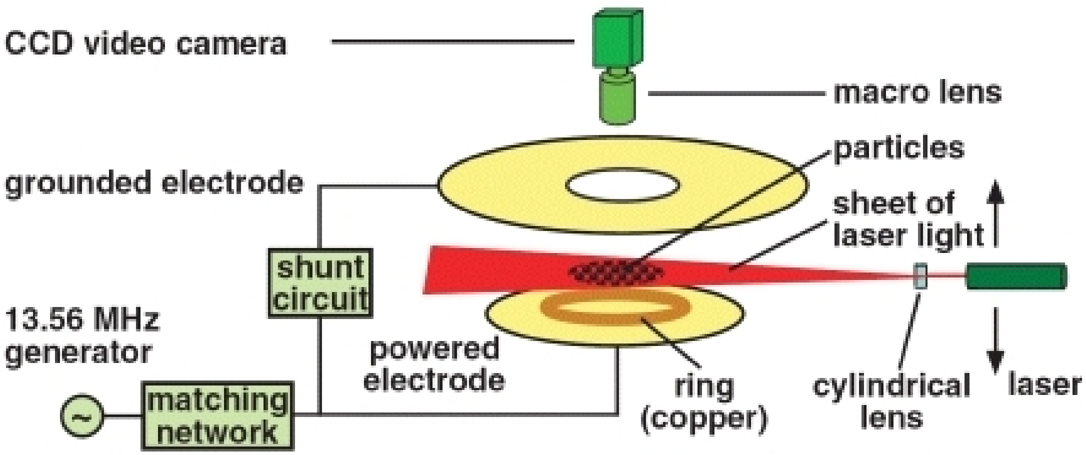
(r: inter-particle distance, T: temperature, λ_D: Debye length).

Phase diagram of (complex) charged matter

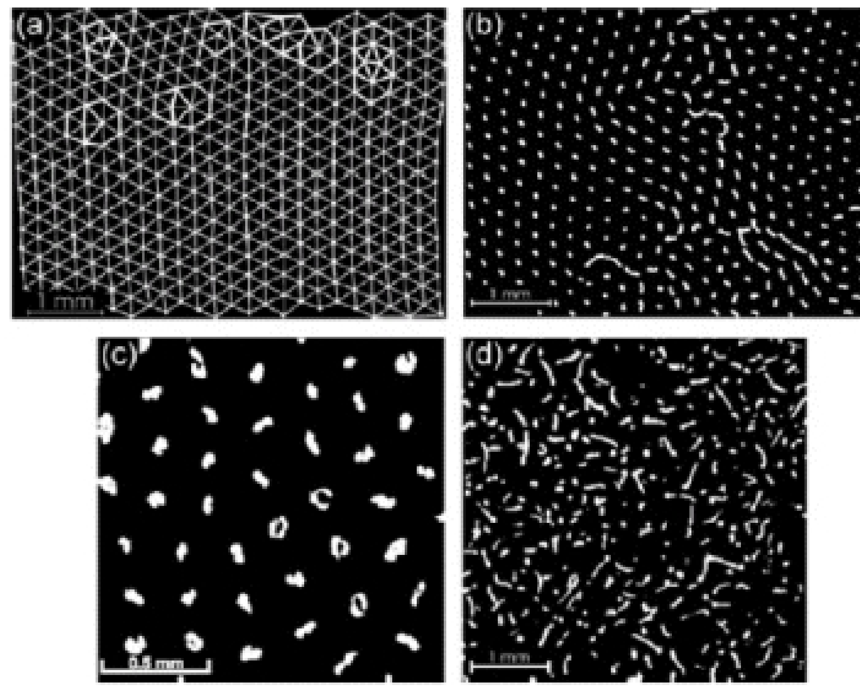


(after G. E. MORFILL, Max Planck Institüt - CIPS).

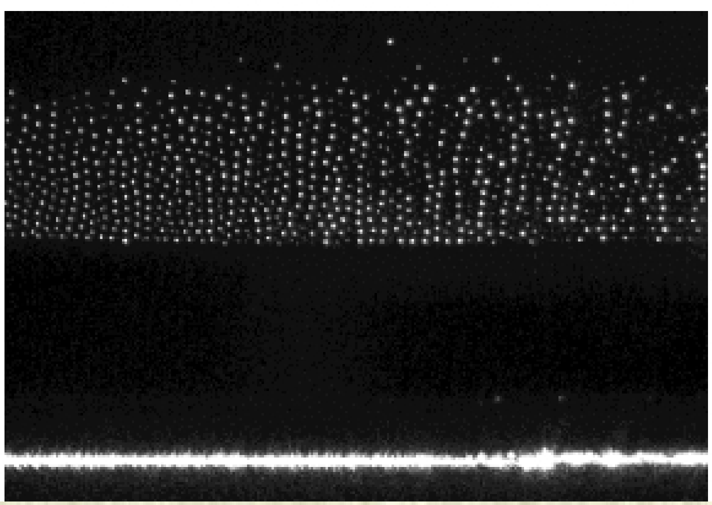
Dust laboratory experiments on Earth:



