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Detection & Controlability Aspects of Intrinsic Localized Modes in Dusty Plasma Crystals

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(Dated: 30-31 May 2005, Paris, France)

ABSTRACT

Intrinsic Localized Modes (ILM) (or Discrete Breathers, DB) are localized oscillatory modes known to occur in atomic or molecular chains characterized by coupling and/or on-site potential nonlinearity. Quasi-crystals of charged mesoscopic dust grains (dust lattices), which have been observed since hardly a decade ago, are an exciting paradigm of such a nonlinear chain. In gas-discharge experiments, these crystals are subject to forces due to an externally imposed electric and/or magnetic field(s), which balance(s) gravity at the levitated equilibrium position, as well as to electrostatic inter-grain interaction forces. Despite the profound role of nonlinearity, which may be due to inter-grain coupling, mode-coupling and to the sheath environment, the elucidation of the nonlinear mechanisms governing dust crystals is still in a preliminary stage. This study is devoted to an investigation, from very first principles, of the existence of discrete localized modes in dust layers. Relying on a set of evolution equation for transverse charged grain displacements, we examine the conditions for the existence and sustainance of discrete localized modes and discuss the dependence of their characteristics on intrinsic plasma parameters. In addition, the possibility of ILM stabilisation via an external force is discussed.

PACS numbers: 52.27.Lw, 52.35.Fp, 52.25.Vy
Keywords: Crystallization of Dust in Plasmas, Nonlinear Control, Discrete Breathers, Intrinsic Localized Modes, Complex Plasmas.

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I. INTRODUCTION

A variety of linear and nonlinear collective effects are known to occur in a dust-contaminated plasma (*dusty plasma*, DP) [1] and relative theoretical research has received new impulse, since roughly a decade ago, thanks to laboratory and space dusty plasma observations. An issue of particular importance in DP research is the formation of strongly coupled DP crystals by highly charged dust grains, typically in the sheath region above a horizontal negatively biased electrode in experiments [1, 2]. Low-frequency oscillations are known to occur [2] in these mesoscopic dust grain quasi-lattices in the longitudinal ($\sim \hat{x}$, in-plane, acoustic mode), horizontal transverse ($\sim \hat{y}$, in-plane, shear mode) and vertical transverse ($\sim \hat{z}$, off-plane, optic-like mode) directions.

Various types of localized (nonlinear) excitations are known from solid state physics to exist in periodic chains (*lattices*)of interacting particles, in addition to propagating vibrations (*phonons*), due to a mutual balance between the intrinsic nonlinearity of the medium and dispersion. Such structures, usually investigated in a continuum approximation (i.e. assuming that the typical spatial variation scale far exceeds the typical lattice scale, e.g. the lattice constant r_0), include non-topological *solitons* (pulses), *kinks* (i.e. shocks or dislocations) and localized modulated envelope structures (*envelope solitons*), and generic nonlinear theories have been developed in order to investigate their relevance in different physical contexts [3]. In addition to these (continuum) theories, which deliberately sacrifice discreteness in the altar of analytical tractability, attention has been paid since more than a decade ago to highly localized (either stationary or propagating) vibrating structures [e.g. *discrete breathers* (DBs), also widely referred to as *intrinsic localized modes* (ILMs)], which owe their very existence to the lattice discreteness itself. Thanks to a few pioneering works [4–8] and a number of studies which followed, many aspects involved in the spontaneous formation, mobility and interaction of DBs are now elucidated, both theoretically and experimentally; see in Refs. [9–11] for a review (also see Refs. [12, 13], with reference to this study).

Despite the fact that nonlinearity is an inherent feature of the dust crystal dynamics (either due to inter-grain electrostatic interactions, to the sheath environment, which is intrinsically anharmonic, or to coupling between different degrees of freedom), our knowledge of nonlinear mechanisms related to dust lattice modes still appears to be in a rather preliminary stage today. Small amplitude localized longitudinal excitations (described by a Boussinesq equation for the longitudinal grain displacement u, or a Korteweg-deVries equation for the density $\partial u/\partial x$) were considered in Refs. [14] and generalized in Ref. [15]. The nonlinear amplitude modulation of longitudinal and transverse (vertical, off-plane) dust lattice waves was recently considered in Refs. [16, 17] and [18, 19] (also see [20]), respectively. In fact, all of these studies rely on a *continuum* description of the dust lattice. On the other hand, the effect of the high discreteness of dust crystals, clearly suggested by experiments [21–24], may play an important role in mechanisms like energy localization, storage and propagation and thus modify the crystal's dynamical response to external excitations (in view of DP application design, e.g.). To the very best of our knowledge, no study has been carried out, from first principles, of the relevance of DB excitations with respect to dust lattice waves, apart from a preliminary investigation (restricted to single-mode transverse dust-breathers), which was recently presented [25]. This text aims in making a first analytical step towards filling this gap, by raising a number of questions which have not been addressed before. This study is neither exhaustive nor complete; it will be complemented by forthcoming work.

