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Modulated Envelope Wavepackets in *Pair-Ion* and *e-p-i* Plasmas

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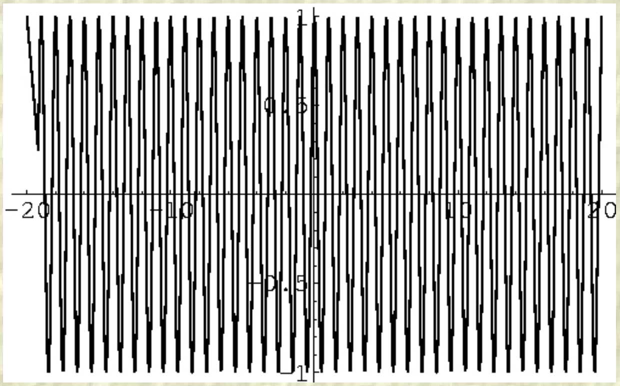
www.tp4.rub.de/∼ioannis/conf/200608-ICTP-oral.pdf

Outline

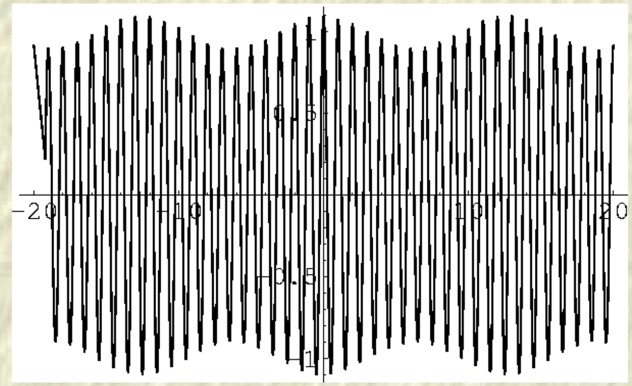
- Introduction
  - *Amplitude Modulation*: definition;
  - Relevance with space and laboratory observations;
  - *Pair-ion and e-p-i plasmas*: Prerequisites.
- Part A: Fluid model for ES waves in *p.p.*
  - The reductive perturbation *(multiple scales)* formalism.
  - *Modulational instability (MI) & envelope excitations.*
- Part B: EM waves in *p.p.*.
- Conclusions.

Intro.: The mechanism of *wave amplitude modulation*

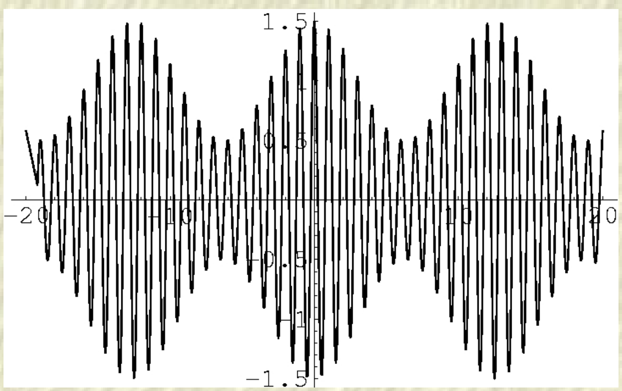
The *amplitude* of a harmonic wave may vary in space and time:



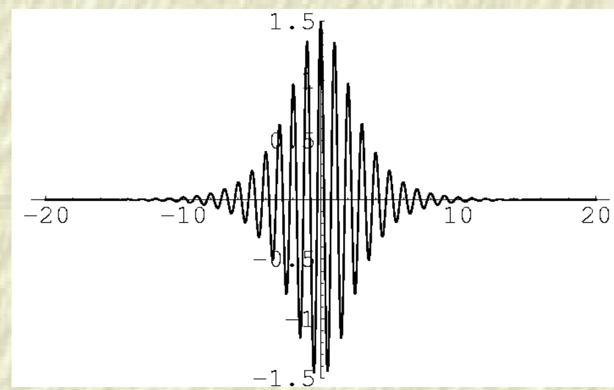
$\longrightarrow$



This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* (modulational instability) or to the formation of *envelope solitons*:



$\longrightarrow$?



Modulated structures occur widely in Nature, e.g. in oceans (freak waves, or rogue waves) ...