top view

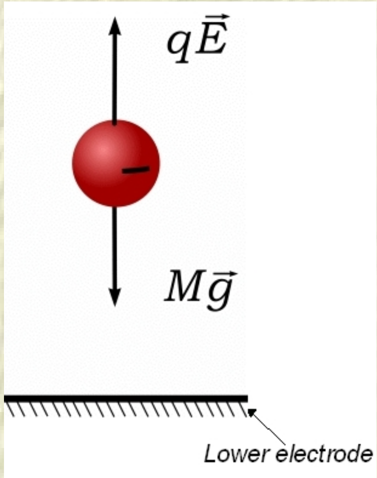


side view



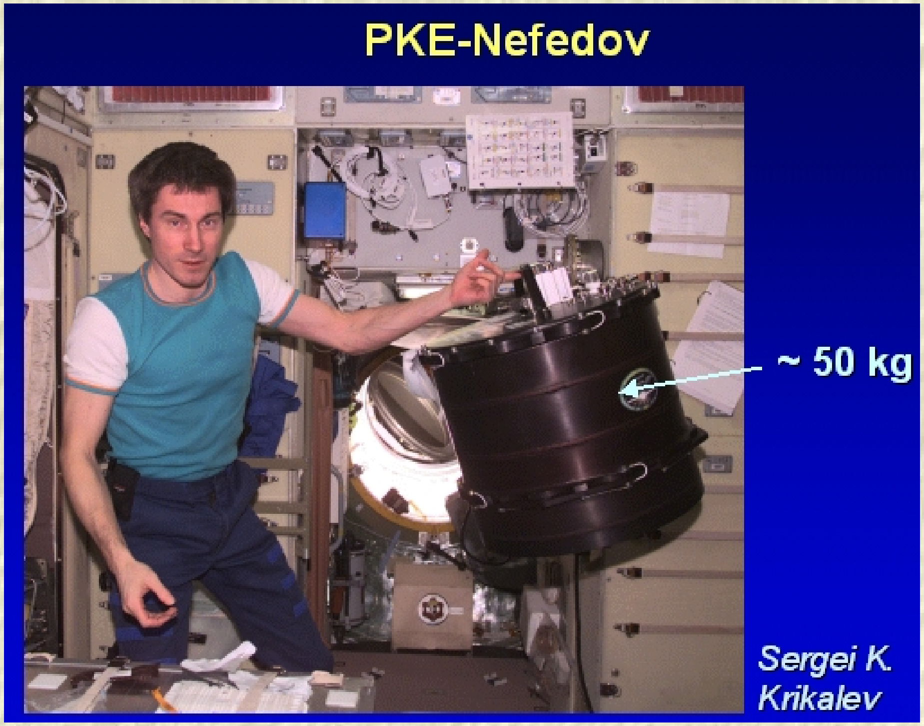
G E Morfill et al,
Plasma Phys. Control. Fusion **44** (2002) B263

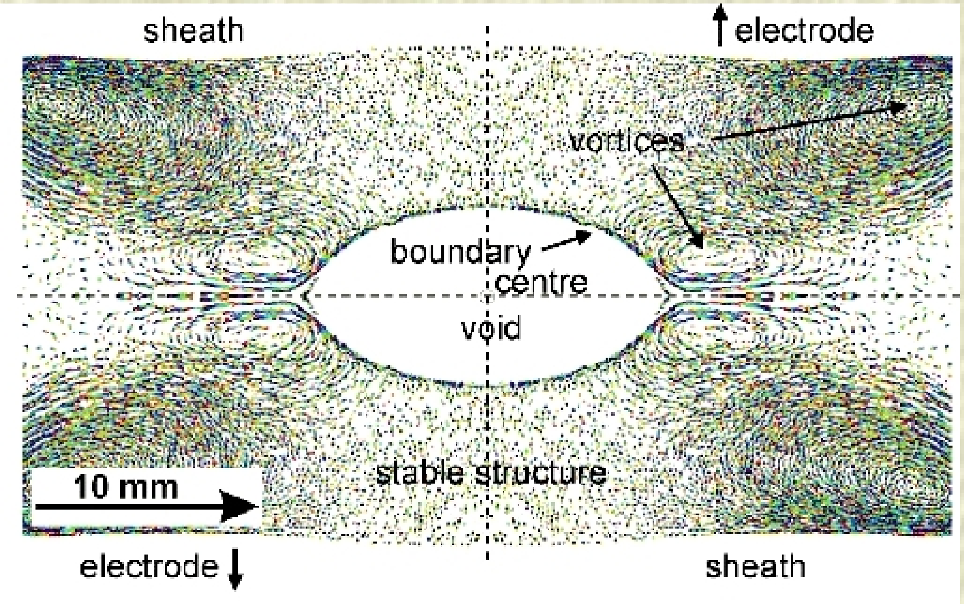
Earth experiments are subject to gravity:



levitation in strong sheath electric field

thus ....: Dust experiments in ISS (International Space Station)





void formed in the middle of discharge under microgravity conditions

(Online data from: Max Planck Institüt - CIPS).

B. Dust ion–acoustic waves (DIAW)

A pedagogical paradigm: consider *Ion - Acoustic Waves* (IAWs):

Ion density n_i (*continuity*) equation:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \, \mathbf{u}_i) = 0 \qquad (1)$$

Mean velocity $\mathbf{u}_i$ equation:

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = -\frac{q_i}{m_i} \nabla \phi - \frac{1}{m_i n_i} \nabla p_i \qquad (2)$$

Pressure p_i equation:

$$\frac{\partial p_i}{\partial t} + \mathbf{u}_i \cdot \nabla p_i = -\gamma \, p_i \, \nabla \cdot \mathbf{u}_i \qquad (3)$$

($\gamma = (f+2)/f = c_P/c_V$: ratio of specific heats e.g. $\gamma = 3$ for 1d, $\gamma = 2$ for 2d, etc.)

The potential ϕ obeys *Poisson's* eq.:

$$\nabla^2 \phi = -4\pi \sum_{\alpha = e,i,d} q_\alpha \, n_\alpha = 4\pi \left(n_{e,0} \, e^{e\phi/k_B T_e} - q_i \, n_i - q_d \, n_{d,0} \right) \qquad (4)$$

Dust ion–acoustic waves *(cont.)*

– Dispersion relation (cf. textbooks):

$$\omega \approx c_{diaw} k \qquad (\text{for} \, \lambda \gg \lambda_{\mathrm{D}})$$

where both $v_{ph} = \omega/k$ and $v_g = \partial\omega/\partial k$ are given by

$$c_{diaw} = \omega_{p,i}\lambda_{D,e} = \left(\frac{n_{i,0}}{n_{e,0}}\right)^{1/2} \left(\frac{k_B T_e}{m_i}\right)^{1/2}$$

– Now, consider the neutrality condition at equilibrium ($Z_i = 1$):

$$n_{i,0}\, q_i - n_{e,0}\, e + n_{d,0}\, q_d = 0 \qquad \Rightarrow \qquad \frac{n_{i,0}}{n_{e,0}} = 1 - s Z_d \frac{n_{d,0}}{n_{e,0}}$$

where $q_d = s Z_d e$; $s = \mathrm{sgn} q_d = q_d/|q_d| = \pm 1$;

– Result: *significant phase velocity modification* of IAWs

$\rightarrow$ DIAWs !!! ($\rightarrow$ *See poster*).

