Nonlinear Schrödinger-type ESW amplitude modulation theory and fluid plasma description:

a theoretical Primer on envelope ES solitons

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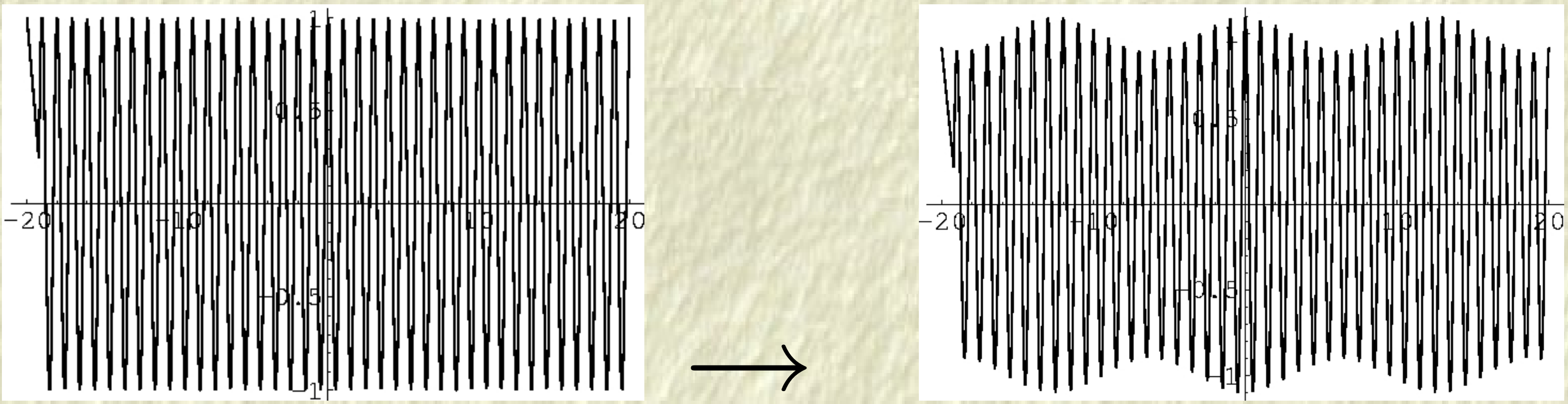
Outline

- Introduction
  - *Amplitude Modulation*: a rapid overview of notions and ideas;
  - Relevance with space and laboratory plasmas;
  - *Intermezzo*: *Dusty Plasmas* (or *Complex Plasmas*).
- The model: electrostatic wave description and formalism
  - A pedagogical paradigm: Ion–acoustic waves (IAWs);
  - Other examples: EAWs, DAWs, ...
- The reductive perturbation *(multiple scales)* technique.
- *Harmonic generation* and *Modulational Instability (MI)*.
- *Envelope excitations*: theory and characteristics.
- Conclusions.

1. Intro. The mechanism of *wave amplitude modulation*

The *amplitude* of a harmonic wave may vary in space and time:



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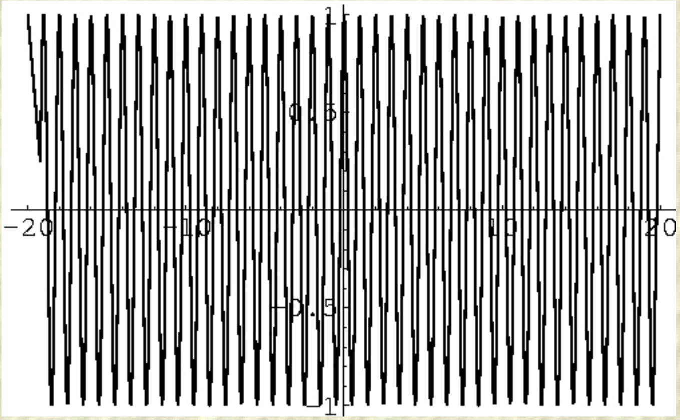

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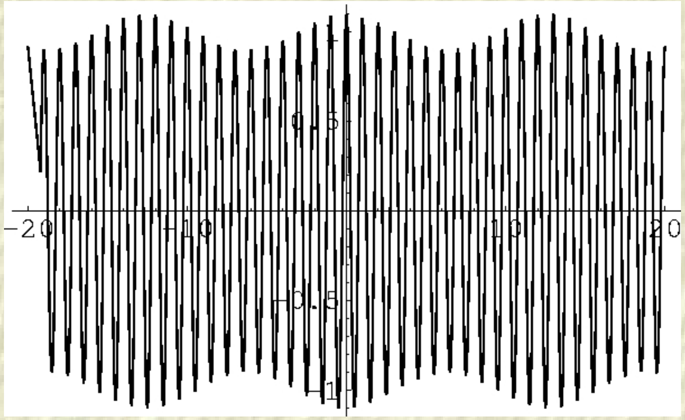

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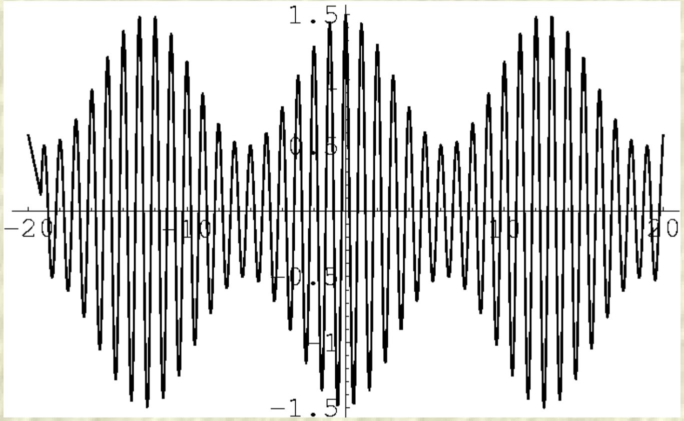

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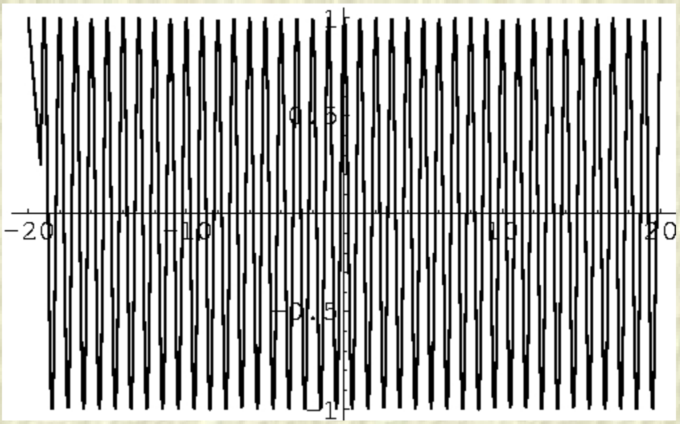

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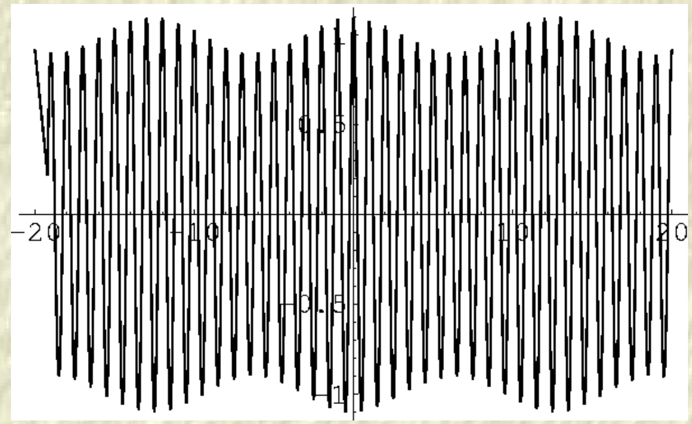
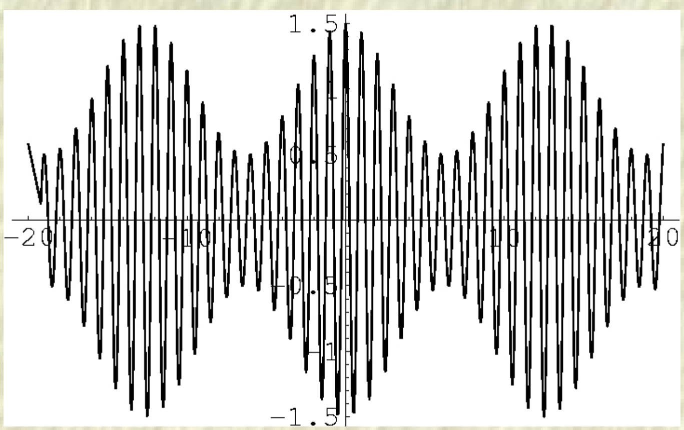


This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* or to the formation of *envelope solitons*:



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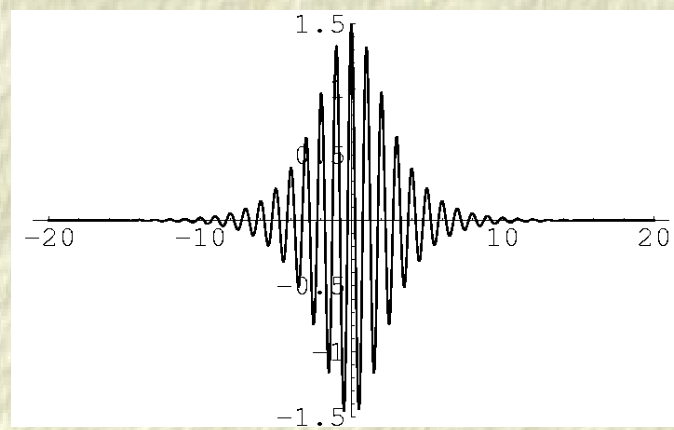
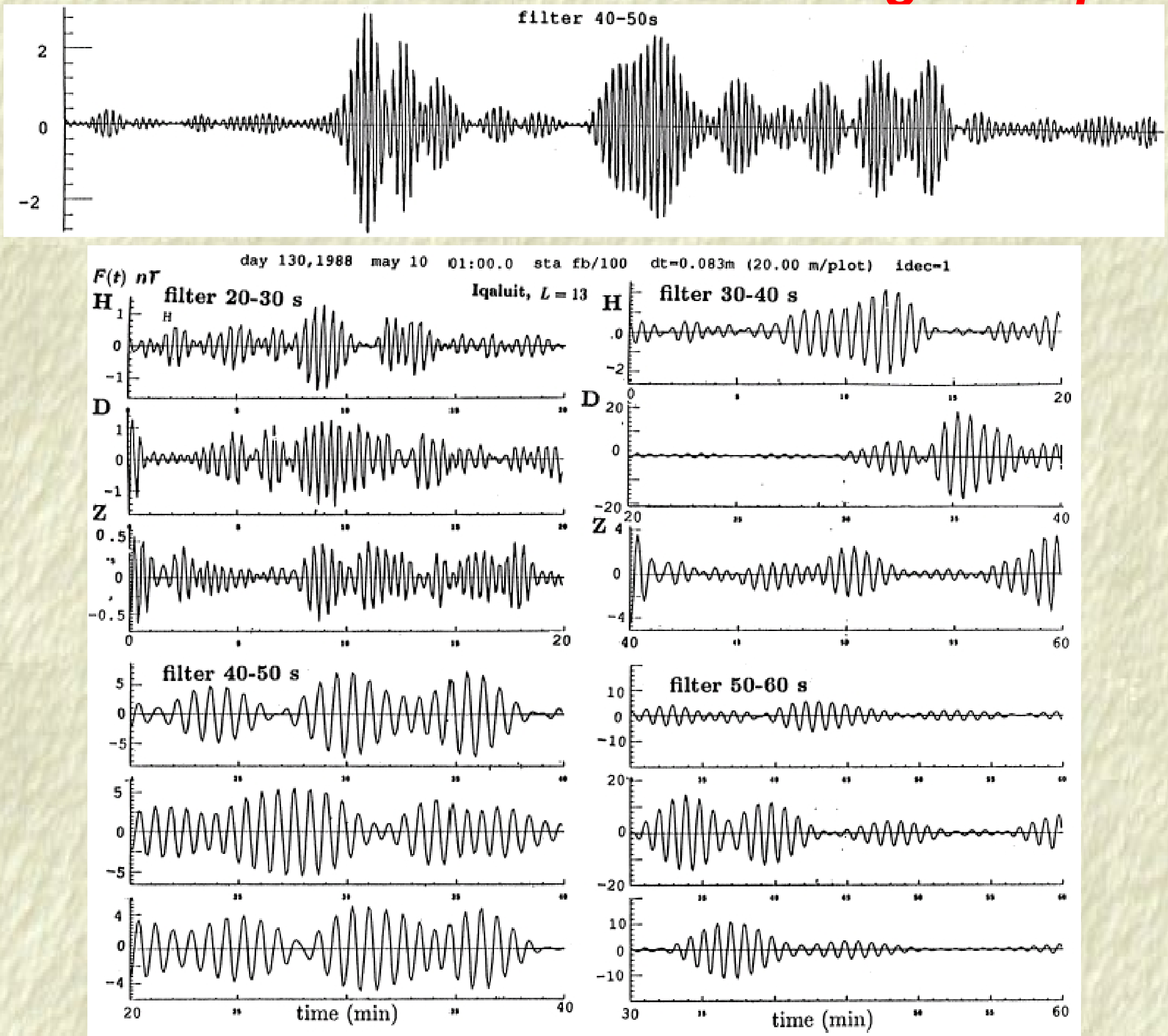
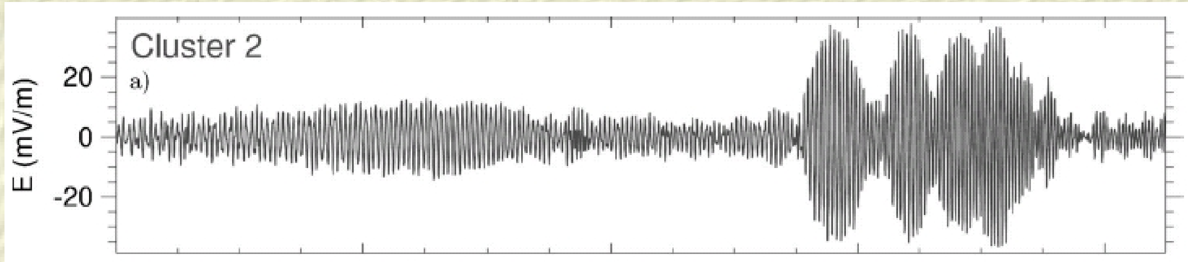


Modulated structures occur in the magnetosphere, ...



(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

..., in satellite (e.g. CLUSTER, FAST, ...) observations:



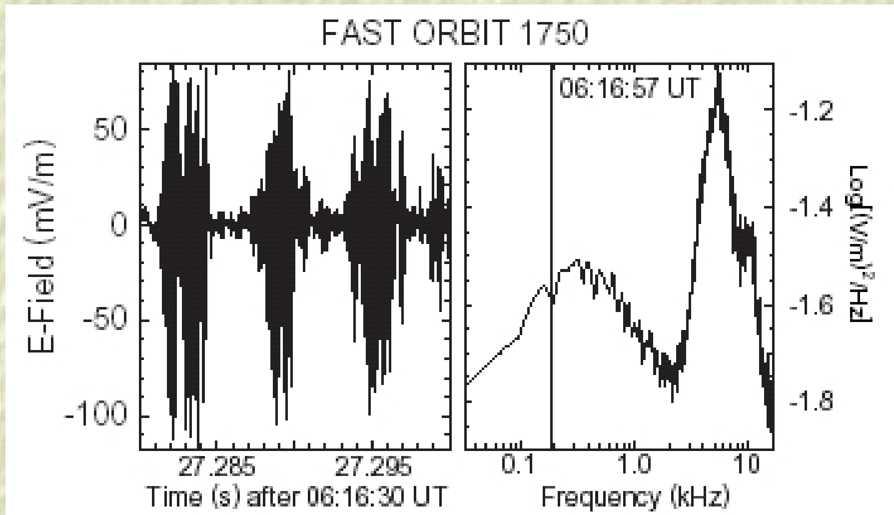


Figure 2. *Left*: Wave form of broadband noise at base of AKR source. The signal consists of highly coherent (nearly monochromatic frequency of trapped wave) wave packets. *Right*: Frequency spectrum of broadband noise showing the electron acoustic wave (at ~ 5 kHz) and total plasma frequency (at ~ 12 kHz) peaks. The broad LF maximum near 300 Hz belongs to the ion acoustic wave spectrum participating in the 3 ms modulation of the electron acoustic waves.

(*) From: O. Santolik *et al.*, *JGR* **108**, 1278 (2003); R. Pottelette *et al.*, *GRL* **26** 2629 (1999).

Modulational instability (MI) **was observed in simulations,**

e.g. early (1972) numerical experiments of EM cyclotron waves:

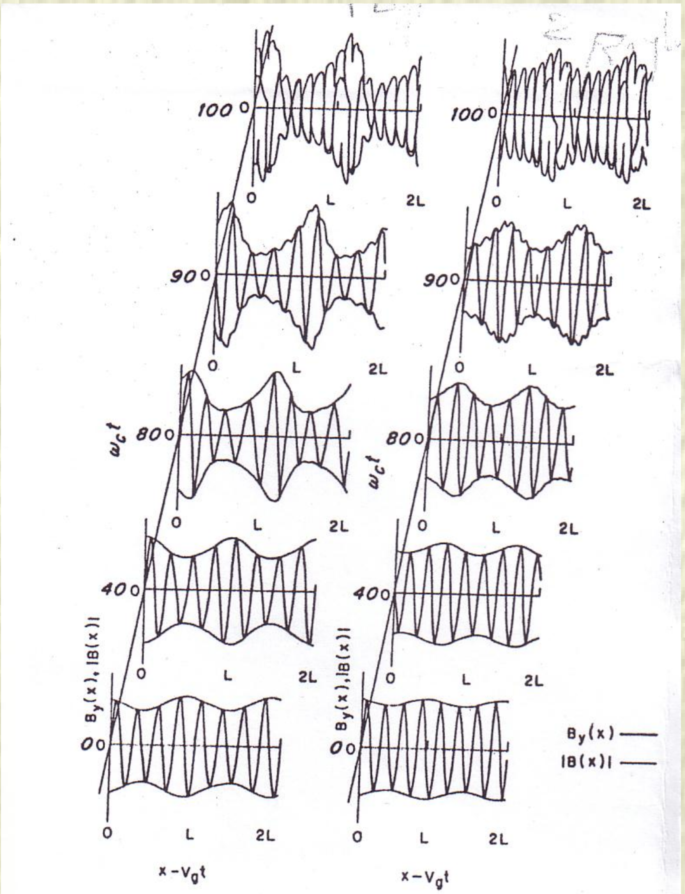


FIG. 2. Spatial variation of the y component of the wave magnetic field $B_y(x)$ (solid curve) and the magnitude of the total field $|B(x)|$ (thin curve) at different time of evolution for cases 1 (left) and 2 (right). Note the harmonic generation in $|B(x)|$ and the sudden collapse of the wave.

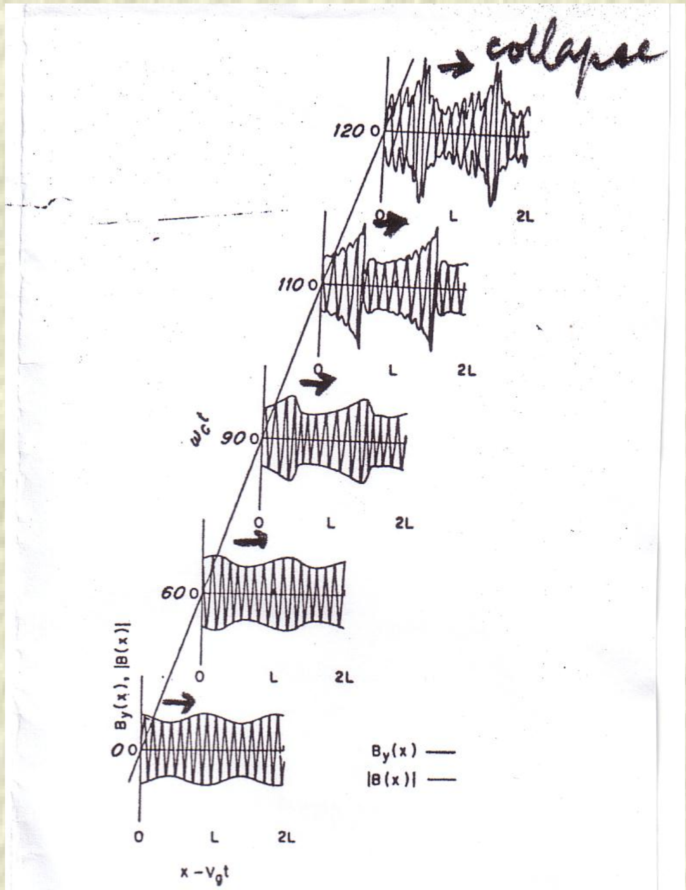


FIG. 3. $B_y(x)$ and $|B(x)|$ for case 4. Note the steepening of the wavefront which leads to the collapse of the wave.

[from: A. Hasegawa, *PRA* **1**, 1746 (1970); *Phys. Fluids* **15**, 870 (1972)].