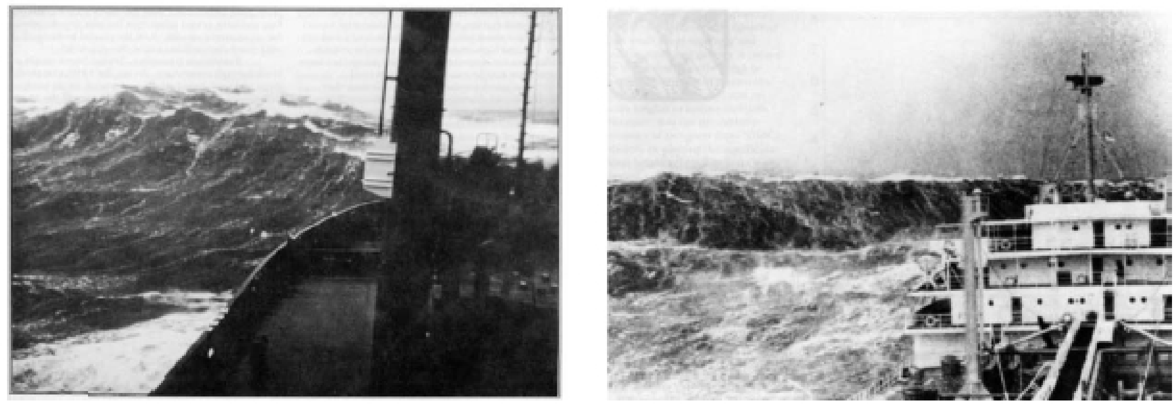


Fig. 2. Various photos of rogue waves.

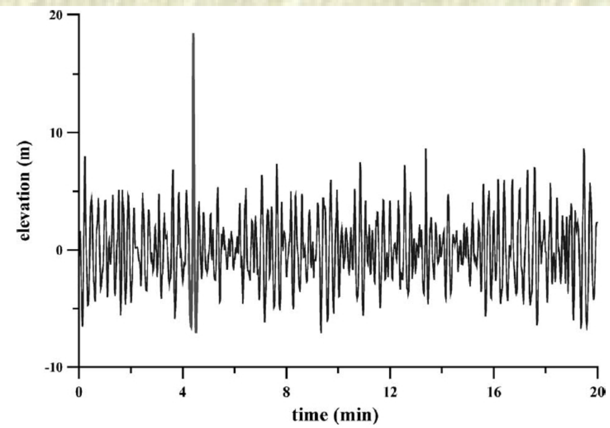


Fig. 3. Time record of the "New Year wave" in the North Sea.

(from: [Kharif & Pelinovsky, Eur. Journal of Mechanics B/Fluids **22**, 603 (2003)])

... during surface wave reconstitution in water basins, ...