C. Dust–acoustic waves (DAW)

(Theoretical prediction: [RAO *et al.* 1990]; experimental confirmation: [BARKAN *et al.* 1995]).

(i) (Linear) characteristics of DAWs

– Dust is the dynamical element; e & i practically in Maxwellian equilibrium:

$$n_e \approx n_{e,0}\, e^{e\phi/k_B T_e}\,, \qquad n_i \approx n_{i,0}\, e^{-Z_i e\phi/k_B T_i}$$

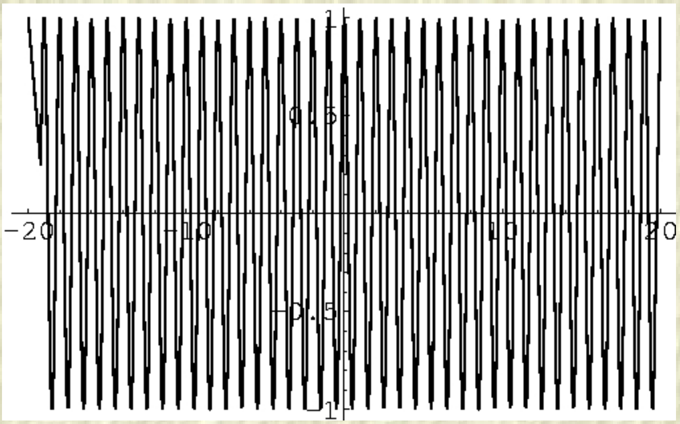
– *Extremely* low frequency waves: $\omega \approx c_{daw} k$ (for $\lambda \gg \lambda_D$) where

$$c_{daw} \sim \left(\frac{n_{d,0}}{n_{i,0}}\right)^{1/2} \left(\frac{k_B T_i}{m_d}\right)^{1/2}$$

(Typically: $\omega \sim 10\,\mathrm{Hz}$; $\lambda \sim 0.5\,\mathrm{mm}$; $c_{daw} \sim 5 - 10\,\mathrm{cm/s}$);

Intermezzo: The mechanism of *wave amplitude modulation*

The *amplitude* of a harmonic wave may vary in space and time:



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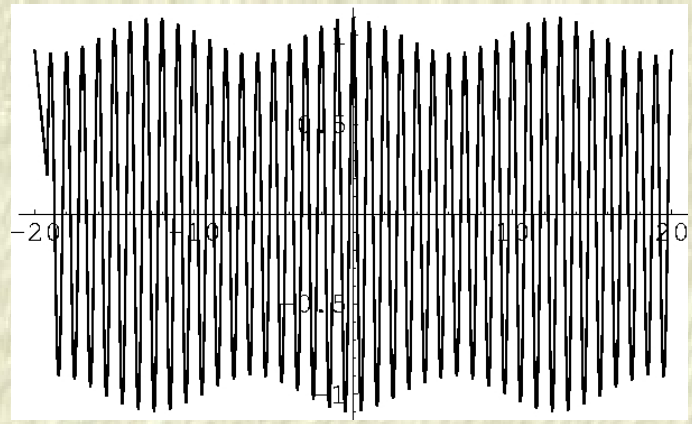
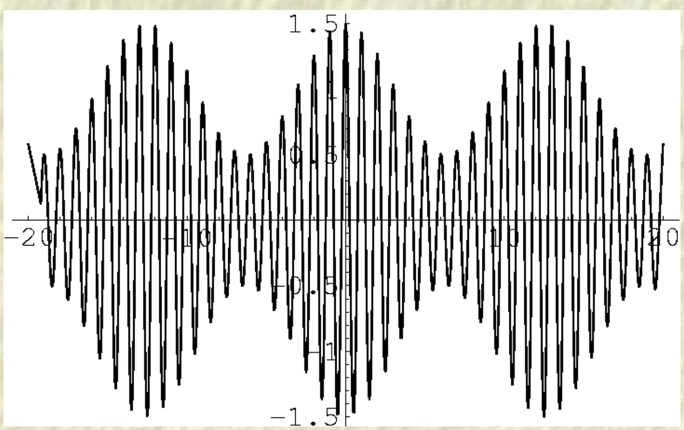
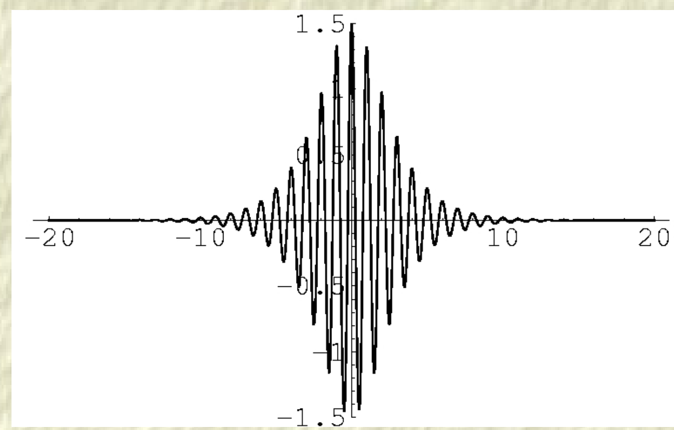