Spontaneous MI **has been observed in experiments,:**

Modulational instability in a multi-component plasma

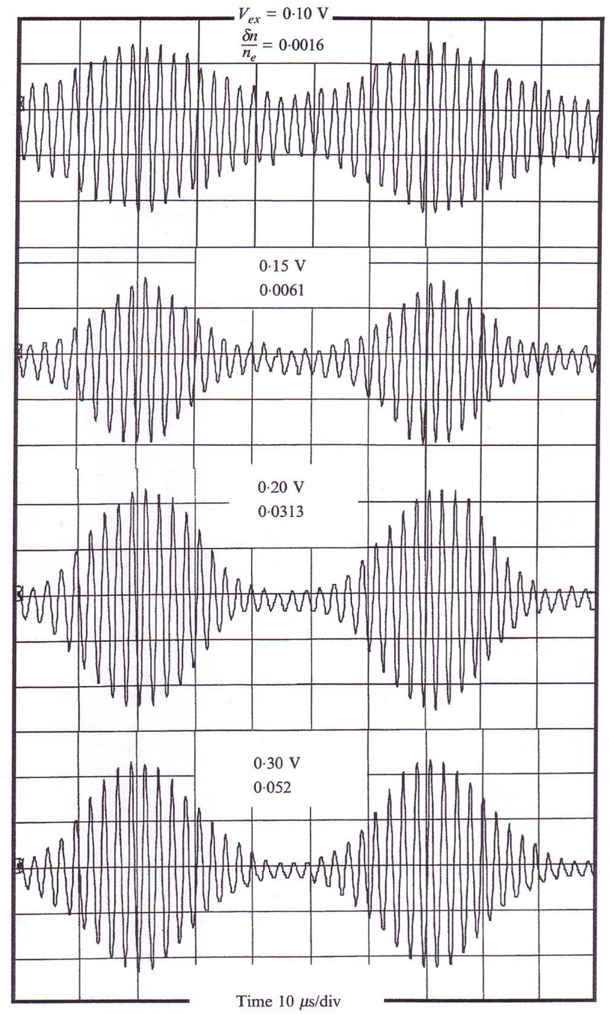


FIGURE 5. Oscilloscope traces of the detected signal for different excitation voltages. The probe was fixed at 14 cm from the grid. f_c = 400 kHz and f_m = 50 kHz.

e.g. on *ion acoustic waves*

[from: Bailung and Nakamura, *J. Plasma Phys.* **50** (2), 231 (1993)].

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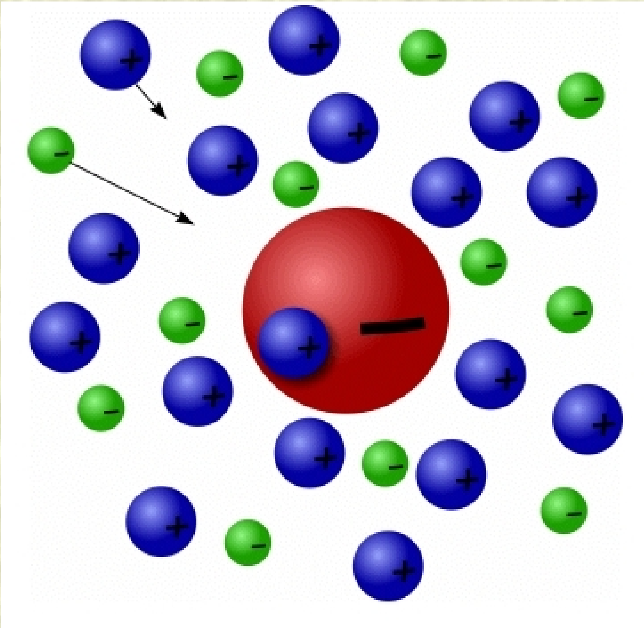
- *Focus*: *electrostatic* waves; e.g. *ion acoustic* (IA), *electron acoustic* (EA), *dust acoustic* (DA) waves, ...

Intermezzo: DP – Dusty Plasmas (or *Complex Plasmas*): definition and characteristics of a focus issue



- Ingredients:
  - *electrons* e^- (charge $-e$, mass m_e),
  - *ions* i^+ (charge $+Z_i e$, mass m_i), *and*
  - charged micro-particles $\equiv$ *dust grains* d (most often d^-):

    charge $Q = \pm Z_d e \sim \pm(10^3 - 10^4)\, e$,

    mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,

    radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu m$.

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– Ion acoustic waves (IAW): ions ($\alpha = i$) in a background of
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– Electron acoustic waves (EAW): electrons ($\alpha = e$) in a background of *stationary* ions ($\alpha' = i$): $n_i = cst.$;

– DAW: dust grains ($\alpha = d$) against *thermalized* electrons and ions ($\alpha' = e, i$): $n_e = n_{e,0}\, e^{e\Phi/K_B T_e}$, $n_i = n_{i,0}\, e^{-Z_i e\Phi/K_B T_i}$.

Fluid moment equations:

Density n_α (*continuity*) equation:

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Pressure p_α equation: [(*) *Cold* vs. *Warm* fluid model]

$$\frac{\partial p_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla p_\alpha \, = \, -\gamma \, p_\alpha \, \nabla \cdot \mathbf{u}_\alpha$$

[$\gamma = (f+2)/f = c_P/c_V$: ratio of specific heats e.g. $\gamma = 3$ for 1d, $\gamma = 2$ for 2d, etc.].

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The potential Φ obeys *Poisson's* eq.:

$$\nabla^2 \Phi \, = \, -4\pi \sum_{\alpha''=\alpha, \{\alpha'\}} q_{\alpha''} \, n_{\alpha''} \, = \, 4\pi \, e \, (n_e - Z_i \, n_i + ...)$$

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Hypothesis: Overall charge *neutrality* at equilibrium: $q_\alpha \, n_{\alpha,0} = -\sum_{\{\alpha'\}} q_{\alpha'} \, n_{\alpha',0}$.

Reduced moment evolution equations:

Defining appropriate scales *(see next slide)* one obtains:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\,\mathbf{u}) = 0\,,$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \;=\; -s\,\nabla\phi - \frac{\sigma}{n}\nabla p\,,$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p \;=\; -\gamma\,p\,\nabla \cdot \mathbf{u}\,;$$

also,

$$\nabla^2\phi \;=\; \phi - \alpha\,\phi^2 + \alpha'\,\phi^3 - s\,\beta\,(n-1)\,; \qquad (1)$$

i.e. *Poisson's Eq. close to equilibrium*: $\phi \ll 1$; $s = \mathrm{sgn} q_\alpha = \pm 1$.

- The dimensionless parameters α, α' and β must be determined exactly for any specific problem. They incorporate all the essential dependence on the plasma parameters.

We have defined the reduced (dimensionless) quantities:

- *particle density*: $n = n_\alpha / n_{\alpha,0}$;
- *mean (fluid) velocity*: $\mathbf{u} = [m_\alpha/(k_B T_*)]^{1/2} \mathbf{u}_\alpha \equiv \mathbf{u}_\alpha / c_*$;
- *dust pressure*: $p = p_\alpha / p_0 = p_\alpha / (n_{\alpha,0} k_B T_*)$;
- *electric potential*: $\phi = Z_\alpha e \Phi / (k_B T_*) = |q_\alpha| \Phi / (k_B T_*)$;
- $\gamma = (f+2)/f = C_P/C_V$ (for f degrees of freedom).

Also, *time* and *space* are scaled over:

- t_0, e.g. the inverse *DP plasma frequency*

$$\omega_{p,\alpha}^{-1} = (4\pi n_{\alpha,0} q_\alpha^2 / m_\alpha)^{-1/2}$$

- $r_0 = c_* t_0$, i.e. an *effective Debye length*

$$\lambda_{D,eff} = (k_B T_* / m_\alpha \omega_{p,\alpha}^2)^{1/2} \, .$$

Finally, $\sigma = T_\alpha / T_*$ is the temperature (ratio).

3. Reductive Perturbation Technique

– 1st step. Define *multiple scales* (*fast* and *slow*) i.e. (in 2d)

$$X_0 = x\,, \quad X_1 = \epsilon\, x\,, \quad X_2 = \epsilon^2\, x\,, \quad \ldots$$
$$Y_0 = y\,, \quad Y_1 = \epsilon\, y\,, \quad Y_2 = \epsilon^2\, y\,, \quad \ldots$$
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and modify operators appropriately:

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial X_0} + \epsilon \frac{\partial}{\partial X_1} + \epsilon^2 \frac{\partial}{\partial X_2} + \quad \ldots$$
$$\frac{\partial}{\partial y} \to \frac{\partial}{\partial Y_0} + \epsilon \frac{\partial}{\partial Y_1} + \epsilon^2 \frac{\partial}{\partial Y_2} + \quad \ldots$$
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– 2nd step. Expand near equilibrium:

$$n_\alpha \approx n_{\alpha,0} + \epsilon\, n_{\alpha,1} + \epsilon^2\, n_{\alpha,2} + \ldots$$

$$\mathbf{u}_\alpha \approx \mathbf{0} + \epsilon\, \mathbf{u}_{\alpha,1} + \epsilon^2\, \mathbf{u}_{\alpha,2} + \ldots$$

$$p_\alpha \approx p_{\alpha,0} + \epsilon\, p_{\alpha,1} + \epsilon^2\, p_{\alpha,2} + \ldots$$

$$\phi \approx 0 + \epsilon\, \phi_1 + \epsilon^2\, \phi_2 + \ldots$$

($p_{\alpha,0} = n_{\alpha,0} k_B T_\alpha$; $\epsilon \ll 1$ is a *smallness parameter*).

Reductive perturbation technique *(continued)*

– 3rd step. Project on Fourier space, i.e. consider $\forall m = 1, 2, ...$

$$S_m = \sum_{l=-m}^{m} \hat{S}_l^{(m)} \, e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \hat{S}_0^{(m)} + 2\sum_{l=1}^{m} \hat{S}_l^{(m)} \cos l(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

for $S_m = (n_m, \{u_{x,m}, u_{y,m}\}, p_m, \phi_m)$, i.e. *essentially*:

$$n_1 = n_0^{(1)} + \tilde{n}_1^{(1)} \cos\theta \,, \qquad n_2 = n_0^{(2)} + \tilde{n}_1^{(2)} \cos\theta + \tilde{n}_2^{(2)} \cos 2\theta, \text{ etc.}$$

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– 4rth step. *Oblique modulation* assumption:
the slow amplitudes $\hat{\phi}_l^{(m)}$, etc. vary *only along* the x-axis:

$$\hat{S}_l^{(m)} = \hat{S}_l^{(m)}(X_j, T_j)\,, \qquad j = 1, 2, ...$$

while the fast carrier phase $\theta = \mathbf{k}\cdot\mathbf{r} - \omega t$ is now:

$$k_x\, x + k_y\, y - \omega t = k\, r \cos\alpha - \omega t\ .$$

First-order solution ($\sim \epsilon^1$)

Substituting and isolating terms in $m = 1$, we obtain:

- The *dispersion relation* $\omega = \omega(k)$:

$$\omega^2 = \omega_{p,\alpha}^2 \frac{k^2}{k^2 + k_D^2} + \gamma\, v_{th}^2\, k^2 \qquad (2)$$

with $k_D = \lambda_D^{-1}$, where

$$\omega_{p,\alpha} = \left(\frac{4\pi n_{\alpha,0} q_\alpha^2}{m_\alpha}\right)^{1/2}, \; \lambda_{D,\alpha} = \left(\frac{k_B T_\alpha}{4\pi n_{\alpha,0} q_\alpha^2}\right)^{1/2}, \; v_{th} = \left(\frac{T_\alpha}{m_\alpha}\right)^{1/2}$$

- The *solution(s)* for the 1st–harmonic amplitudes (e.g. $\propto \phi_1^{(1)}$):

$$n_1^{(1)} = s\,\frac{1+k^2}{\beta}\phi_1^{(1)} = \frac{1}{\gamma}\,p_1^{(1)} = \frac{k}{\omega\,\cos\theta}\,u_{1,x}^{(1)} = \frac{k}{\omega\,\sin\theta}\,u_{1,y}^{(1)} \qquad (3)$$

Second-order solution ($\sim \epsilon^2$)

❑ From $m = 2, l = 1$, we obtain the relation:

$$\frac{\partial \psi}{\partial T_1} + v_g \frac{\partial \psi}{\partial X_1} = 0 \qquad (4)$$

where

– $\psi = \phi_1^{(1)}$ is the potential correction ($\sim \epsilon^1$);

– $v_g = \frac{\partial \omega(k)}{\partial k_x}$ is the group velocity along $\hat{x}$;

– the wave's envelope satisfies: $\psi = \psi(\epsilon(x - v_g t)) \equiv \psi(\zeta)$.

