Extended modulational instability criteria in asymmetric coupled nonlinear Schrödinger equations

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The amplitude evolution of two nonlinearly interacting waves is considered, via a set of coupled nonlinear Schrödinger-type equations (CNLSE). The dynamical profile is determined by an interplay between the wave dispersion laws and nonlinearity, via a set of self-nonlinearity and coupling coefficients. No symmetry assumption is *a priori* made on the magnitude and/or sign of relevant coefficients. A generalized dispersion relation is obtained, relating the frequency and wave-number of small perturbations near a coupled monochromatic (Stokes') wave state.

The analytical investigation focuses on two parts. First, adopting the (sole) hypothesis that the two group velocities are equal, a set of explicit (in)stability criteria are obtained for an imaginary part of the amplitude perturbation frequency to occur. These criteria extend the conditions previously obtained for symmetric CNLSE, as e.g. in nonlinear optics. Second, the role of the group velocity misfit is qualitatively pointed out, by considering the full (asymmetric) system of CNLSE in a depictive manner [1].

The analysis reveals a number of possibilities. Two (individually) stable systems may be destabilized due to coupling. Unstable systems may, when coupled, present an enhanced instability growth rate, for an extended wave number range of values. Distinct (up to three) unstable wavenumber windows may arise simultaneously. A stable coupled wave system may be destabilized under the sheer influence of a group velocity mismatch.

Distinct case studies are considered. The analysis is applied to a pair of interacting Bose-Einstein condensates (BECs), where nonlinearity and coupling potentials are related to scattering lengths (positive/negative for repulsive/attractive BEC interactions); criteria for the stability of oscillating BEC pairs are obtained and discussed [2]. The relevance of this investigation to a set of interacting ocean surface water waves in 2D – presumably leading to freak wave formation – is briefly discussed [3].

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