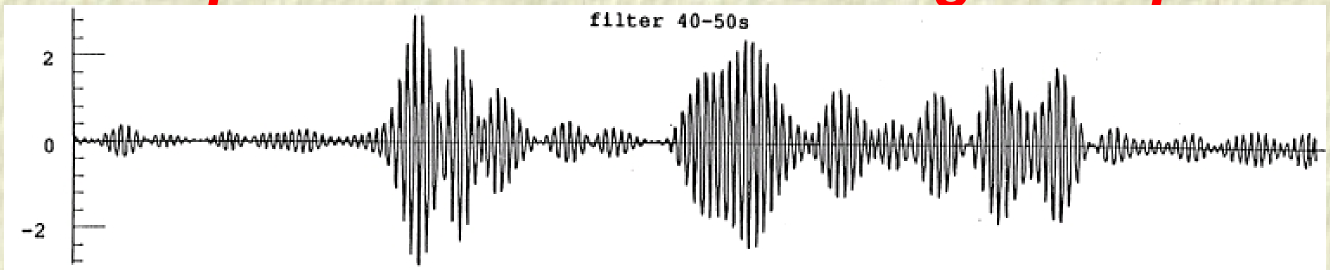
This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* or formation of *envelope solitons*:

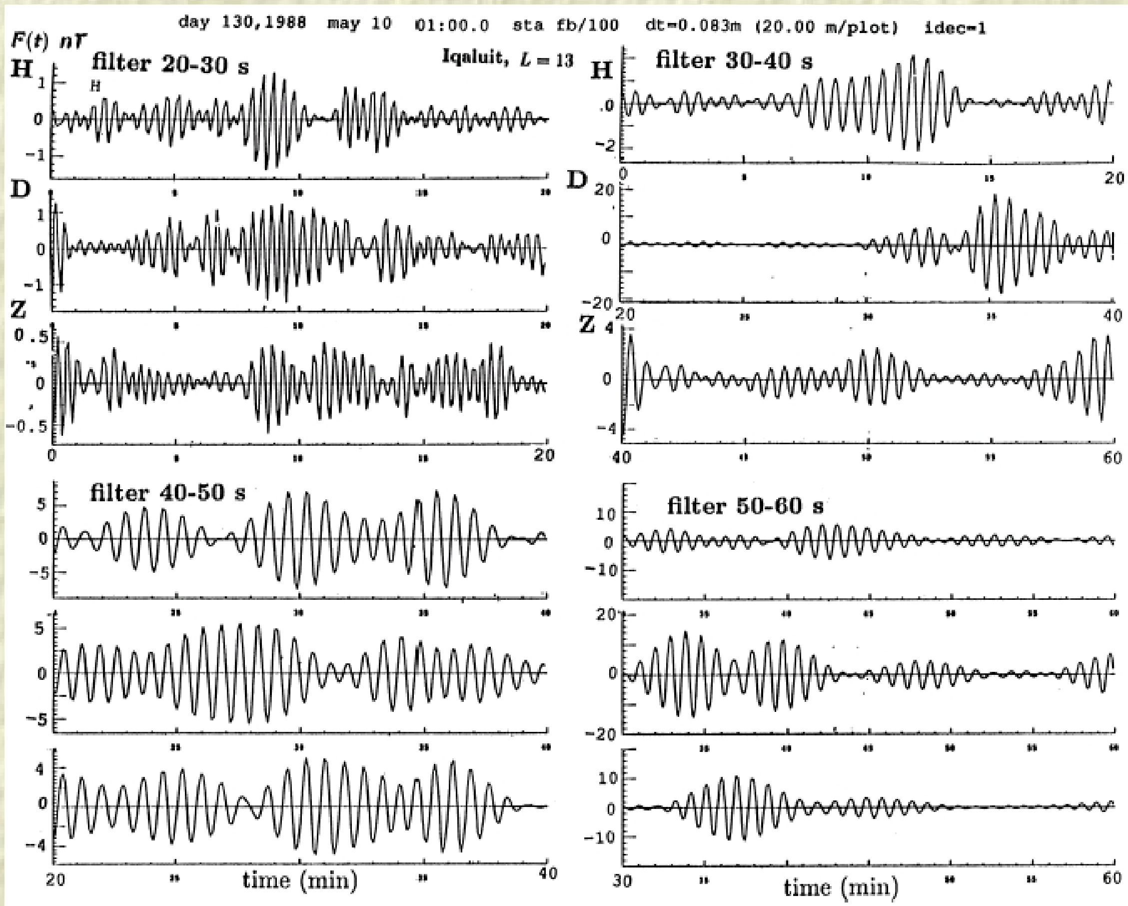


⟶ ?



Envelope structures in the magnetosphere





(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

A dust–fluid model for DAWs

Density n_d (*continuity*) equation:

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \, \mathbf{u}_d) = 0 \tag{5}$$

Mean velocity $\mathbf{u}_d$ equation:

$$\frac{\partial \mathbf{u}_d}{\partial t} + \mathbf{u}_d \cdot \nabla \mathbf{u}_d = -\frac{q_d}{m_d} \nabla \phi - \frac{1}{m_d n_d} \nabla p_d \tag{6}$$

Pressure p_d equation: [(*) *Warm* vs. *Cold* Fluid model]

$$\frac{\partial p_d}{\partial t} + \mathbf{u}_d \cdot \nabla p_d = -\gamma \, p_d \nabla \cdot \mathbf{u}_d \tag{7}$$

($\gamma = (f+2)/f = c_P/c_V$: ratio of specific heats e.g. $\gamma = 3$ for 1d, $\gamma = 2$ for 2d, etc.)