❑ The solution, up to $\sim \epsilon^2$, is of the form:

$$\phi \approx \epsilon \, \psi \, \cos\theta \, + \, \epsilon^2 \left[\phi_0^{(2)} + \phi_1^{(2)} \cos\theta + \phi_2^{(2)} \cos 2\theta \right] + \mathcal{O}(\epsilon^3) \, ,$$

etc. ($+$ similar expressions for n_d, u_x, u_y, p_d): $\rightarrow$ *Harmonics!*.

Third-order solution ($\sim \epsilon^3$)

- Compatibility equation (from $m = 3, l = 1$), in the form of:

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0\,. \qquad (5)$$

  i.e. a *Nonlinear Schrödinger–type Equation* (NLSE) .

- Variables: $\zeta = \epsilon(x - v_g t)$ and $\tau = \epsilon^2\,t$;

- *Dispersion coefficient* P:

$$P = \frac{1}{2}\frac{\partial^2 \omega}{\partial k_x^2} = \frac{1}{2}\left[\omega''(k)\cos^2\alpha + \omega'(k)\,\frac{\sin^2\alpha}{k}\right]; \qquad (6)$$

- *Nonlinearity coefficient* Q: ...
  A *(lengthy!)* function of k, angle α and T_e, T_i, ... $\rightarrow$ *(omitted)*.

4. Modulational (in)stability analysis

- The NLSE admits the *harmonic wave solution*:

$$\psi = \hat{\psi}\, e^{iQ|\hat{\psi}|^2\tau} + \mathrm{c.c.}$$

- *Perturb* the amplitude by setting: $\hat{\psi} = \hat{\psi}_0 + \epsilon\, \hat{\psi}_{1,0} \cos(\tilde{k}\zeta - \tilde{\omega}\tau)$

- We obtain the *(perturbation)* dispersion relation:

$$\tilde{\omega}^2 = P^2\, \tilde{k}^2 \left(\tilde{k}^2 - 2\frac{Q}{P}|\hat{\psi}_{1,0}|^2 \right). \qquad (7)$$

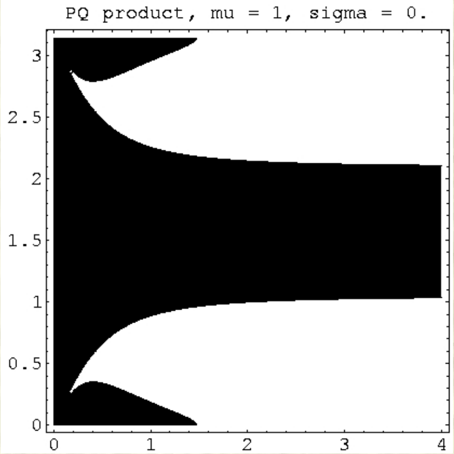
- If $PQ < 0$: the amplitude ψ is *stable* to external perturbations;

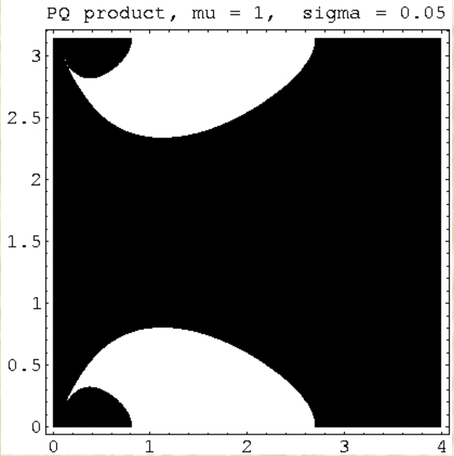
- If $PQ > 0$: the amplitude ψ is *unstable* for $\tilde{k} < \sqrt{2\frac{Q}{P}}|\psi_{1,0}|$.

Stability profile (IAW): Angle α versus wavenumber k

Typical values: $Z_i = +1$ (hydrogen plasma), $\gamma = 2$.

– *Ion-acoustic waves*; cold ($\sigma = 0$) vs. warm ($\sigma \neq 0$) fluid:





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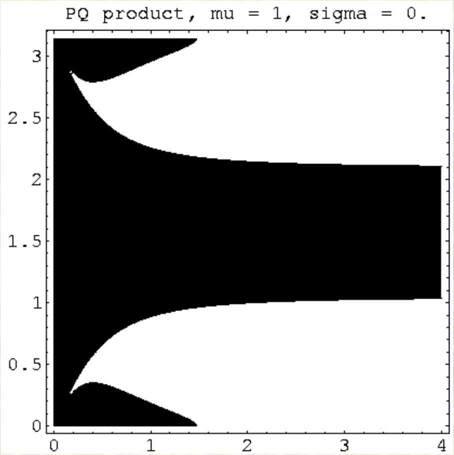
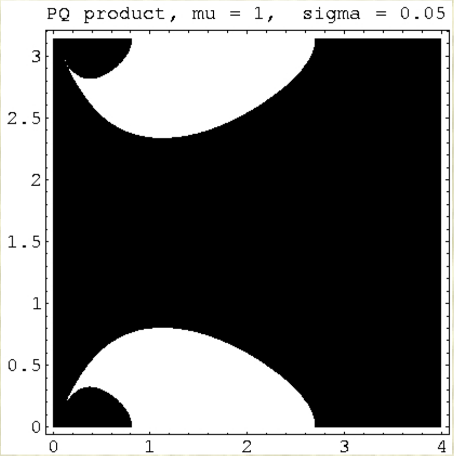
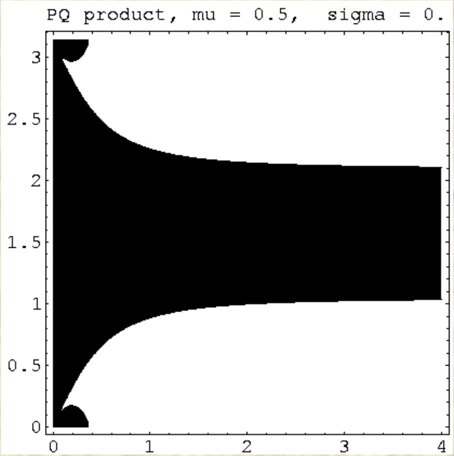
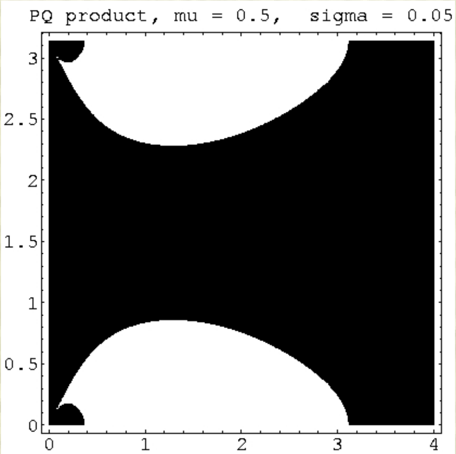




– *Dust-ion acoustic waves*, i.e. in the presence of *negative dust* ($n_{d,0}/n_{i,0} = 0.5$):

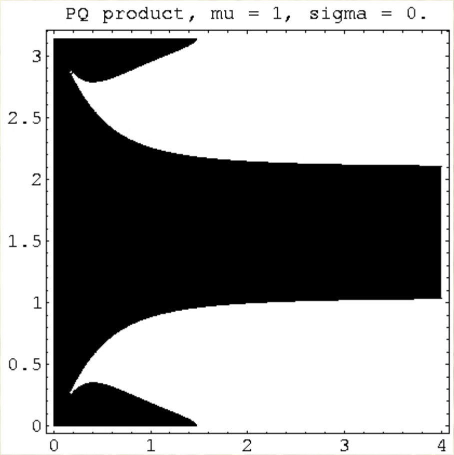


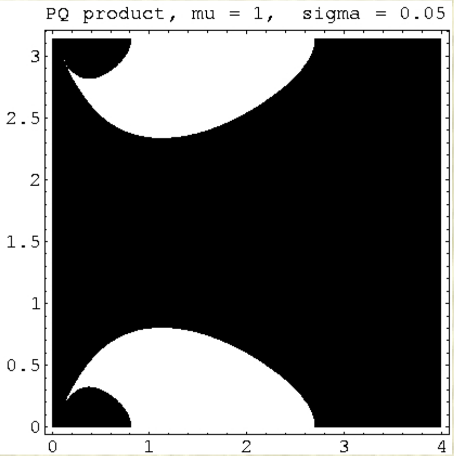


Stability profile (IAW): Angle α versus wavenumber k

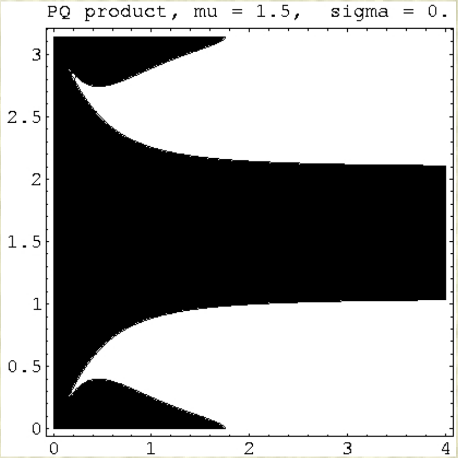
Typical values: $Z_i = +1$ (hydrogen plasma), $\gamma = 2$.