II. THE MODEL

We shall consider the vertical (off-plane, $\sim \hat{z}$) grain displacement in a dust crystal (assumed quasi-one-dimensional: identical grains of charge q and mass M are situated at $x_n = n\, r_0,$ where $n = ..., -1, 0, 1, 2, ...$), by taking into account the intrinsic nonlinearity of the sheath electric (and/or magnetic) potential. The in-plane (longitudinal, acoustic, $\sim \hat{x}$ and shear, optical, $\sim \hat{y}$) degrees of freedom are assumed suppressed; this situation is indeed today realized in appropriate experiments, where an electric potential (via a thin wire) [21] or a coherent light (laser) impulse [22–24] is used to trigger transverse dust grain oscillations, while (a) confinement potential(s) ensure(s) the chain's in-plane stability.

A. Equation of motion

The vertical grain displacement obeys an equation in the form [18, 19]

$$\frac{d^2\delta z_n}{dt^2} + \nu \frac{d\delta z_n}{dt} + \omega_0^2\, (\,\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\, \delta z_n + \alpha\, (\delta z_n)^2 + \beta\, (\delta z_n)^3 = 0\,, \qquad (1)$$

where $\delta z_n(t) = z_n(t) - z_0$ denotes the small displacement of the $n-$th grain around the (levitated) equilibrium position z_0, in the transverse ($z-$) direction. The characteristic frequency $\omega_0 = \left[-q\Phi'(r_0)/(M r_0)\right]^{1/2}$ results from the dust grain (electrostatic) interaction potential $\Phi(r)$, e.g. for a Debye-Hückel potential [26]: $\Phi_D(r) = (q/r)\, e^{-r/\lambda_D}$, one has: $\omega_{0,D}^2 = q^2/(Mr_0^3)\,(1 + r_0/\lambda_D)\,\exp(-r_0/\lambda_D)$, where λ_D denotes the effective DP Debye radius [1]. The damping coefficient ν accounts for dissipation due to collisions between dust grains and neutral atoms. The gap frequency ω_g and the nonlinearity coefficients α, β are defined via the overall vertical force:

$$F(z) = F_{e/m} - Mg \approx -M[\omega_g^2 \delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3] + \mathcal{O}[(\delta z_n)^4]\,, \tag{2}$$

which has been expanded around z_0 by formally taking into account the (anharmonicity of the) local form of the sheath electric (follow exactly the definitions in Ref. [18], not reproduced here) and/or magnetic [27] field(s), as well as, possibly, grain charge variation due to charging processes [19]. Recall that the electric/magnetic levitating force(s) $F_{e/m}$ balance(s) gravity at z_0. Notice the difference in structure from the usual nonlinear Klein-Gordon equation used to describe one-dimensional oscillator chains — cf. e.g. Eq. (1) in Ref. [6]: TDLWs (*'phonons'*) in this chain are stable only in the presence of the field force $F_{e/m}$.

For convenience, we may re-scale the time and vertical displacement variables over appropriate quantities, i.e. the characteristic (single grain) oscillation period ω_g^{-1} and the lattice constant r_0, respectively, viz. $t = \omega_g^{-1}\tau$ and $\delta z_n = r_0 q_n$; Eq. (1) is thus expressed as:

$$\frac{d^2 q_n}{d\tau^2} + \epsilon(\, q_{n+1} + \, q_{n-1} - 2\, q_n) + \, q_n + \alpha'\, q_n^2 + \beta'\, q_n^3 = 0\,, \tag{3}$$

where the (dimensionless) damping term, now expressed as $(\nu/\omega_g)dq_n/d\tau \equiv \nu' \dot{q}_n$, will be henceforth omitted in the left-hand side. The coupling parameter $\epsilon = \omega_0^2/\omega_g^2$ measures the strength of the inter-grain interactions (with respect to the single-grain vertical vibrations); this is typically a *small* parameter, in real experiments (see below). The nonlinearity coefficients are now: $\alpha' = \alpha r_0/\omega_g^2$ and $\beta' = \beta r_0^2/\omega_g^2$.