The potential ϕ obeys *Poisson's* eq.:

$$\nabla^2 \phi = -4\pi \sum_{\alpha=e,i,d} q_\alpha \, n_\alpha = 4\pi \, (n_e \, e - q_i \, n_i - q_d \, n_d) \tag{8}$$

(ii) Amplitude modulation – reductive perturbation theory

– 1st step. Define *multiple scales* (*fast* and *slow*) i.e. (in 2d)

$$X_0 = x\,, \quad X_1 = \epsilon\, x\,, \quad X_2 = \epsilon^2\, x\,, \quad \ldots$$
$$Y_0 = y\,, \quad Y_1 = \epsilon\, y\,, \quad Y_2 = \epsilon^2\, y\,, \quad \ldots$$
$$T_0 = t\,, \quad T_1 = \epsilon\, t\,, \quad T_2 = \epsilon^2\, t\,, \quad \ldots \qquad (9)$$

– 2nd step. Expand near equilibrium:

$$n_d \approx n_{d,0} + \epsilon\, n_{d,1} + \epsilon^2\, n_{d,2} + \ldots$$
$$\mathbf{u_d} \approx \mathbf{0} + \epsilon\, \mathbf{u_{d,1}} + \epsilon^2\, \mathbf{u_{d,2}} + \ldots$$
$$p_d \approx p_{d,0} + \epsilon\, p_{d,1} + \epsilon^2\, p_{d,2} + \ldots$$
$$\phi \approx 0 + \epsilon\, \phi_1 + \epsilon^2\, \phi_2 + \ldots$$

($\epsilon \ll 1$ is a smallness parameter).

Reductive perturbation technique *(continued)*

– 3rd step. Project on Fourier space, i.e. consider $\forall m = 1, 2, ...$

$$S_m = \sum_{l=-m}^{m} \hat{S}_l^{(m)} e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \hat{S}_0^{(m)} + 2\sum_{l=1}^{m} \hat{S}_l^{(m)} \cos l(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

for $S_m = n_m, \{u_{x,m}, u_{y,m}\}, p_m, \phi_m,$ i.e. *essentially*:

$$n_1 = n_0^{(1)} + \tilde{n}_1^{(1)} \cos\theta\,, \qquad n_2 = n_0^{(2)} + \tilde{n}_1^{(2)} \cos\theta + \tilde{n}_2^{(2)} \cos 2\theta, \text{ etc.}$$

– 4rth step. *Oblique modulation* assumption:
the slow amplitudes $\hat{\phi}_l^{(m)}$, etc. vary *only along* the x-axis:

$$\hat{S}_l^{(m)} = \hat{S}_l^{(m)}(X_j, T_j)\,, \qquad j = 1, 2, ...$$

while the fast carrier phase $\theta = \mathbf{k}\cdot\mathbf{r} - \omega t$ is now:

$$k_x\, x + k_y\, y - \omega t = k\, r \cos\alpha - \omega t.$$

First-order solution ($\sim \epsilon^1$)

Substituting and isolating terms in $m = 1$, we obtain:

- The *dispersion relation* $\omega = \omega(k)$:

$$\omega^2 = \omega_{p,d}^2 \frac{k^2}{k^2 + k_{D,eff}^2} + \gamma\, v_{th}^2\, k^2 \qquad (10)$$

with $k_{D,eff} = \lambda_{D,eff}^{-1} = (\lambda_{D,e}^{-2} + \lambda_{D,i}^{-2})^{1/2}$, where

$$\omega_{p,d} = \left(\frac{4\pi n_{d,0} q_d^2}{m_d} \right)^{1/2}, \; \lambda_{D,\alpha} = \left(\frac{k_B T_\alpha}{4\pi n_{\alpha,0} q_\alpha^2} \right)^{1/2}, \; (\alpha = e, i)$$

- The *solution(s)* for the 1st–harmonic amplitudes (e.g. $\propto \phi_1^{(1)}$):

$$n_1^{(1)} = s \frac{1 + k^2}{\beta} \phi_1^{(1)} = \frac{1}{\gamma} p_1^{(1)} = \frac{k}{\omega \, \cos\theta} u_{1,x}^{(1)} = \frac{k}{\omega \, \sin\theta} u_{1,y}^{(1)} \qquad (11)$$

Second-order solution ($\sim \epsilon^2$)

- From $m = 2, l = 1$, we obtain the relation:

$$\frac{\partial \psi}{\partial T_1} + v_g \frac{\partial \psi}{\partial X_1} = 0 \qquad (12)$$

where
– $\psi = \phi_1^{(1)}$ is the potential correction ($\sim \epsilon^1$);
– $v_g = \frac{\partial \omega(k)}{\partial k_x}$ is the group velocity along $\hat{x}$;
– the wave's envelope satisfies: $\psi = \psi(\epsilon(x - v_g t)) \equiv \psi(\zeta)$.

- The solution, up to $\sim \epsilon^2$, is of the form:

$$\phi \approx \epsilon \psi \cos\theta + \epsilon^2 (\phi_0^{(2)} + \phi_1^{(2)} \cos\theta + \phi_2^{(2)} \cos 2\theta) + \mathcal{O}(\epsilon^3) \, ,$$

etc. (similar expressions for n_d, u_x, u_y, p_d).

Third-order solution ($\sim \epsilon^3$)

- Compatibility equation (from $m = 3, l = 1$), in the form of:

$$i\,\frac{\partial\psi}{\partial\tau} + P\,\frac{\partial^2\psi}{\partial\zeta^2} + Q\,|\psi|^2\,\psi = 0\,. \tag{13}$$

  i.e. a *Nonlinear Schrödinger–type Equation* (NLSE) .

- Variables: $\zeta = \epsilon(x - v_g t)$ and $\tau = \epsilon^2\,t$;

- *Dispersion coefficient* P:

$$P = \frac{1}{2}\frac{\partial^2\omega}{\partial k_x^2} = \frac{1}{2}\left[\omega''(k)\cos^2\alpha + \omega'(k)\,\frac{\sin^2\alpha}{k}\right]; \tag{14}$$

- *Nonlinearity coefficient* Q: ...
  A *(lengthy!)* function of k, angle α and T_e, T_i, ... $\rightarrow$ *poster/text*.