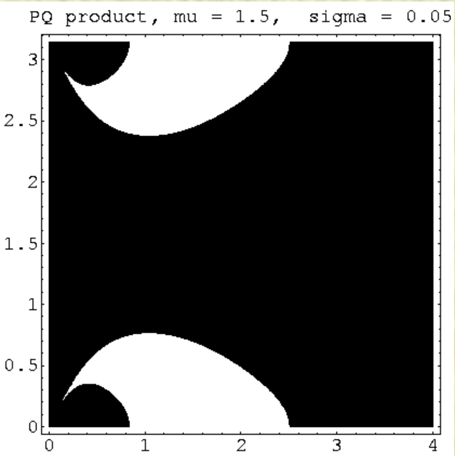
– *Ion-acoustic waves*; cold ($\sigma = 0$) vs. warm ($\sigma \neq 0$) fluid:





– *Dust-ion acoustic waves*, i.e. in the presence of *positive dust* ($n_{d,0}/n_{i,0} = 0.5$):

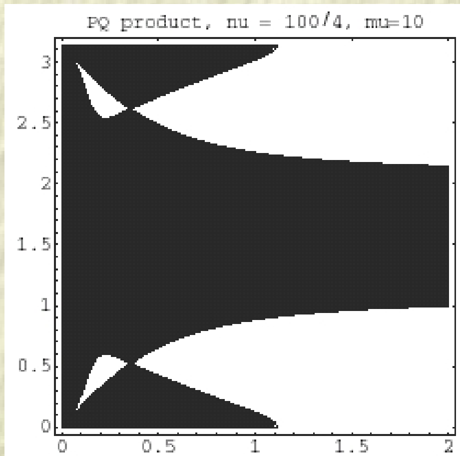


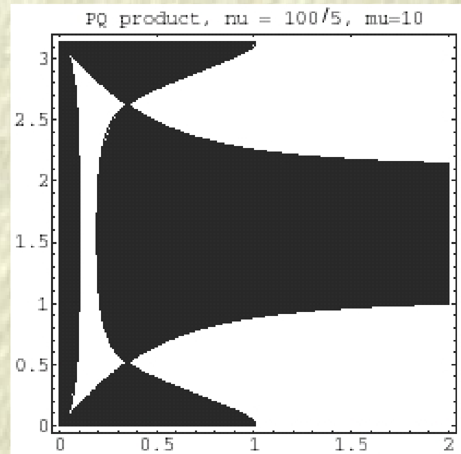


Stability profile (IAW/EAW): Angle α versus wavenumber k

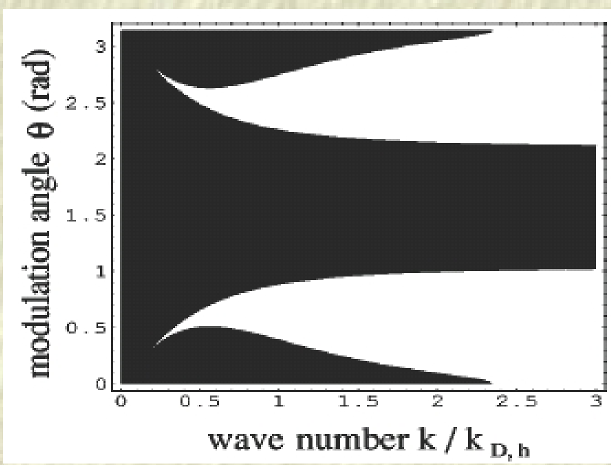
Typical values: $Z_i = +1$ (hydrogen plasma), $\gamma = 2$.

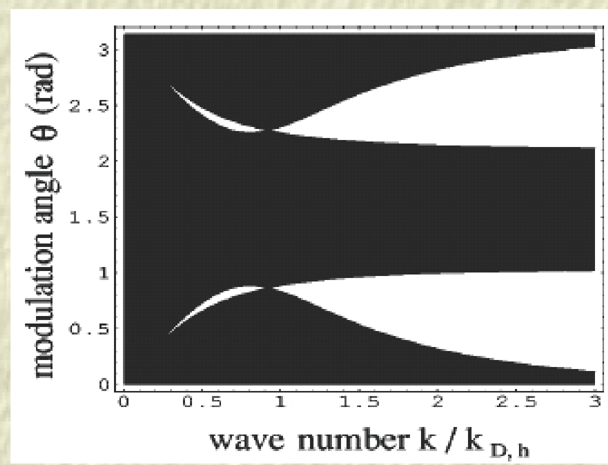
– *Ion acoustic waves*, in the presence of *2 electron populations*:





– *Electron acoustic waves* (*+ cold electrons*):

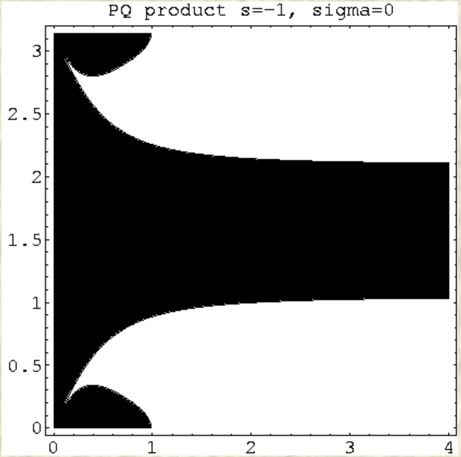


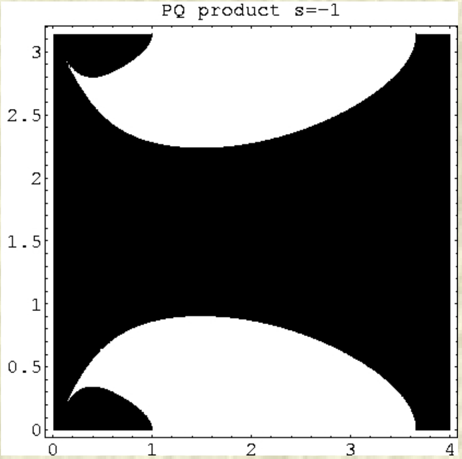


Stability profile (DAW): Angle α versus wavenumber k

Typical values: $Z_d/Z_i \approx 10^3$, $T_e/T_i \approx 10$, $n_{d,0}/n_{i,0} \approx 10^{-3}$, $\gamma = 2$.

– *Negative dust*: $s = -1$; cold ($\sigma = 0$) vs. warm ($\sigma \neq 0$) fluid:





Stability profile (DAW): Angle α versus wavenumber k

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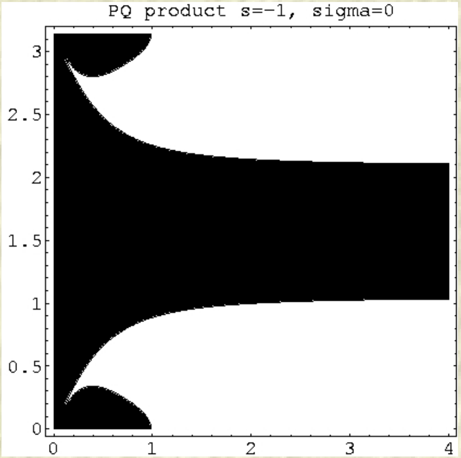
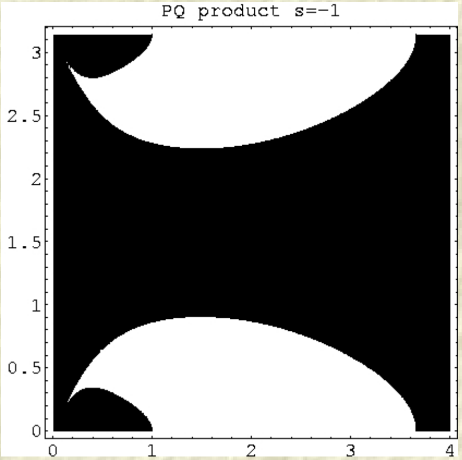
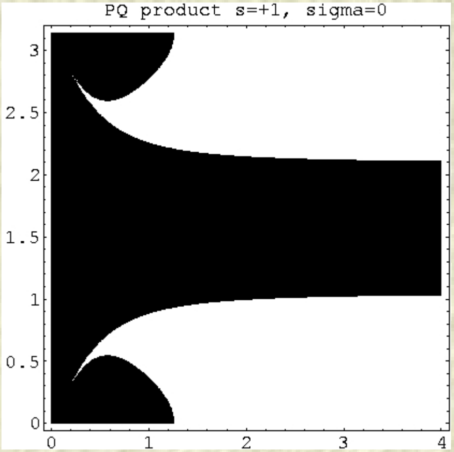
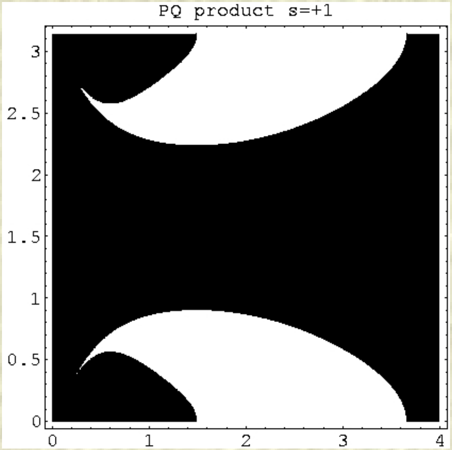
– *Negative dust*: $s = -1$; cold ($\sigma = 0$) vs. warm ($\sigma \neq 0$) fluid:





– The same plot for *positive dust* ($s = +1$):





5. Localized envelope excitations (solitons)

❏ The NLSE:

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0$$

accepts various soliton solutions: $\psi = \rho\, e^{i\Theta}\,;$

the *total* electric potential is then: $\phi \approx \epsilon\,\rho\,\cos(\mathbf{kr} - \omega t + \Theta)$

where the amplitude ρ and phase correction Θ depend on ζ, τ.

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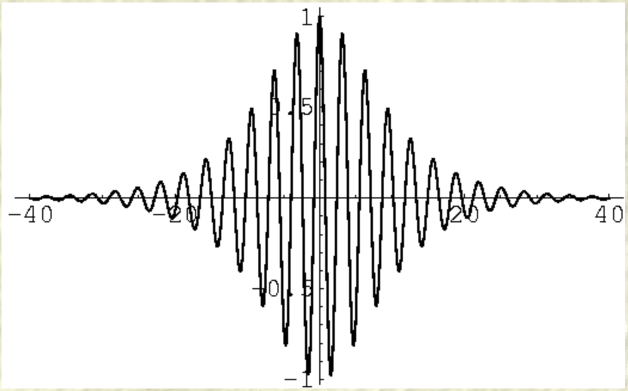
  the *total* electric potential is then: $\phi \approx \epsilon\, \rho\, \cos(\mathbf{kr} - \omega t + \Theta)$
  where the amplitude ρ and phase correction Θ depend on ζ, τ.