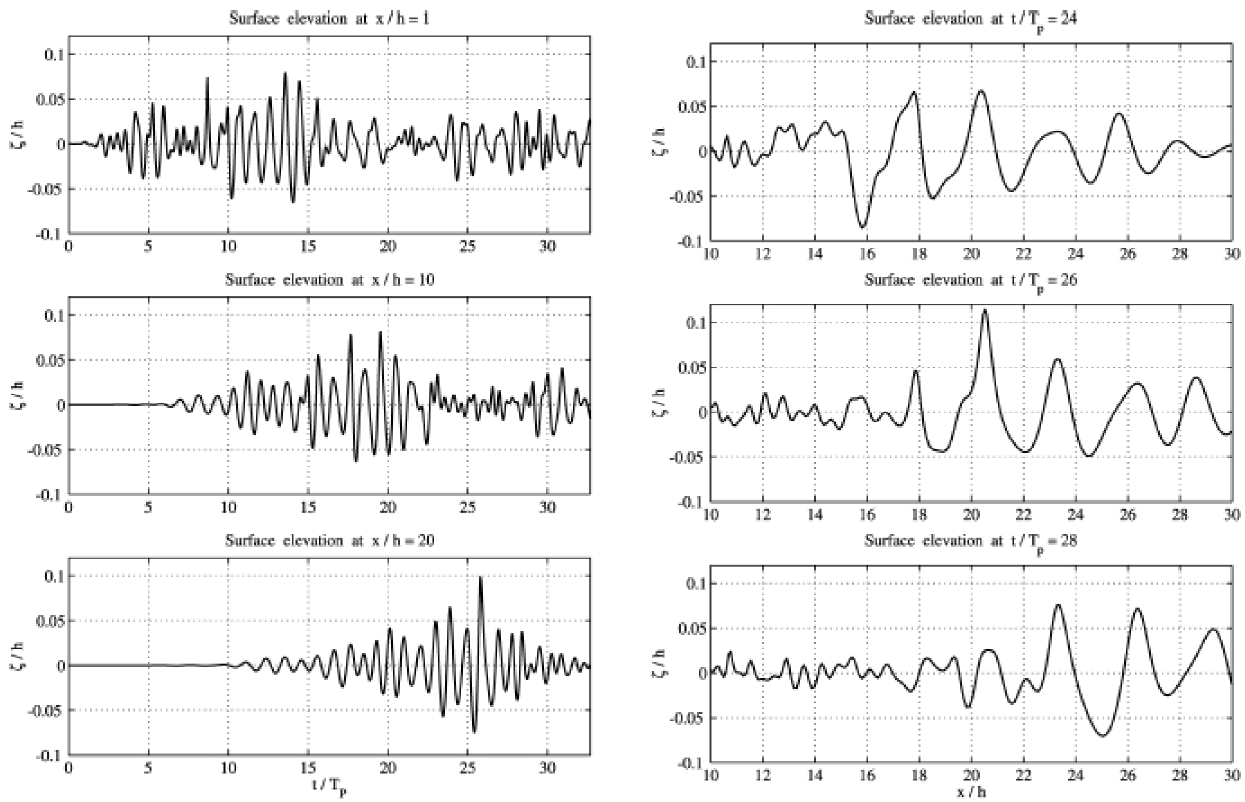
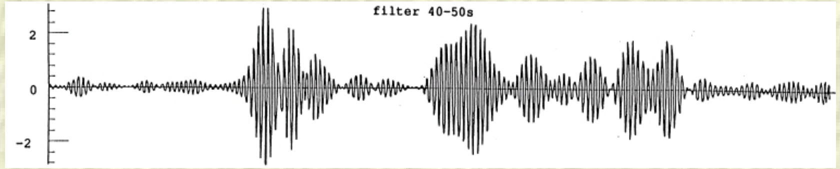
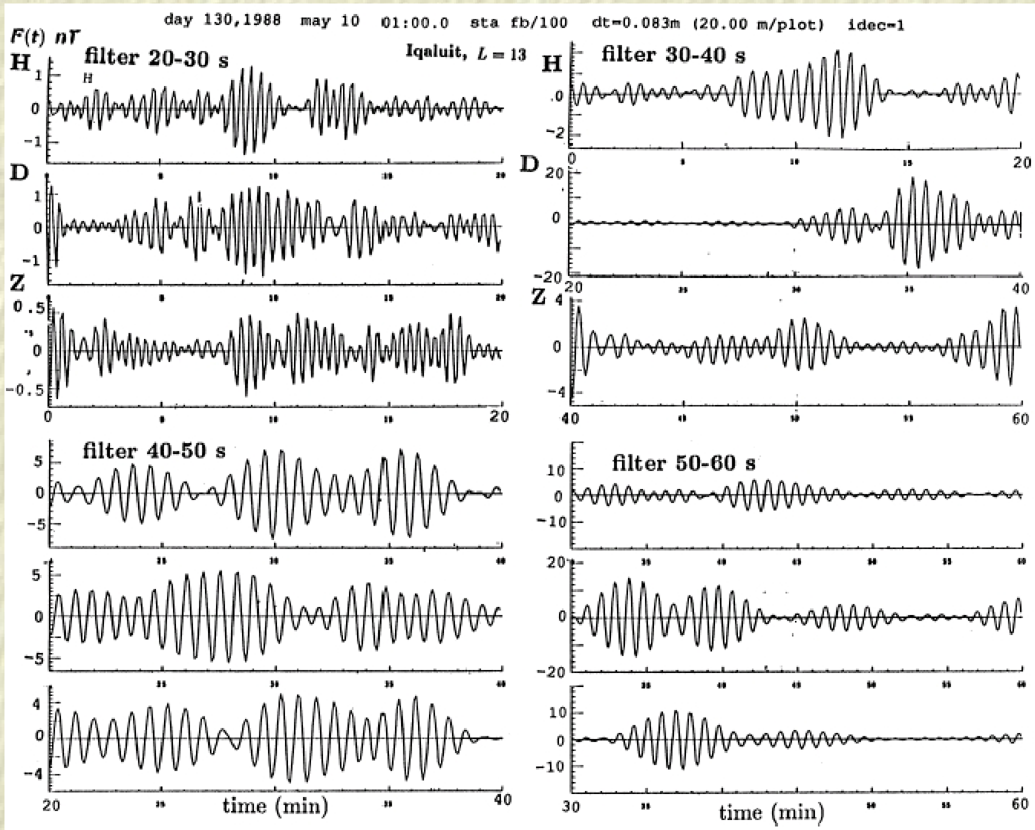


Fig. 16. Evolution of rogue wave sequence—registrations at $x = 5$ m, 50 m and 100 m (left) as well as wave profiles at $t = 75$ s, 81 s and 87 s (right hand side) (water depth $h = 5$ m, $T_p = 3.13$ s).

(from: [Klauss, Applied Ocean Research **24**, 147 (2002)])

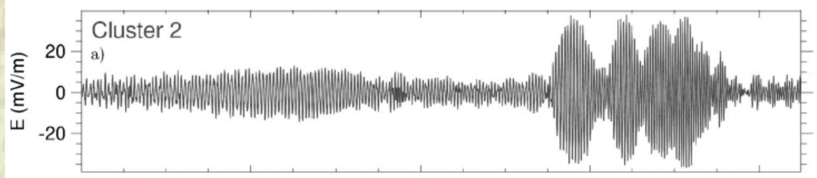
..., in EM field measurements in the magnetosphere, ...