Modulational (in)stability analysis

- The NLSE admits the *harmonic wave solution*:

$$\psi = \hat{\psi}\, e^{iQ|\hat{\psi}|^2\tau} + \text{c.c.}$$

- *Perturb* the amplitude by setting: $\hat{\psi} = \hat{\psi}_0 + \epsilon\, \hat{\psi}_{1,0} \cos(\tilde{k}\zeta - \tilde{\omega}\tau)$

- We obtain the *(perturbation)* dispersion relation:

$$\tilde{\omega}^2 = P^2\, \tilde{k}^2 \left(\tilde{k}^2 - 2\frac{Q}{P} |\hat{\psi}_{1,0}|^2 \right). \qquad (15)$$

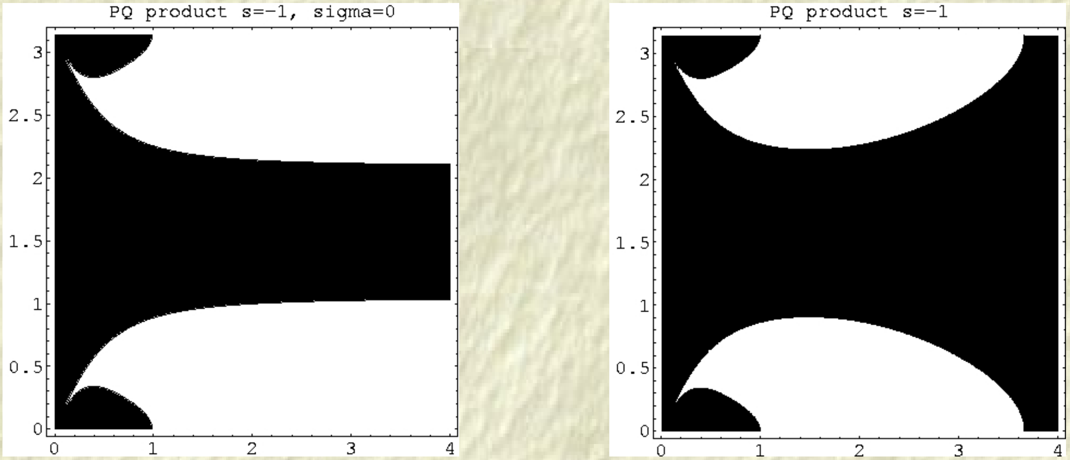
- If $PQ < 0$: the amplitude ψ is *stable* to external perturbations;

- If $PQ > 0$: the amplitude ψ is *unstable* for $\tilde{k} < \sqrt{2\frac{Q}{P}} |\psi_{1,0}|$;

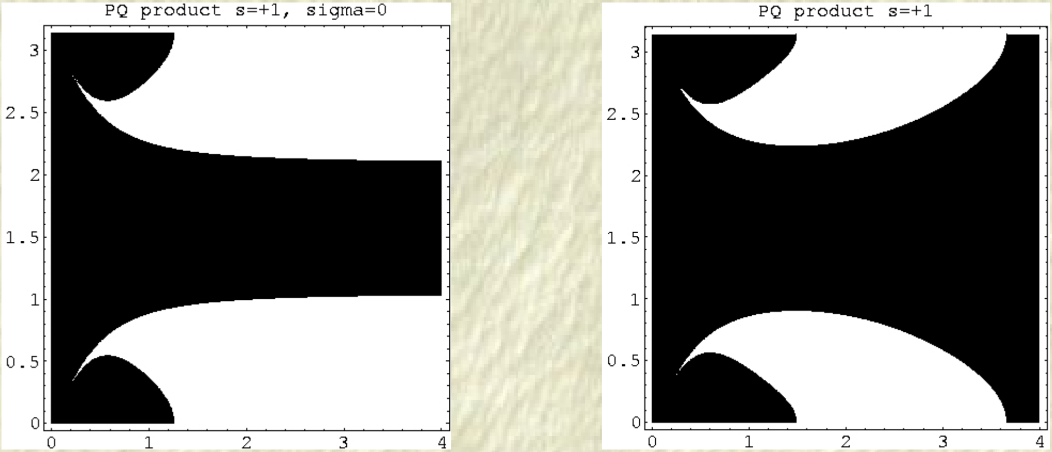
Stability profile: Angle α versus wavenumber k

Typical values: $Z_d/Z_i \approx 10^3$, $T_e/T_i \approx 10$, $n_{d,0}/n_{i,0} \approx 10^{-3}$, $\gamma = 2$.

– *Negative dust*: $s = -1$; cold ($\sigma = 0$) vs. warm ($\sigma \neq 0$) fluid:



– The same plot for *positive dust* ($s = +1$):



(iii) Localized envelope excitations (solitons)

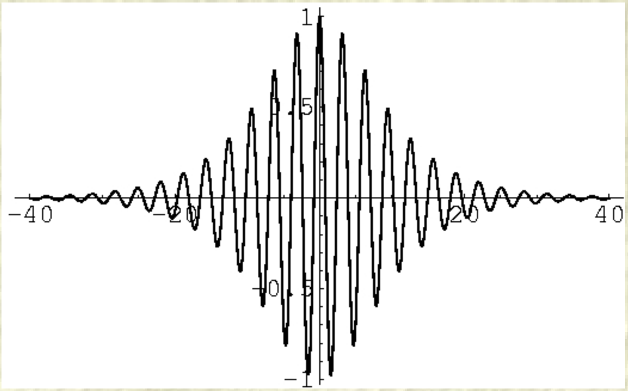
- The NLSE accepts various soliton solutions: $\psi = \rho\, e^{i\Theta}$;
  the *total* electric potential is then: $\phi \approx \epsilon\, \rho\, \cos(\mathbf{kr} - \omega t + \Theta)$
  where the amplitude ρ and phase correction Θ depend on ζ, τ.