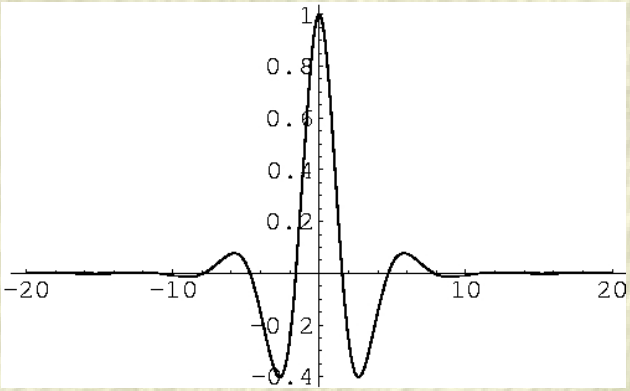
- Bright–type envelope soliton (pulse):

$$\rho = \rho_0\, \mathrm{sech}\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta - (\Omega + \frac{1}{2}v^2)\tau\right]. \quad (8)$$

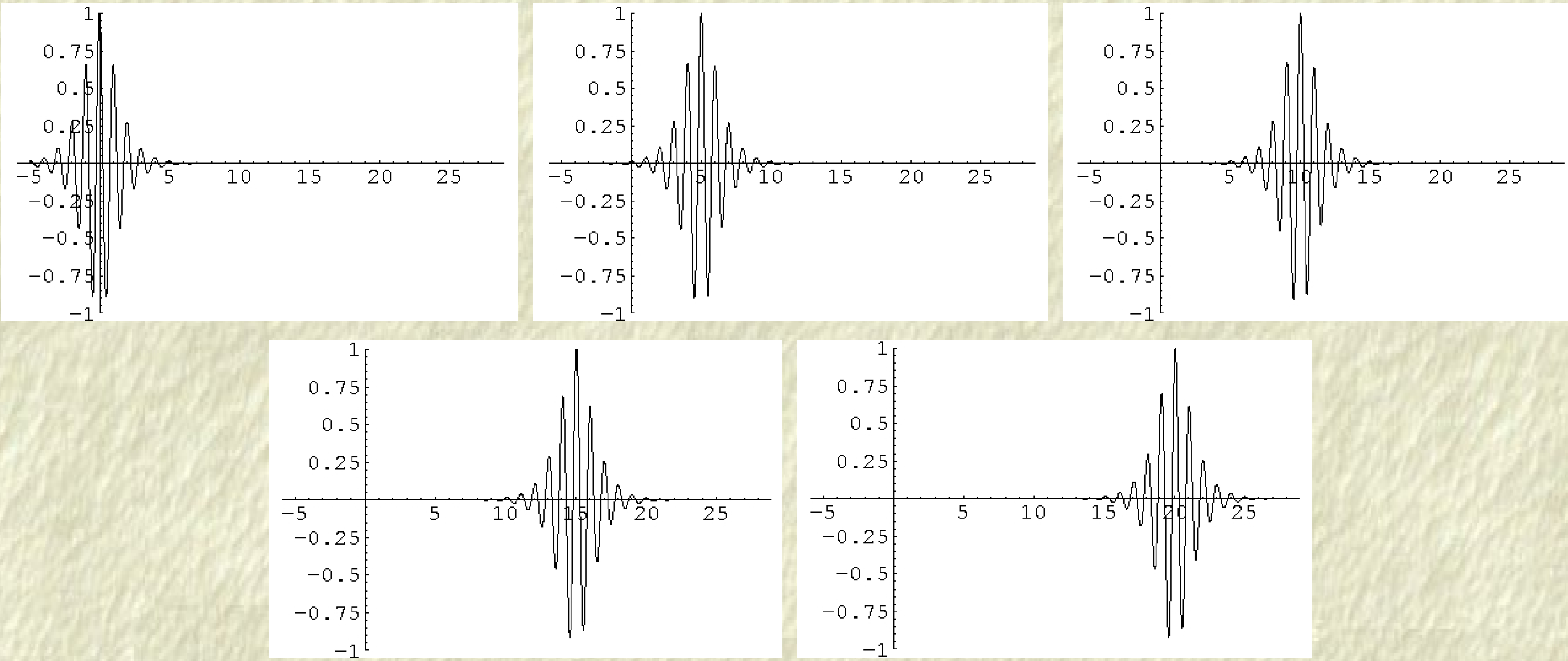
$$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$$

This is a propagating (and *oscillating*) localized pulse:

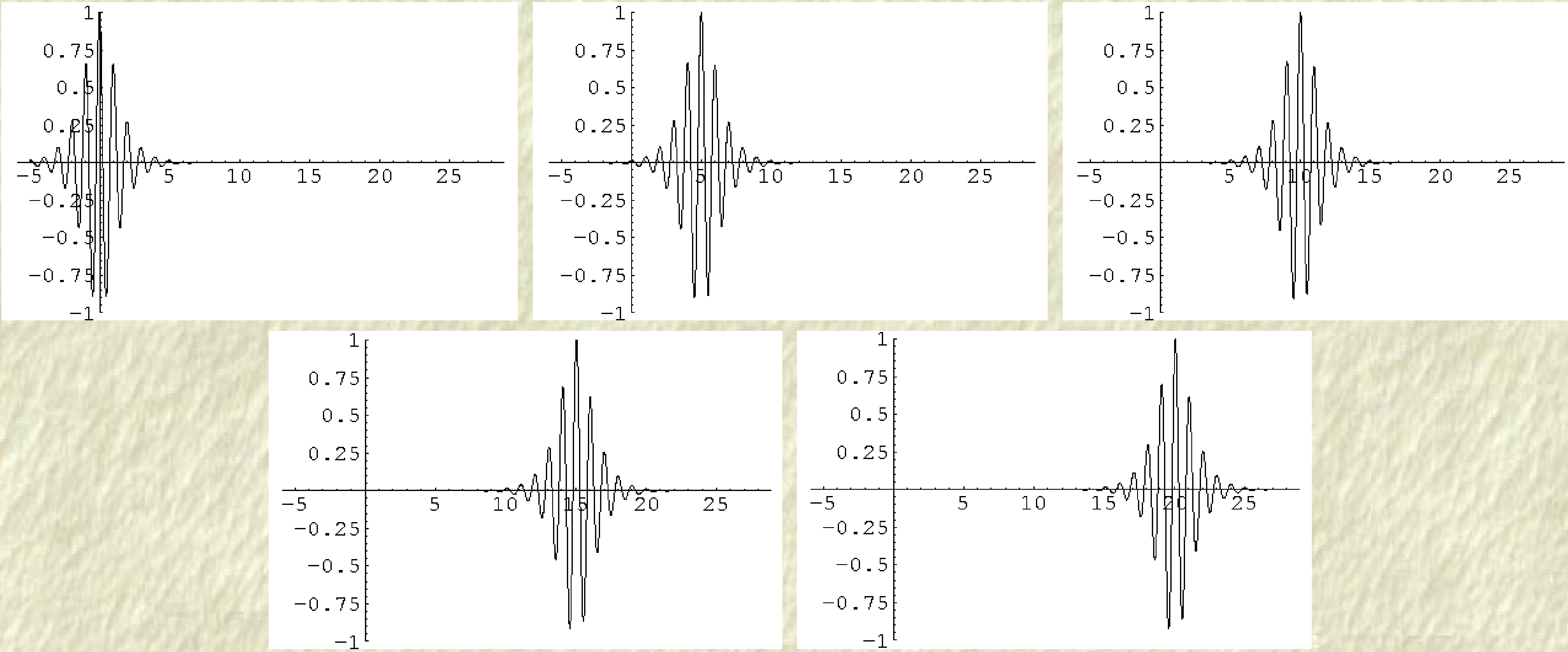




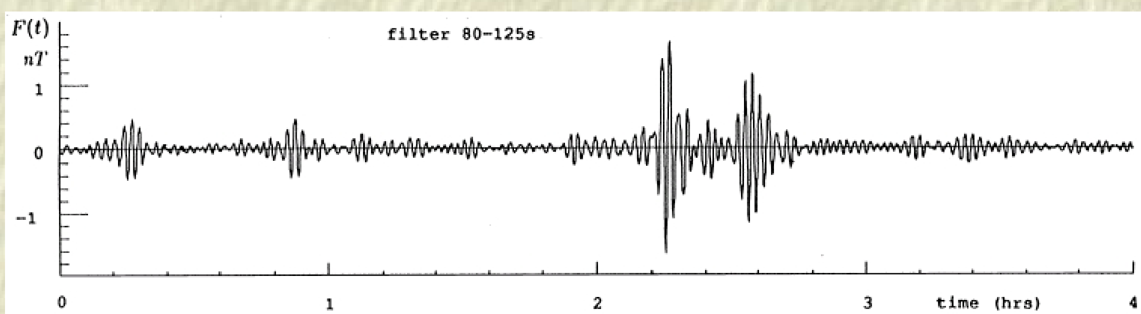
Propagation of a bright envelope soliton (pulse)



Propagation of a bright envelope soliton (pulse)

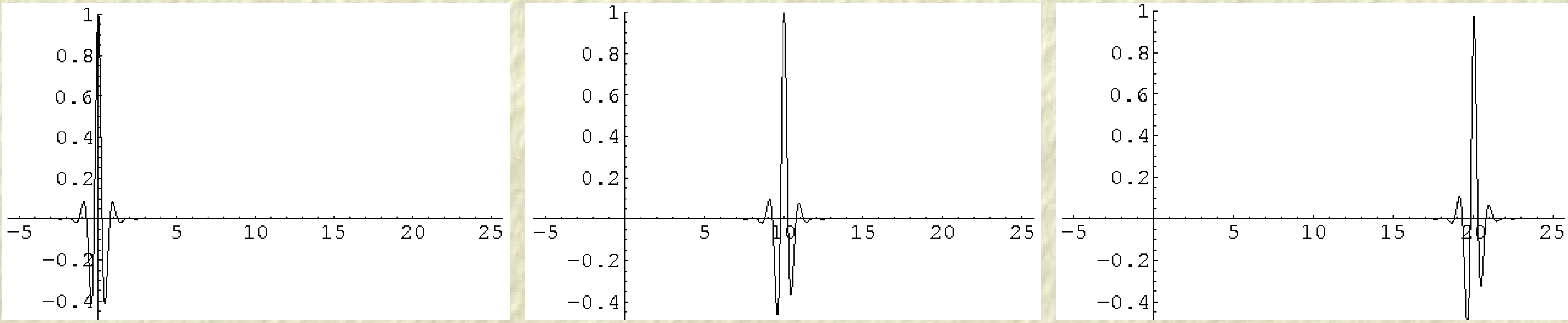


Cf. electrostatic plasma wave data from satellite observations:

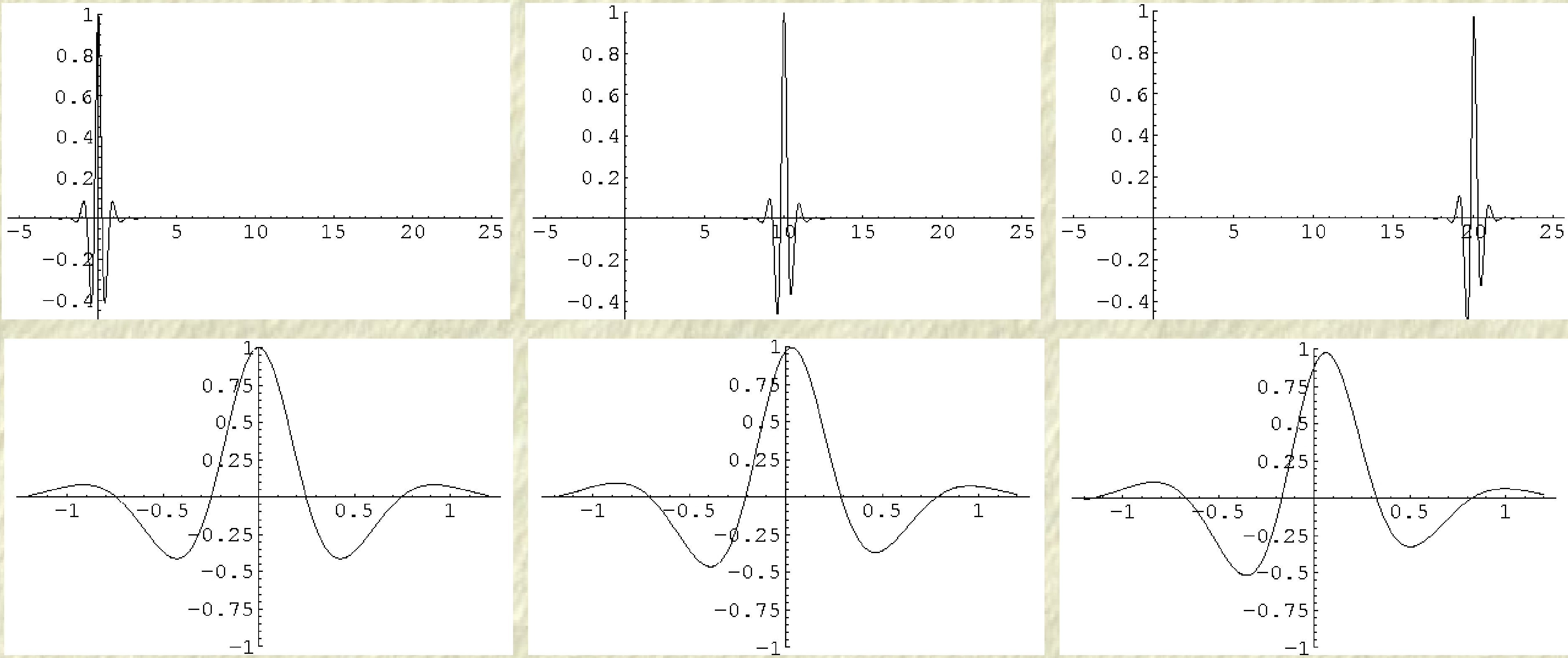


(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

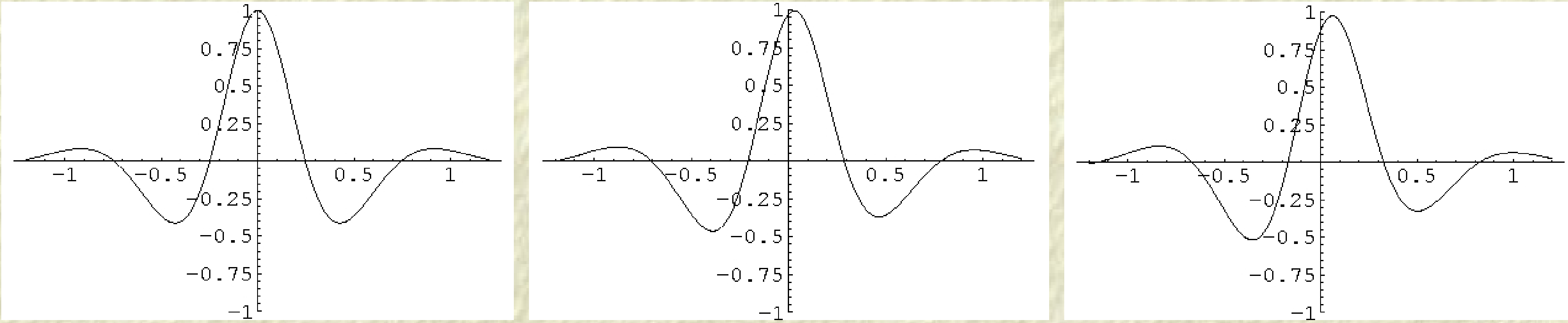
Propagation of a bright envelope soliton (continued...)



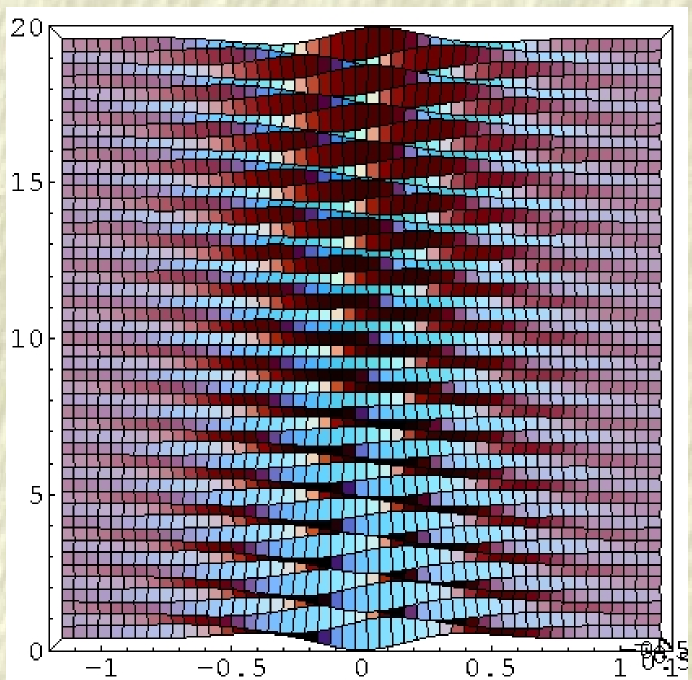
Propagation of a bright envelope soliton (continued...)



Propagation of a bright envelope soliton (continued...)



Rem.: *Time-dependent phase* → *breathing* effect (at rest frame):



Localized envelope excitations *(part 2)*

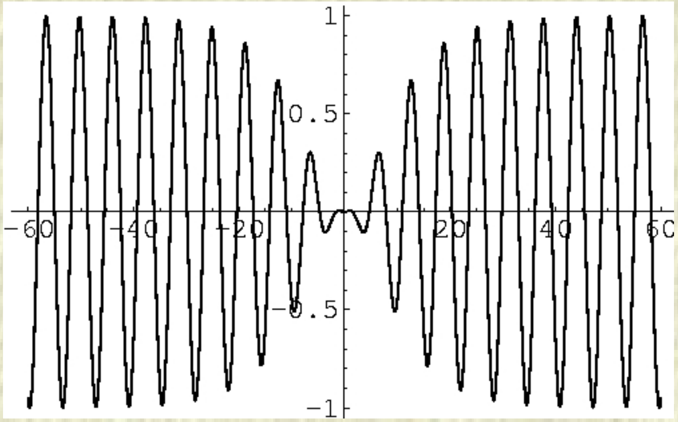
❑ Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{\rho_1}} \qquad (9)$$

This is a *propagating* *localized* hole (zero density void):



Localized envelope excitations *(part 3)*

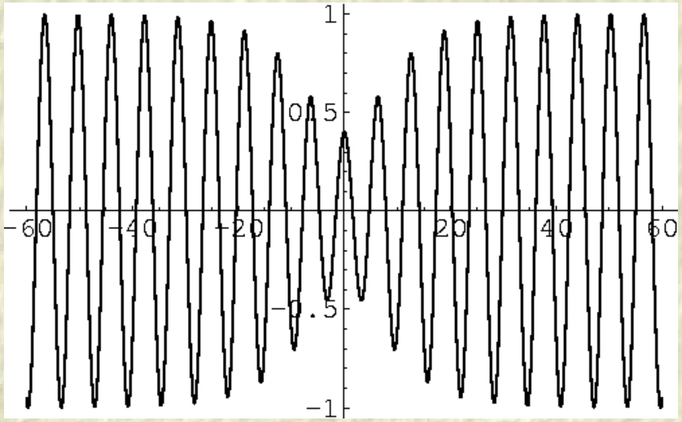
❑ Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - a^2 \operatorname{sech}^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{a\rho_2} \qquad (10)$$

This is a propagating *(non zero-density)* void:



6. Conclusions

- *Amplitude Modulation* (due to carrier self-interaction) is an inherent feature of electrostatic (ES) plasma mode dynamics;

- ES waves may undergo spontaneous *modulational instability*; this is an intrinsic feature of nonlinear dynamics, which ...

- ... may lead to the formation of *envelope localized structures* (envelope solitons), in account for *energy localization* phenomena widely observed in space and laboratory.

- The RP analytical framework permits modeling of these mechanisms in terms of intrinsic physical (plasma) parameters.
  $\rightarrow$ *a small step towards understanding the nonlinear behaviour of Plasmas*.

Thank You !

Ioannis Kourakis
Padma Kant Shukla

Acknowledgments:

Material from:

I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **10** (9), 3459 (2003);
idem, *PRE*, **69** (3), 036411 (2003).
idem, *J. Phys. A* , **36** (47), 11901 (2003).
idem, *European Phys. J. D*, **28**, 109 (2004).

Available at: `www.tp4.rub.de/~ioannis`

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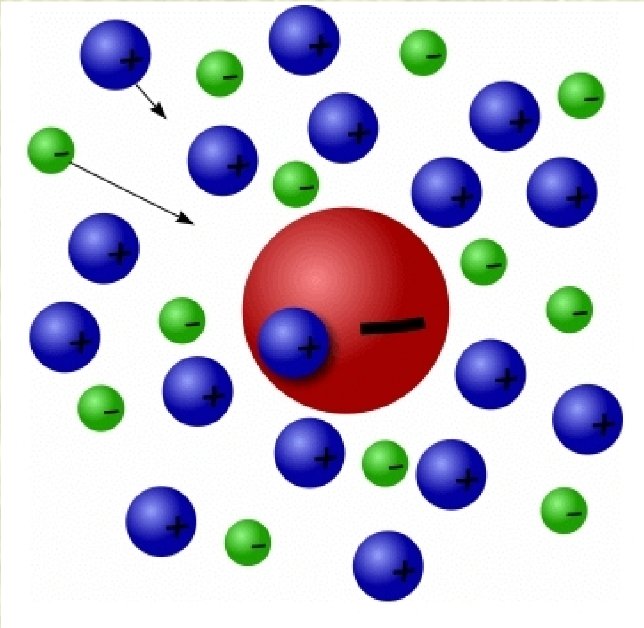
Appendix: DP – Dusty Plasmas (or *Complex Plasmas*): definition and characteristics



- Ingredients:
  - *electrons* e^- (charge $-e$, mass m_e),
  - *ions* i^+ (charge $+Z_i e$, mass m_i), *and*
  - charged micro-particles $\equiv$ *dust grains* d (most often d^-):

    charge $Q = \pm Z_d e \sim \pm(10^3 - 10^4)\, e$,

    mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,

    radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu$m.

Origin: Where does the dust come from?

- *Space:* cosmic debris (silicates, graphite, amorphous carbon), comet dust, man-made pollution (Shuttle exhaust, satellite remnants), ...

- *Atmosphere:* extraterrestrial dust (meteorites): $\geq 2 \cdot 10^4$ tons a year (!)(*), atmospheric pollution, chemical aerosols, ...

- *Fusion reactors:* plasma-surface interaction, carbonaceous particulates resulting from wall erosion-created debris (graphite, CFCs: Carbon Fiber Composites, ...)