(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

..., in satellite (e.g. CLUSTER, FAST, ...) observations:



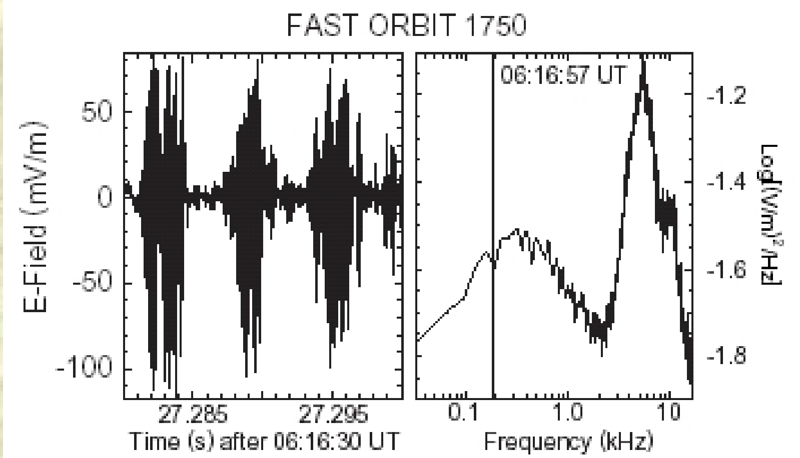


Figure 2. *Left*: Wave form of broadband noise at base of AKR source. The signal consists of highly coherent (nearly monochromatic frequency of trapped wave) wave packets. *Right*: Frequency spectrum of broadband noise showing the electron acoustic wave (at $\sim$ 5 kHz) and total plasma frequency (at $\sim$ 12 kHz) peaks. The broad LF maximum near 300 Hz belongs to the ion acoustic wave spectrum participating in the 3 ms modulation of the electron acoustic waves.

(*) From: O. Santolik *et al.*, *JGR* **108**, 1278 (2003); R. Pottelette *et al.*, *GRL* **26** 2629 (1999).

Modulational instability (MI) **was observed in simulations,**

e.g. early (1972) numerical experiments of EM cyclotron waves:

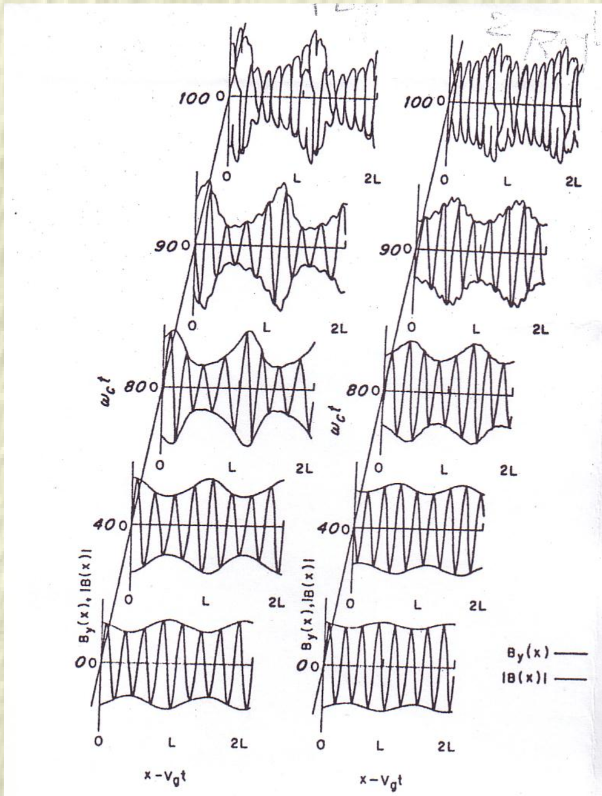


FIG. 2. Spatial variation of the y component of the wave magnetic field $B_y(x)$ (solid curve) and the magnitude of the total field $|B(x)|$ (thin curve) at different time of evolution for cases 1 (left) and 2 (right). Note the harmonic generation in $|B(x)|$ and the sudden collapse of the wave.