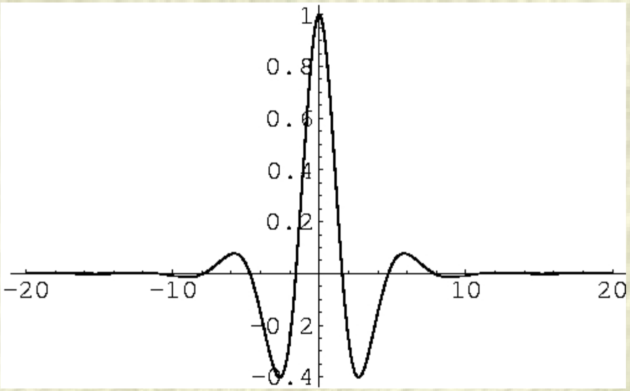
- Bright–type envelope soliton (pulse):

$$\rho = \rho_0\, sech\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta\, - (\Omega + \frac{1}{2}v^2)\tau\right]. \quad (16)$$

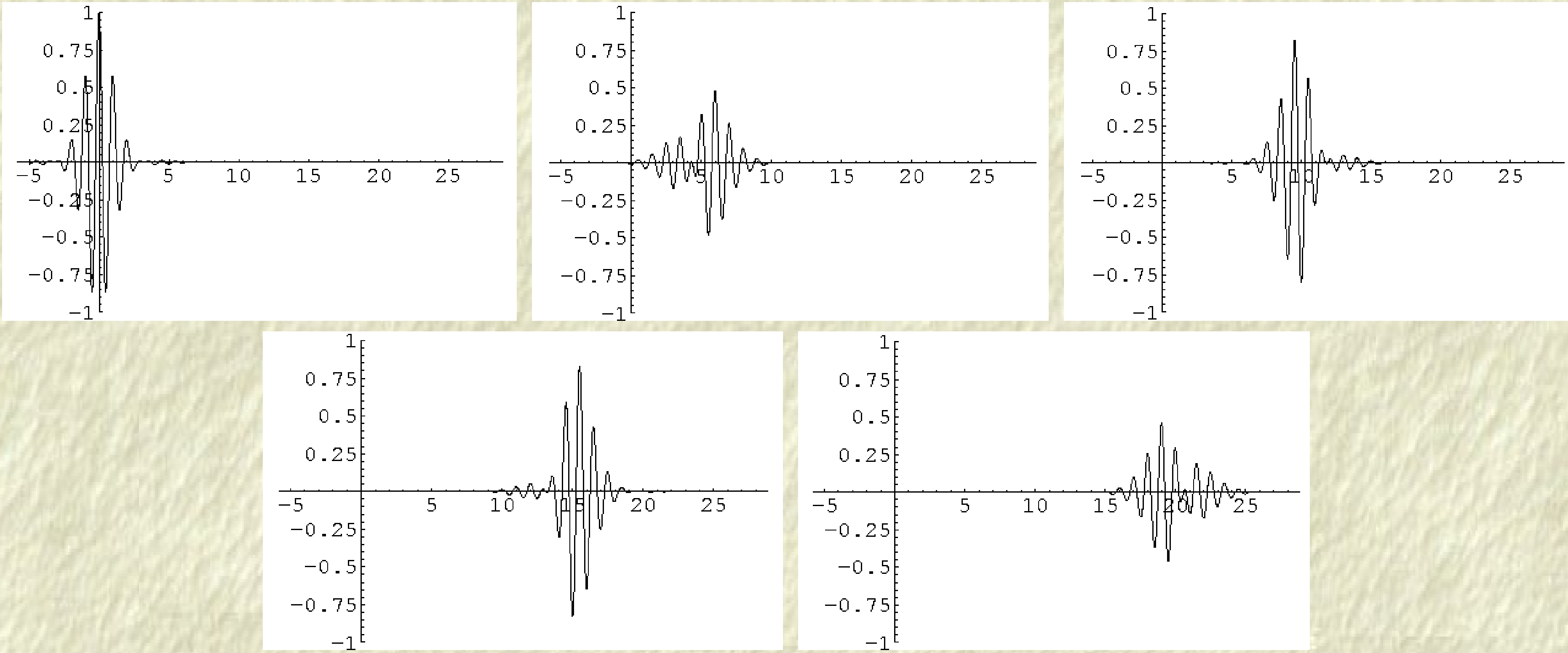
$$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$$

This is a propagating (and *oscillating*) localized pulse:

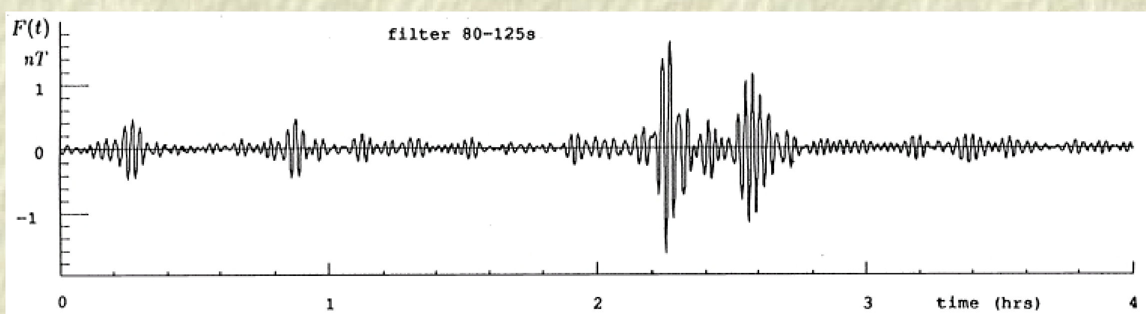




Propagation of a bright envelope soliton (pulse)



Cf. electrostatic plasma wave data from satellite observations:



(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

Localized envelope excitations *(part 2)*

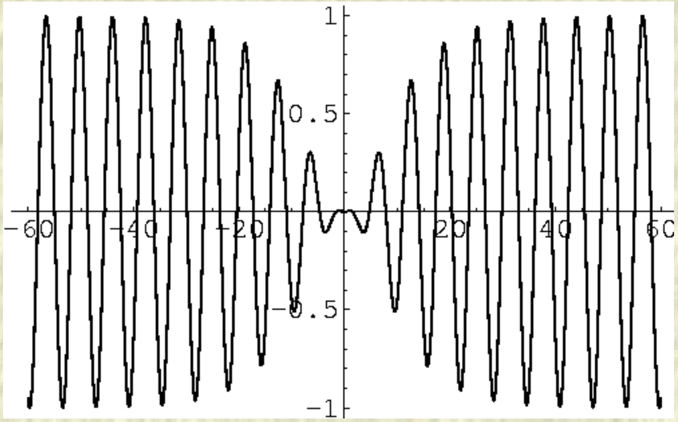
- Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{\rho_1}} \qquad (17)$$