Eq. (3) will be the basis of the analysis that will follow. Note that the primes in α' and β' will henceforth be omitted.

B. The model Hamiltonian

In order to relate our physical problem to existing generic models from solid state physics, it is appropriate to consider the equation of motion (1) as it may be derived from a Hamiltonian function, which here reads:

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m_j} + V(q_j) - \frac{\epsilon}{2}(q_j - q_{j-1})^2 \right]. \tag{4}$$

Here, p_j obviously denotes the (classical) momentum $p_j = M\dot{q}_j$. The substrate potential, related to the sheath plasma environment, is of the form:

$$V(q_j) = \frac{1}{2}\, q_j^2 + \frac{\alpha}{3}\, q_j^3 + \frac{\beta}{4}\, q_j^4\,. \tag{5}$$

The coupling parameter ϵ takes *small* numerical values (cf. below), accounting for the high lattice discreteness anticipated in this study. The minus sign preceding it denotes the inverse dispersive character of (linear excitations propagating in) the system; see the discussion below. Upon setting $\epsilon \to -\epsilon$, the 'traditional' (discretized) *nonlinear Klein-Gordon* model is recovered [28].

It should be noted that both experimental [21] and ab initio (numerical) [29] studies suggest that dust crystals are embedded in an *nonlinear* on-site (sheath) potential V, in the vertical direction, which is (possibly strongly) asymmetric around the origin, i.e. *not* an *even* function of q_j. This implies a finite value of the cubic anharmonicity parameter α, thus invalidating models involving *even* potential forms – e.g. $V(q_j) \sim q_j^2/2 + \beta q_j^4/4$ – in our case.

III. LINEAR WAVES

Retaining only the linear contribution and considering oscillations of the type, $\delta z_n \sim \exp[i\,(knr_0 - \omega t)] + c.c.$ (complex conjuguate) in Eq. (1), one obtains the well known transverse dust lattice (TDL) wave optical-mode-like dispersion relation

$$\omega^2 = \omega_g^2 - 4\omega_0^2 \sin^2\left(\frac{kr_0}{2}\right), \tag{6}$$

i.e.

$$\tilde{\omega}^2 = 1 \, - 4\epsilon \, \sin^2(\tilde{k}/2) \, . \tag{7}$$

See that the wave frequency $\omega \equiv \tilde{\omega}\omega_g$ *decreases* with increasing wavenumber $k = 2\pi/\lambda \equiv \tilde{k}/r_0$ (or decreasing

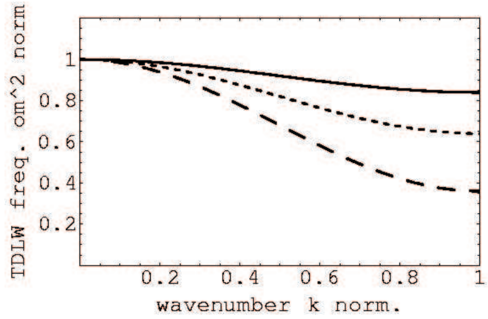

FIG. 1: The dispersion relation of the TDL excitations: frequency ω (normalized over ω_g) versus wavenumber k. The value of ω_0/ω_g ($\sim$ coupling strength) increase from top to bottom. Note that upper (less steep, continuous) curve is more likely to occur in a real (weakly-coupled) DP crystal.