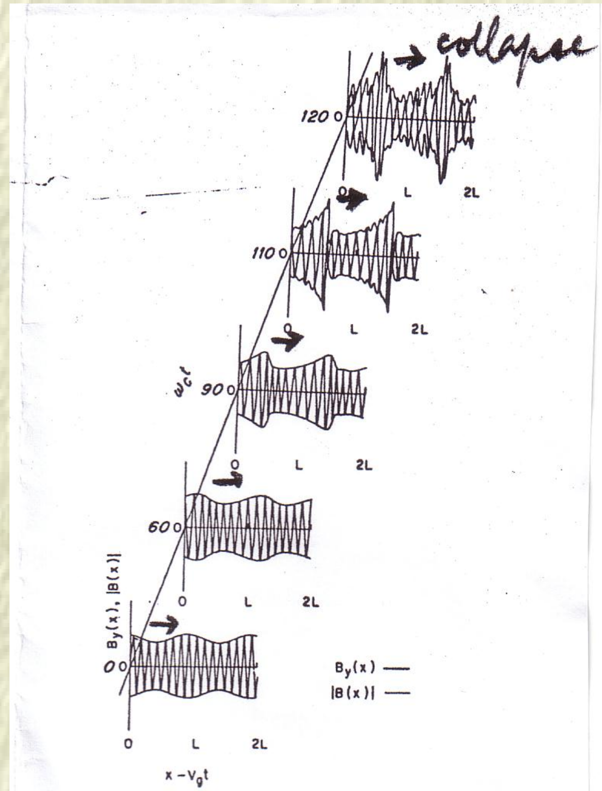


FIG. 3. $B_y(x)$ and $|B(x)|$ for case 4. Note the steepening of the wavefront which leads to the collapse of the wave.

[from: A. Hasegawa, *PRA* **1**, 1746 (1970); *Phys. Fluids* **15**, 870 (1972)].

Spontaneous MI has been observed in experiments,:

Modulational instability in a multi-component plasma

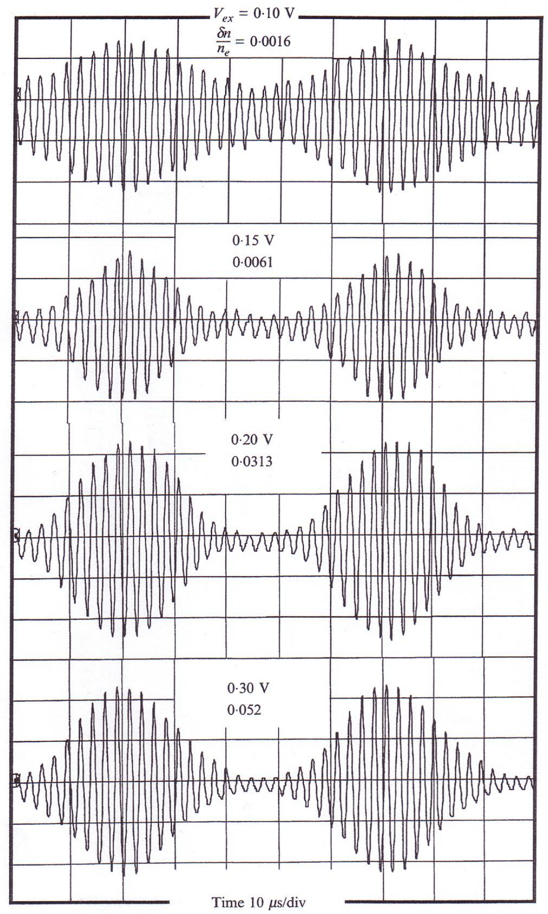


FIGURE 5. Oscilloscope traces of the detected signal for different excitation voltages. The probe was fixed at 14 cm from the grid. f_c = 400 kHz and f_m = 50 kHz.

e.g. on *ion acoustic waves*

[from: Bailung and Nakamura, *J. Plasma Phys.* **50** (2), 231 (1993)].

Questions to be addressed in this brief presentation:

- The Formalism: How can one describe the (slow) evolution (*modulation*) of a wave *amplitude* in space and time?
- Can *Modulational Instability* (MI) of plasma "fluid" modes be predicted by a simple, tractable analytical model?
- Can *envelope modulated localized structures* (such as those observed in space and laboratory plasmas) be modeled by an exact theory?
- *Focus*: Modulated *electrostatic* (ES) and *electromagnetic* (EM) waves in *pair plasmas.*

Pair-ion plasmas: **prerequisites (1)**

❑ Electron-ion plasmas:

– *electrons* e^- (charge $-e$, mass m_e),

– *ions* i^+ (charge $+Z_i e$, mass $m_i \gg m_e$),

– ...

❑ Intrinsic features (that we have long *taken for granted*):

— *Distinct electron/ion frequency scales*, e.g.

$$\omega_{p,s} = \left(\frac{4\pi n_s q_s^2}{m_s}\right)^{1/2}, \qquad \omega_{c,s} = \frac{q_s B}{m_s c} \qquad (s = e, i)$$

hence $\omega_{p,e} \gg \omega_{p,i}$, $\omega_{c,e} \gg \omega_{c,i}$.

— Longevity (recombination neglected, no overall density variation).