This is a *propagating localized hole* (zero density void):



Localized envelope excitations *(part 3)*

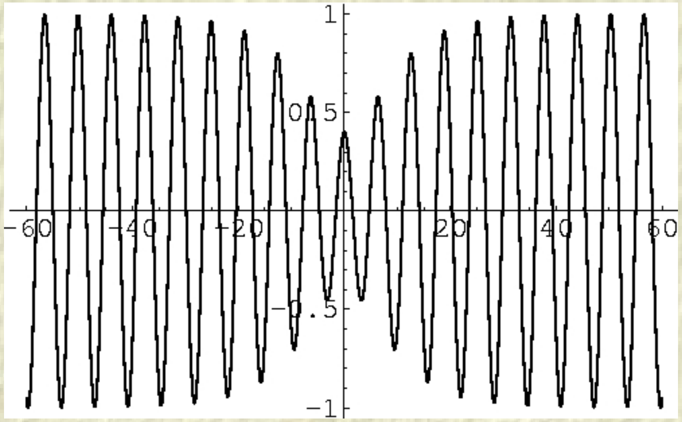
- Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - a^2\, sech^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

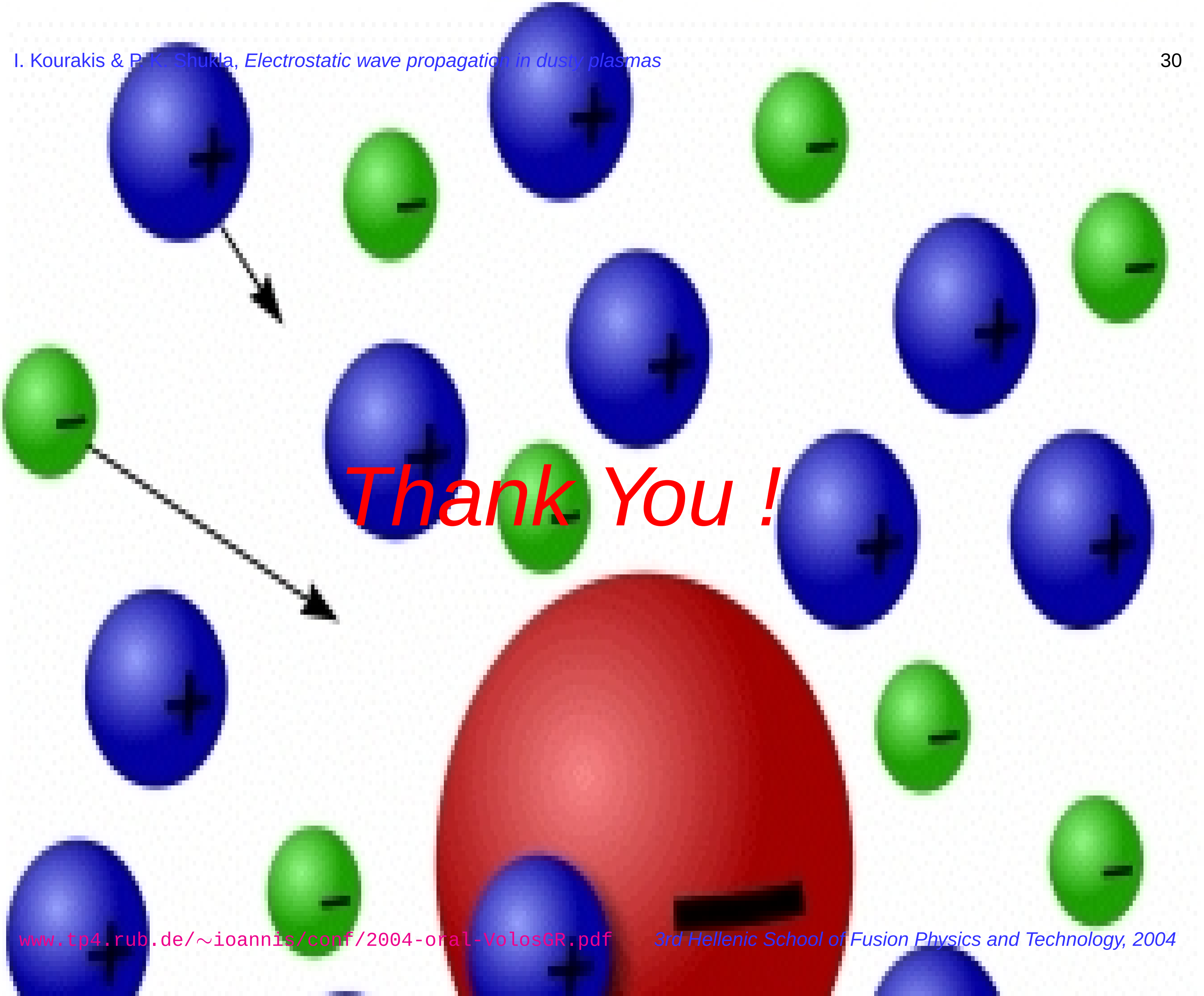
$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{a\rho_2} \qquad (18)$$

This is a propagating *(non zero-density)* void:



E. Conclusions

- *Dusty* (or *Complex*) *plasmas* occur widely in space, laboratory and controlled nuclear fusion environments;
- *In fusion devices*, harnessing the dynamics of dust is among the important issues in large device (e.g. ITER) design;
- Electrostatic waves propagating in dusty plasmas are characterized by *modulational instability*; this is an intrinsic feature of nonlinear dynamics, which may lead to the ...
- ... formation of *envelope localized structures* (envelope solitons), and thus ...
- ... explain *energy localization* phenomena observed in DP
$\longrightarrow$ *a small step towards understanding Complex Plasmas*.



Thank You !