- *Laboratory:* (man-injected) melamine–formaldehyde particulates (**) injected in *rf* or *dc* discharges; 3d ($=$ multiple 2d layers) or 1d (by appropriate experimental setting) crystals.

Sources: [P. K. Shukla & A. Mamun 2002], (*) [DeAngelis 1992], (**) [G. E. Morfill *et al.* 1998]

Some unique features of *the Physics of* Dusty Plasmas:

- Complex plasmas are *overall charge neutral*; most (sometimes *all*!) of the negative charge resides on the microparticles;
- The microparticles can be *dynamically dominant*: mass density $\approx 10^2$ times higher than the neutral gas density and $\approx 10^6$ times higher than the ion density !
- Studies in *slow motion* are possible due to high M i.e. *low Q/M ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!) $\rightarrow$ video;
- Dust charge ($Q \neq \mathrm{const.}$) is now a dynamical variable, associated to a new *collisionless damping* mechanism;

(...continued) More "heretical" features are:

- Important *gravitational* (compared to the *electrostatic*) interaction effects; gravito-plasma physics; gravito-electrodynamics; Jeans-type (gravitational) plasma instabilities etc. [Verheest PPCF 41 A445, 1999]

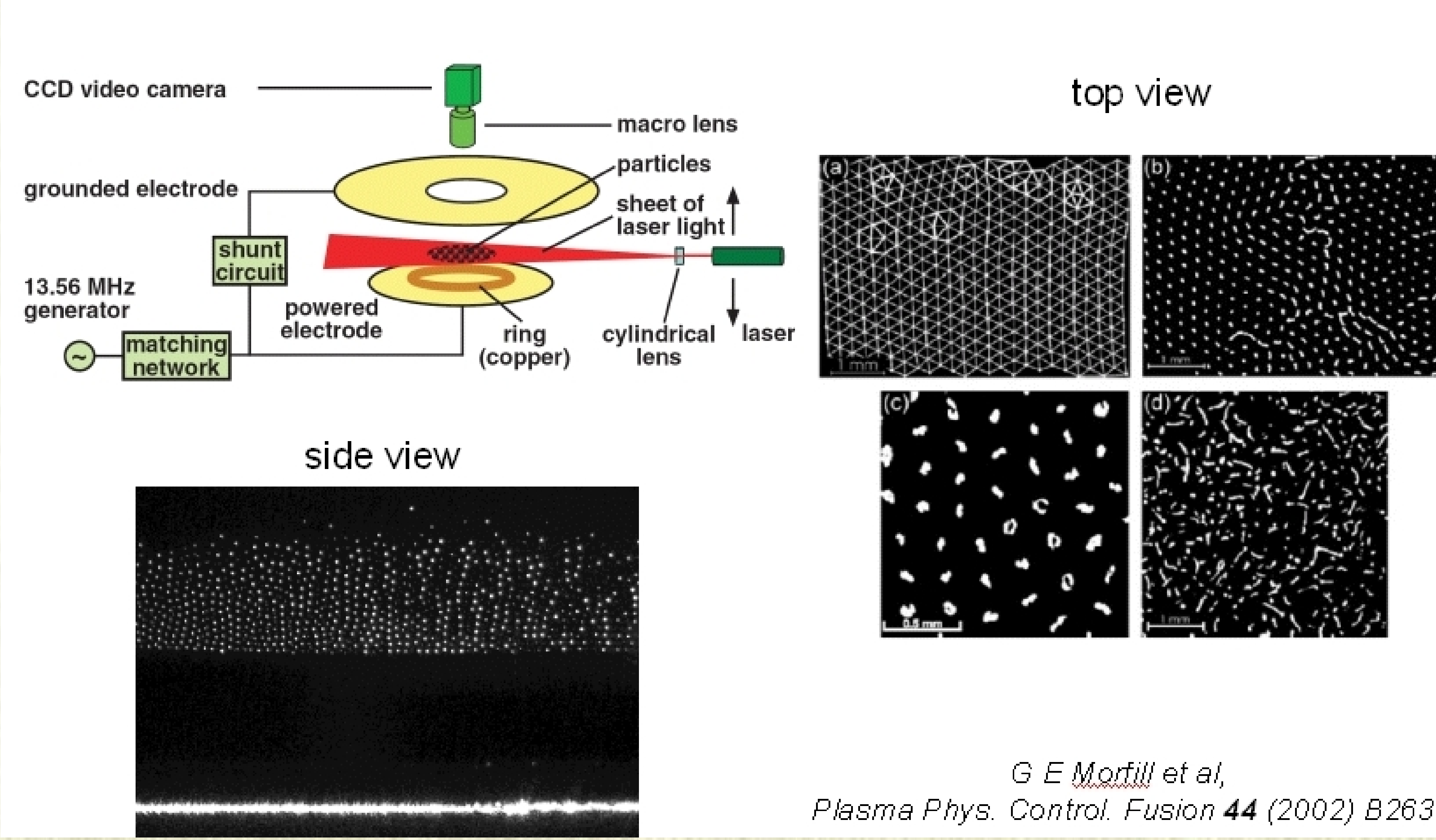
- Complex plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [IKEZI 1986]) states, depending on the value of the *effective coupling (plasma) parameter* Γ;

$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \sim \frac{Q^2}{r\,T} e^{-r/\lambda_D}$$

(r: inter-particle distance, T: temperature, λ_D: Debye length).

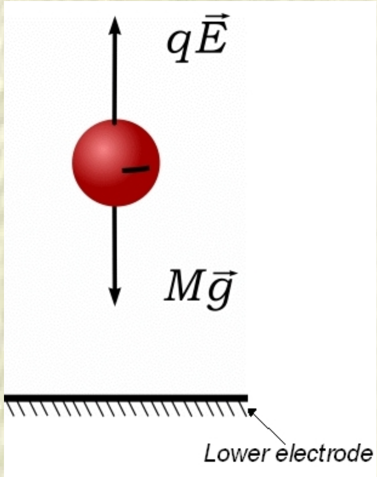
Cf.: Lecture given by *Tito Mendonça* (Sat. July 17, 2004).

Dust laboratory experiments on Earth:



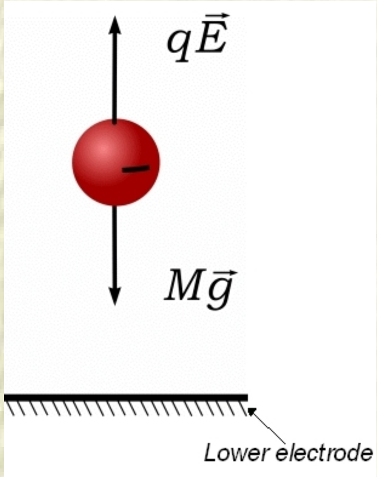
G E Morfill et al,
Plasma Phys. Control. Fusion **44** (2002) B263

Earth experiments are subject to gravity:



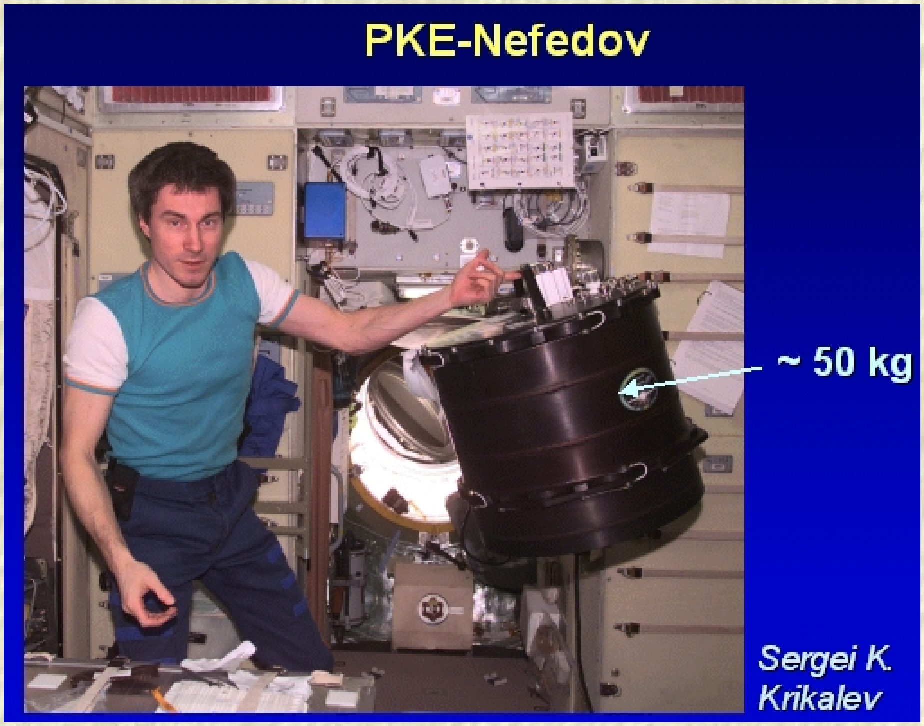
levitation in strong sheath electric field

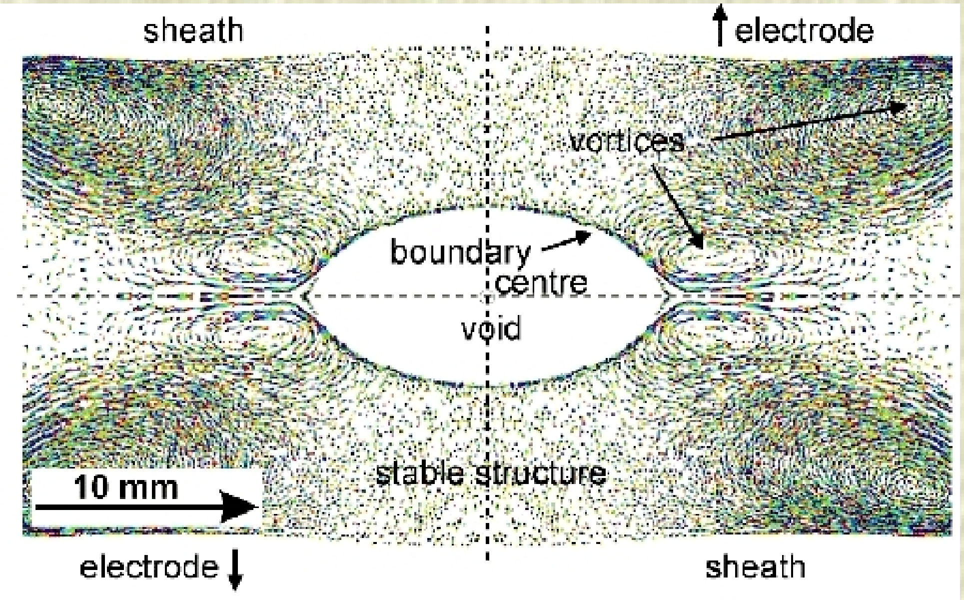
Earth experiments are subject to gravity:



levitation in strong sheath electric field

thus ....: Dust experiments in ISS (International Space Station)





void formed in the middle of discharge under microgravity conditions

(Online data from: Max Planck Institüt - CIPS).