Pair-ion plasmas: prerequisites (2)

- Pair-ion plasmas:
  - *Positive ions* i^+ (charge $+Ze$, mass m),
  - *Negative ions* i^- (charge $-Ze$, mass m),
  - ... (heavier ions, in a multi-component eg. *e-p-i* composition).
- *No (pair-ion) frequency separation*: $\omega_{p,+} \approx \omega_{p,-} \quad \omega_{c,+} = \omega_{c,-}$.
- New Physics:

— *Novel (linear) ES/EM mode profile*

[Iwamoto PRE 1989, Stewart & Laing JPP 1992, Zank & Greaves PRE 1995].

— No Faraday rotation.

→ Talk(s) (and Lecture Notes) by F. Verheest and H. Saleem.

Pair-ion plasmas: prerequisites (3)

- Magnetized *electron-positron (e-p)* and e-p-i plasmas exist:
  - – in *pulsar magnetospheres* [Ginzburg 1971, Michel RMP 1982],
  - – in *bipolar outflows (jets) in active galactic nuclei (AGN)* [Miller 1987, Begelman RMP 1984]
  - – at *the center of our own galaxy* [Burns 1983],
  - – in the *early universe* [Hawking 1983],
  - – in *inertial confinement fusion schemes* [Liang *et al.* PRL 1998]
  - – in (very sophisticated, yet short-lived) experiments [Greaves, Surko *et al.* PoP 1994, Zhao *et al.* PoP 1996].

- *Pair-ion plasmas (p.p.)* have been formed in laboratory,
  - – in recent *fullerene ion* ($C_{60}^{\pm}$) experiments [Oohara & Hatakeyama PRL 2003].

Part A: Two-fluid model for ES waves in pair plasma or e-p-i plasma

Fluid Eqs. (for $j = 1^+, 2^-$):

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j \; = \; -s_j \frac{Ze}{m} \nabla \phi - \frac{1}{m n_j} \nabla p_j$$

$$p_j = C n_j^\gamma \, , \qquad p_{j,0} = n_{j,0} k_B T_j \, , \qquad \gamma = 1 + 2/f \, , \qquad s_j = q_j/|q_j| = \pm 1$$

Poisson's eq.

$$\nabla^2 \Phi \; = \; -4\pi \sum_s q_s \, n_s \; = \; 4\pi \, e \, (Z \, n_- - Z \, n_+ - s_3 Z_3 \, n_3)$$

Neutrality hypothesis: $Z n_{+,0} - Z n_{-,0} + s_3 Z_3 \, n_3 = 0$ (n_3 =cst.).

$3^\pm$: a massive (*immobile*) background species, eg. $3 = i^+$ in *epi* plasmas.

"Pure" p.p.: $n_3 = 0$, i.e. $n_{+,0} = n_{-,0}$, whereas $e^- p^+ i^+$ or $X^+ X^- d^\pm$: $n_3 \neq 0$.

Reductive Perturbation Technique

– 1st step. Define *multiple scales* (*fast* and *slow*) i.e. (in 2d)

$$\mathbf{r}_0 = \mathbf{r}\,, \quad \mathbf{r}_1 = \epsilon\,\mathbf{r}\,, \quad \mathbf{r}_2 = \epsilon^2\,\mathbf{r}\,, \quad \ldots$$

$$T_0 = t\,, \quad T_1 = \epsilon\,t\,, \quad T_2 = \epsilon^2\,t\,, \quad \ldots$$

$$\mathbf{r} = (x, y, z) \tag{1}$$

– 2nd step. Expand near equilibrium:

$$n_j \approx n_{j,0} + \epsilon\,n_{j,1} + \epsilon^2\,n_{j,2} + \ldots$$

$$\mathbf{u}_j \approx \mathbf{0} + \epsilon\,\mathbf{u}_{j,1} + \epsilon^2\,\mathbf{u}_{j,2} + \ldots$$

$$\phi \approx 0 + \epsilon\,\phi_1 + \epsilon^2\,\phi_2 + \ldots$$

$(\epsilon \ll 1)$.

Reductive perturbation technique *(continued)*

– 3rd step. Project on Fourier space, i.e. consider $\forall m = 1, 2, ...$

$$S_m = \sum_{l=-m}^{m} \hat{S}_l^{(m)} \, e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \hat{S}_0^{(m)} + 2\sum_{l=1}^{m} \hat{S}_l^{(m)} \cos l(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

for $S_m \in (n_m, \mathbf{u}_m, \phi_m)$, i.e. *essentially*:

$$n_1 = n_0^{(1)} + \tilde{n}_1^{(1)} \cos\theta \,, \qquad n_2 = n_0^{(2)} + \tilde{n}_1^{(2)} \cos\theta + \tilde{n}_2^{(2)} \cos 2\theta, \text{ etc.}$$

Reductive perturbation technique *(continued)*

– 3rd step. Project on Fourier space, i.e. consider $\forall m = 1, 2, ...$

$$S_m = \sum_{l=-m}^{m} \hat{S}_l^{(m)} e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \hat{S}_0^{(m)} + 2\sum_{l=1}^{m} \hat{S}_l^{(m)} \cos l(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