wavelength λ), implying that transverse vibrations propagate as a *backward wave*: the group velocity $v_g = \omega'(k)$ and the phase velocity $\omega_{ph} = \omega/k$ have opposite directions (this behaviour has been observed in recent experiments). The modulational stability profile of these linear waves (depending on the plasma parameters) was investigated in Refs. [18, 19]. Notice the natural *gap frequency* $\omega(k=0) = \omega_g = \omega_{max}$, corresponding to an overall motion of the chain's center of mass, as well as the *cutoff frequency* $\omega_{min} = (\omega_g^2 - 4\omega_0^2)^{1/2} \equiv \omega_g \, (1 - 4\epsilon^2)^{1/2}$ (obtained at the end of the first Brillouin zone $k = \pi/r_0$) which is *absent in the continuum limit*, viz. $\omega^2 \approx \omega_g^2 - \omega_0^2 \, k^2 \, r_0^2$ (for $k \ll r_0^{-1}$); obviously, the study of wave propagation in this ($k \lesssim \pi/r_0$) region invalidates the continuum treatment employed so far in literature. The essential feature of discrete dynamics, to be retained here, is the (narrow) bounded TDLW (*'phonon'*) frequency band, limited in the interval $\omega \in [(\omega_g^2 - 4\omega_0^2)^{1/2}, \omega_g]$; note that one thus naturally obtains the stability constraint: $\omega_0^2/\omega_g^2 = \epsilon < 1/4$ (so that $\omega \in \Re \quad \forall k \in [0, \pi/r_0]$).

We needn't go into further details concerning the linear regime, since it is covered in the literature. We shall, instead, see what happens if the *nonlinear* terms are retained, in this discrete description.

IV. EXISTENCE OF DISCRETE BREATHERS - ANALYSIS

We are interested in the (possibility for the) existence of multi-mode breathers, i.e. localized (discrete) excitations in the form:

$$q_n(\tau) = \sum_{m=-\infty}^{\infty} A_n(m) \exp(im\omega\tau) \, , \tag{8}$$

with $A_n(m) = A_n^*(-m)$ for reality and $|A_n(m)| \to 0$ as $n \to \pm\infty$, for localization.

A. The formalism

Inserting Eq. (8) in the equation of motion (3), one obtains a (numerable) set of algebraic equations in the form:

$$\begin{aligned} A_{n+1}(m) + A_{n-1}(m) + C_m A_n(m) \; &= \; -\frac{\beta}{\epsilon} \sum_{m_1} \sum_{m_2} \sum_{m_3} A_n(m_1) A_n(m_2) A_n(m_3) \\ &\quad - \frac{\alpha}{\epsilon} \sum_{m_4} \sum_{m_5} A_n(m_4) A_n(m_5) \, , \end{aligned} \tag{9}$$

where the dummy indices m_j ($j = 1, 2, ..., 5$) satisfy $m_1 + m_2 + m_3 = m_4 + m_5 = m$; we have defined:

$$C_m = -\left(2 - \frac{1 - m^2\omega^2}{\epsilon}\right) . \tag{10}$$

In order to be more precise and gain in analytical tractability (yet somewhat losing in generality), one may assume that the contribution of higher (for $m \geq 2$) frequency harmonics may be neglected. Eq. (8) then reduces to:

$$q_n(t) \approx 2A_n(1)\cos\omega\tau \, + A_n(0)\,. \tag{11}$$

Note the zeroth-harmonic (mean displacement) term, for $n = 0$, which is due to the cubic term ($\sim \alpha$, above), and should vanish for $\alpha = 0$. The system (9) thus becomes (for $m = 0, 1$):

$$\begin{aligned} A_{n+1}(1) + A_{n-1}(1) + C_1 A_n(1) &= -2\frac{\alpha}{\epsilon}A_n(1)A_n(0) - \frac{\beta}{\epsilon}\left[A_n(1)A_n^2(0) + 3A_n^2(1)A_n(-1)\right] \\ A_{n+1}(0) + A_{n-1}(0) + C_0 A_n(0) &= -2\frac{\alpha}{\epsilon}A_n(1)A_n(-1) \, - 6\frac{\beta}{\epsilon}\,A_n(0)A_n(1)A_n(-1)\,, \end{aligned} \tag{12}$$

i.e., setting $A_n(1) = A_n(-1) = A_n$ and $A_n(0) = B_n$, viz. $q_n(t) = 2A_n\cos\omega\tau \, + B_n$:

$$\begin{aligned} A_{n+1} + A_{n-1} + C_1 A_n &= -2\frac{\alpha}{\epsilon}A_n B_n - \frac{\beta}{\epsilon}\left(A_n B_n^2 + 3A_n^3\right) \\ B_{n+1} + B_{n-1} + C_0 B_n &= -2\frac{\alpha}{\epsilon}A_n^2 \, - 6\frac{\beta}{\epsilon}\,A_n^2 B_n\,. \end{aligned} \tag{13}$$