– 4rth step. (for multi-dimensional propagation) *Modulation obliqueness*:

the slow amplitudes $\hat{\phi}_l^{(m)}$, etc. vary *only along* the x-axis:

$$\hat{S}_l^{(m)} = \hat{S}_l^{(m)}(X_j, T_j)\,, \qquad\qquad j = 1, 2, ...$$

while the fast carrier phase $\theta = \mathbf{k}\cdot\mathbf{r} - \omega t$ is now (in 2d):

$$k_x\, x + k_y\, y - \omega t = k\, r \cos\alpha - \omega t \quad .$$

→ *Poster on oblique modulation of ES waves by R. Esfandyari-Kalejahi et al.*

First-order solution ($\sim \epsilon^1$)

- *Dispersion relation* $\omega = \omega(k)$, for $\omega \leftarrow \frac{\omega}{\omega_{p,-}}$, $\quad k \leftarrow k\lambda_{D,-} = \frac{k\,T_-^{1/2}}{m^{1/2}\,\omega_{p,-}}$:

$$\omega_1^2 \approx c_s k^2\,, \qquad \omega_2^2 \approx \omega_0^2 + c_s k^2\,,$$

where: $\beta = n_{+,0}/n_{-,0}\,, \qquad \sigma = T_+/T_- \qquad (\beta \to 1$: p.p.)

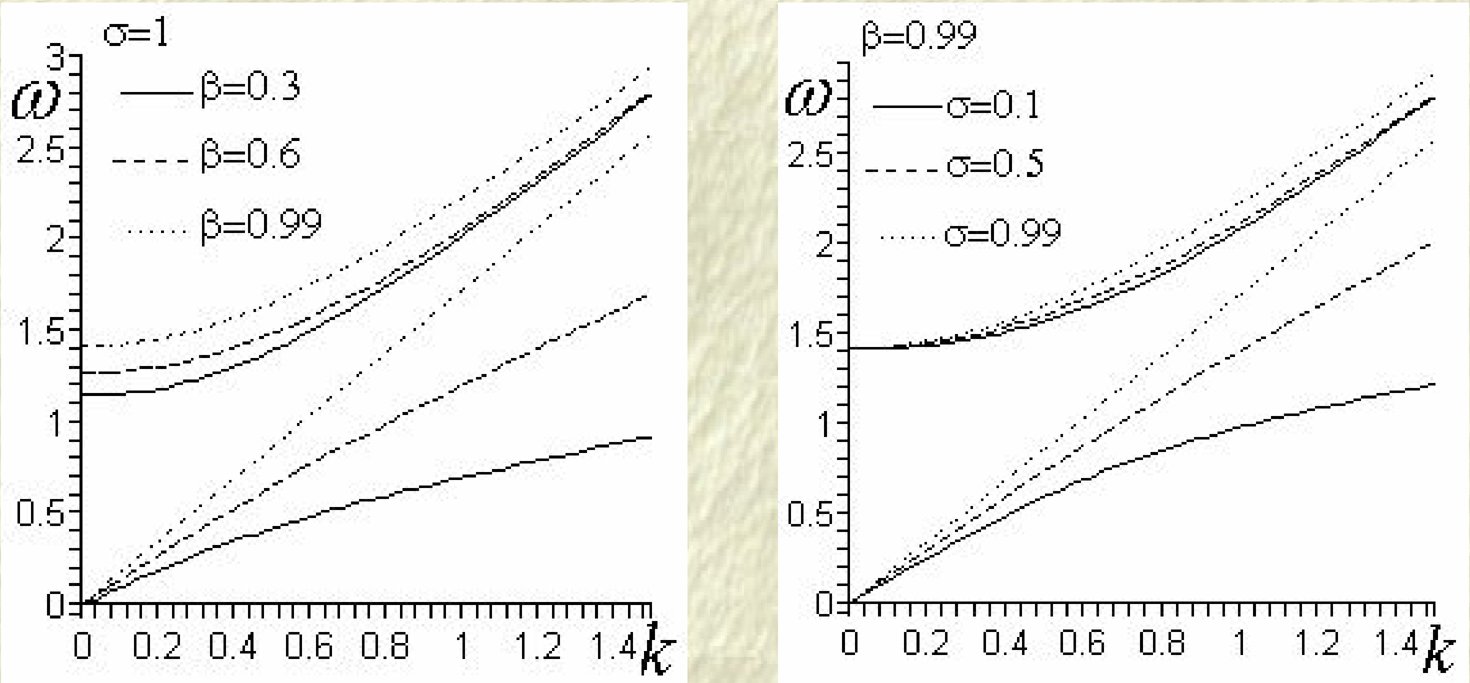
and

$$\omega_0^2 = (1+\beta)\omega_{p,-}^2\,, \qquad c_s^2 = 3\beta \frac{1+\sigma\beta}{1+\beta}\frac{T_-}{m}\,.$$