We see that the amplitudes A_n (B_n) of the first (zeroth) harmonic terms, corresponding to the $n-$th site, will be given by the iterative solution of Eqs. (13) [or, of Eqs. (9), should higher harmonics m be considered]. In specific, one may express (13) as:

$$\begin{aligned} a_{n+1} &= -c_n - C_1 a_n + 2\frac{\alpha}{\epsilon}a_n b_n + \frac{\beta}{\epsilon}\left(a_n b_n^2 + 3a_n^3\right) \equiv f_1(a_n, b_n, c_n, d_n) \\ b_{n+1} &= -d_n - C_0 b_n + 2\frac{\alpha}{\epsilon}a_n^2 \, + 6\frac{\beta}{\epsilon}\,a_n^2 b_n \equiv f_0(a_n, b_n, c_n, d_n) \\ c_{n+1} &= a_n \\ d_{n+1} &= b_n\,, \end{aligned} \tag{14}$$

and then iterate, for a given initial condition $(a_1, b_1, c_1, d_1) = (A_1, B_1, A_0, B_0)$, the map defined by (14).

At this stage, one needs to determine whether the fixed point of the 4-dimensional map (14) [or of the complete 4N-dimensional map corresponding to (9), in general] is hyperbolic, and examine the dimensionality of its stable and unstable manifolds. It is known [12, 13] that the existence of discrete breathers is associated with homoclinic orbits, implying a saddle point at the origin.

Let us now linearize the map (14) near the fixed point $(a_1, b_1, c_1, d_1) = (0, 0, 0, 0) \equiv \mathbf{0}_4$, by setting e.g. $(a_n, b_n, c_n, d_n) = (\xi_1, \xi_2, \xi_3, \xi_4)_n^T \equiv \mathbf{\Xi}_n \in \Re^4$, where $\xi_{j,n} \ll 1$ ($j = 1, ..., 4$). One thus obtains the matrix relation:

$$\mathbf{\Xi}_{n+1} = \mathbf{M}\,\mathbf{\Xi}_n\,, \tag{15}$$

where $\mathbf{M}$ is the matrix:

$$\mathbf{M} = \begin{pmatrix} -C_1 & 0 & -1 & 0 \\ 0 & -C_0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\,. \tag{16}$$

Now, it is a trivial algebraic exercise to show that the characteristic polynomial $p(\lambda) \equiv Det(\mathbf{M} - \lambda\mathbf{I})$ of this matrix may be factorized as:

$$p(\lambda) \;=\; (\lambda^2 + C_0\lambda + 1)\,(\lambda^2 + C_1\lambda + 1) \equiv p_0(\lambda)p_1(\lambda)\,,$$

implying the existence of 4 eigenvalues, say $\lambda_{1,2,3,4}$, such that $p_0(\lambda_{1,2}) = p_0(\lambda_{3,4}) = 0$. One may check that the condition for all eigenvalues to be real and different, hence for $\mathbf{0}_4$ to be a saddle point, amounts to the constraint: $|C_{0,1}| > 2$, i.e. $C_0 \notin [-2, 2]$ *and* $C_1 \notin [-2, 2]$. Recalling that

$$C_1 = (1 - 2\epsilon - \omega^2)/\epsilon\,, \qquad C_0 = (1 - 2\epsilon)/\epsilon, \tag{17}$$

from (10), one finds the (simultaneous) constraints: $1 - 4\epsilon > 0$ and $(1 - \omega^2)(1 - \omega^2 - 4\epsilon) > 0$. One immediately sees that the former (i.e. $\epsilon < 1/4$) corresponds to the linear stability condition mentioned above, while the latter amounts to the requirement that the breather frequency should lie outside the 'phonon band', viz. $\omega^2/\omega_g^2 \notin [1 - 4\epsilon, 1]$.

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It is straightforward to show that in case one considers the complete multi-mode map, defined by Eq. (9), one obtains an analogous factorizable characteristic polynomial for the $4N \times 4N$ matrix $\mathbf{M}$, viz. $p(\lambda) = \prod_m p_m(\lambda)$. The same analysis then leads to the hyperbolicity criterion:

$$|C_m| < 2 \qquad m = 0, 1, 2, ...$$

One thus recovers, in addition to the first of the above constraint ($\epsilon < 1/4$), the condition: $m\omega/\omega_g \notin (1 - 4\epsilon, 1)^{1/2}$ ($\forall m = 0, 1, 2, ...$), which coincides with the – physically meaningful – non-breather-phonon-resonance condition found via different analytical methods [8–10]. We see that the breather frequency, as well as all its multiples (harmonics) should lie outside the allowed linear vibration frequency band, otherwise the breather may enter in resonance with the linear TDLW (*'phonon'*) dispersion curve, resulting in its being decomposed into a superposition of linear excitations (and hence de-localized).

B. Numerical analysis

At this stage, one is left with task of finding the numerical values of A_n, B_n [cf. (13)] for a given homoclinic orbit; these may then be used as an initial condition, in order to solve the equation (13) numerically, by considering a given number of particles N and harmonic modes m_{max} (viz. $m = 0, 1, 2, ..., m_{max}$). One thus obtains a given set of numerical values for u_n ($n = 1, 2, ..., N$), which constitute the numerical solution for the anticipated breather excitation. The stability of the solution thus obtained, say $\hat{q}_n$, my be checked by directly substituting with $q_n = \hat{q}_n + \xi_n$ (for $n = -N, ..., 0, ..., N$) into the initial equation of motion (3).

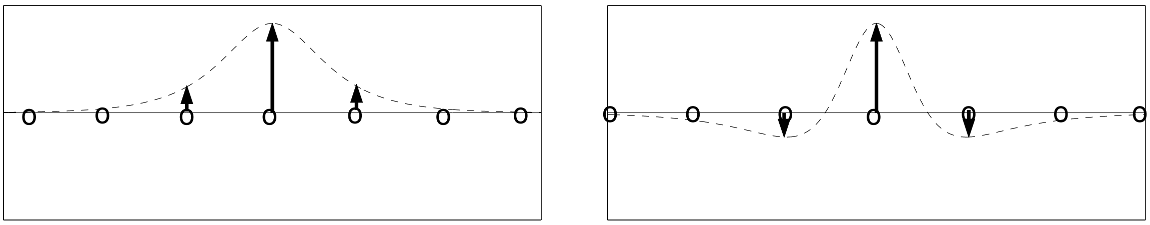

FIG. 2: Localized discrete breather dust lattice excitations; the successive lattice site displacements are depicted at maximum amplitude: (a) odd-parity solution; (b) even-parity solution.

This details of such a numerical scheme is being elaborated, and the detailed results will be reported in an extended paper, in preparation.

V. BREATHER CONTROL.

The stability of a breather excitation may be *controlled* via external feedback, as known from one-dimensional discrete solid chains [33], [34]. The method consists in using the knowledge of a reference state (unstable breather), say $\delta z_n^{(0)} = \hat{z}_n(t)$, e.g. obtained via an investigation of the homoclinic orbits of the 2d map obeyed by the main Fourier component [9], and then perturbing the evolution equation (1) by adding a term $+K[\hat{z}_n(t) - \delta z_n]$ in the right-hand side (*rhs*), in order to stabilize breathers via tuning of the control parameter K. This method relies on the application of the *continuous feedback control* (*cfc*) formalism (see the Refs. in [33], [34]). Alternatively, as argued in [34], a more efficient scheme should instead involve a term $+Ld[\hat{z}_n(t) - \delta z_n]/dt$ in the *rhs* of Eq. (1) (*dissipative cfc*), whence the damping imposed results in a higher convergence to the desired solution $\hat{z}_n(t)$.

In order to demonstrate the method let us use the celebrated model by MacKay and Aurby [7] which is the historic example shown to possess ILM of the breather type. That entails setting $\alpha = 0$ in the Hamiltonian 4 mentioned above. Figure 3 illustrates the controllability of an unstable multi-breather present in 4 setting ($\alpha = 0$), via the application of a dissipative continuous feedback control (CFC)scheme. Its shape was calculated using the method of homoclinic orbits in Fourier space [13]. The CFC drives the system through a transcritical bifurcation to a stable region where now the same shape of the breather remains stable. Details and the effect of possible fluctuation of the control parameter can be found in [33], [34].

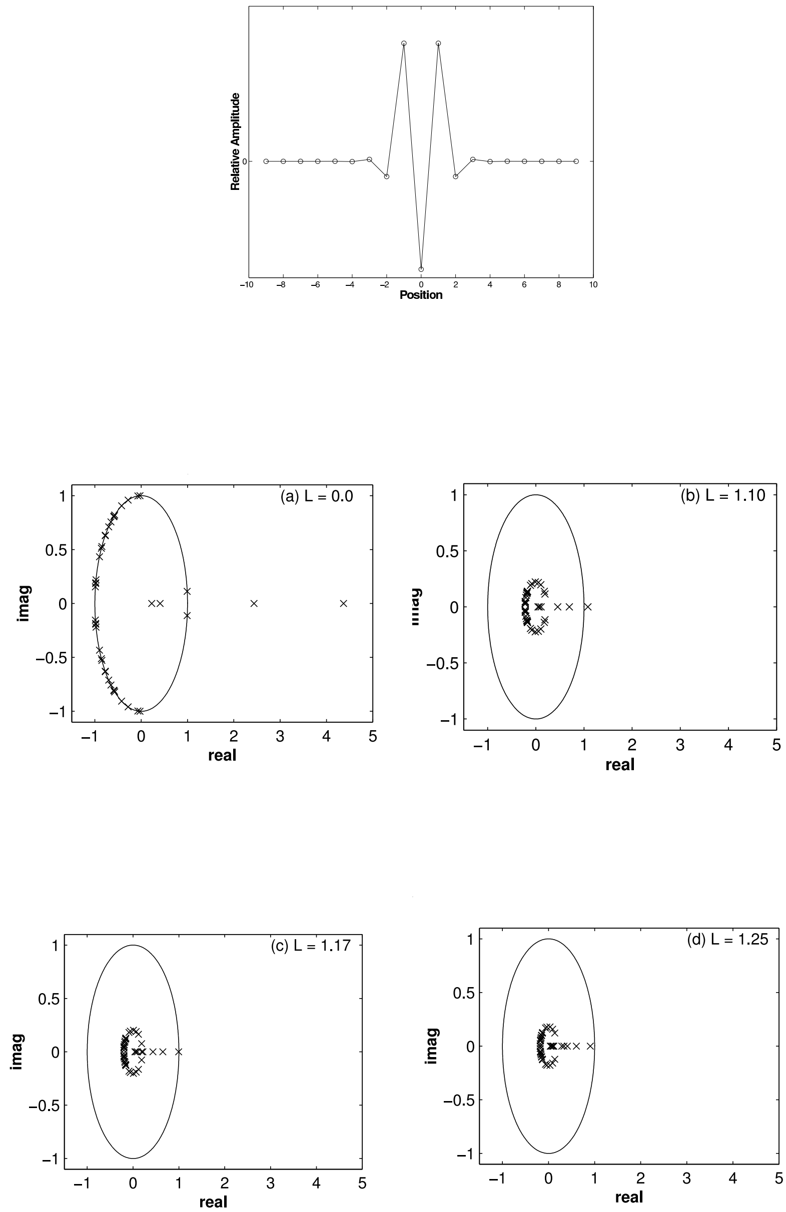

FIG. 3: Successive values of the eigenvalues (Floquet multipliers) of the monodromy matrix for the multi-breather depicted on the top of the figure. As the (dissipative) control parameter L increases in strength all eigenvalues eventualy pass inside the unit circle through a transcritical bifurcation. That means than the breather can indeed be stabilized via CFC.

VI. CONCLUSIONS - DISCUSSION

We have investigated, from first principles, the possibility of existence of localized discrete breather-type excitations associated with vertical dust grain motion in a dust mono-layer, which is assumed to be one-dimensional. It may be noted that the localized structures presented here, owe their existence to the intrinsic lattice discreteness in combination with the nonlinearity of the plasma sheath. Both are experimentally tunable physical mechanisms, so our results may be investigated (and will hopefully be verified) by appropriately designed experiments. The experimental confirmation of their existence in dust crystals appears as a promising field, which may open new directions e.g. in the design of controling schemes for sustaining unstable breathers relevant to selected applications.

Acknowledgments

V. B. was supported by *ESA's PRODEX C-90105 project for microgravity research*, I. K. and P.K.S. were supported by the *SFB591 (Sonderforschungsbereich) – Universelles Verhalten gleichgewichtsferner Plasmen: Heizung, Transport und Strukturbildung* German government Programme. V.B. wishes to thank T. Bountis (CRANS, Univ. of Patras, Greece) and G. Nicolis (CeNoLi, ULB) for many insightful discussions. I.K. is indebted to T. Bountis (CRANS, Univ. of Patras, Greece), S. Flach (MPIPKS, Dresden, Germany) and V. Koukouloyannis (AUTh, Thessaloniki, Greece) for a number of elucidating discussions.

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