- The *solution(s)* for the 1st–harmonic amplitudes (e.g. $\propto \phi_1^{(1)}$):

$$n_{+,1}^{(1)} = \frac{\beta k^2}{\omega^2 - 3\sigma\beta^2 k^2}\phi_1^{(1)} = \frac{\beta k}{\omega}\,u_{+,1}^{(1)}\,, \qquad n_{-,1}^{(1)} = -\frac{k^2}{\omega^2 - 3k^2}\phi_1^{(1)} = \frac{k}{\omega}\,u_{-,1}^{(1)}$$

Dispersion relation vs. parameters $\beta = n_{+,0}/n_{-,0}$, and $\sigma = T_{+}/T_{-}$



[from: Esfandyari, Kourakis, Mehdipoor & Shukla, sub *JPA: Math. Phys.* (2006)].

Second-order solution ($\sim \epsilon^2$)

- From $m = 2, l = 1$, we obtain the relation:

$$\frac{\partial \psi}{\partial T_1} + v_g \frac{\partial \psi}{\partial X_1} = 0 \tag{2}$$

where
– $\psi = \phi_1^{(1)}$ is the potential correction ($\sim \epsilon^1$);

– $v_g = \frac{\partial \omega(k)}{\partial k_x}$ is the group velocity along $\hat{x}$;

– the wave's envelope satisfies: $\psi = \psi(\epsilon(x - v_g t)) \equiv \psi(\zeta)$.

- The solution, up to $\sim \epsilon^2$, is of the form:

$$\phi \approx \epsilon \, \psi \, \cos\theta \; + \; \epsilon^2 \left[\phi_0^{(2)} + \phi_1^{(2)} \cos\theta + \phi_2^{(2)} \cos 2\theta \right] + \mathcal{O}(\epsilon^3) \, ,$$

($+$ similar expressions for $n_{+/-}$ and $\mathbf{u}_{+/-}$) $\rightarrow$ *Harmonic generation!*.

Third-order solution ($\sim \epsilon^3$)

- Compatibility equation (from $m = 3, l = 1$), in the form of:

$$i \frac{\partial \psi}{\partial \tau} + P \frac{\partial^2 \psi}{\partial \zeta^2} + Q \, |\psi|^2 \, \psi = 0 \, .$$

i.e. a *Nonlinear Schrödinger–type Equation* (NLSE) .

- Variables: $\zeta = \epsilon(x - v_g t)$ and $\tau = \epsilon^2 \, t$;

- *Dispersion coefficient* P:

$$P = \frac{1}{2} \frac{\partial^2 \omega}{\partial k_x^2} = \frac{1}{2} \left[\omega''(k) \cos^2 \alpha + \omega'(k) \frac{\sin^2 \alpha}{k} \right] ; \qquad (3)$$

- *Nonlinearity coefficient* Q: ... $\rightarrow$*(omitted)*

  $=$ A *(lengthy!)* function of k, angle α and plasma parameters.

NLSE Story 1: Modulational (in)stability analysis

- The NLSE admits the *harmonic wave solution*:

$$\psi = \hat{\psi}\, e^{iQ|\hat{\psi}|^2\tau} + \text{c.c.}$$

- *Perturb* the amplitude by setting: $\hat{\psi} = \hat{\psi}_0 + \epsilon\,\hat{\psi}_{1,0}\cos(\tilde{k}\zeta - \tilde{\omega}\tau)$

- We obtain the *(perturbation)* dispersion relation:

$$\tilde{\omega}^2 = P^2\,\tilde{k}^2\left(\tilde{k}^2 - 2\frac{Q}{P}|\hat{\psi}_{1,0}|^2\right). \qquad (4)$$

- If $PQ < 0$: the amplitude ψ is *stable* to external perturbations;

- If $PQ > 0$: the amplitude ψ is *unstable* for $\tilde{k} < \sqrt{2\frac{Q}{P}}|\psi_{1,0}|$.

NLSE Story 2: Localized envelope excitations (envelope solitons)

- The NLSE:

$$i\,\frac{\partial\psi}{\partial\tau} + P\,\frac{\partial^2\psi}{\partial\zeta^2} + Q\,|\psi|^2\,\psi = 0$$

accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}$;
the *total* electric potential is then: $\phi \approx \epsilon\,\rho\,\cos(\mathbf{kr} - \omega t + \Theta)$ where the amplitude ρ and phase correction Θ depend on ζ, τ.

- If $PQ > 0$: *Bright* solitons (envelope pulses);

- If $PQ < 0$: *Dark (black/grey)* solitons (envelope holes).

Localized envelope excitations (solitons) for $PQ > 0$

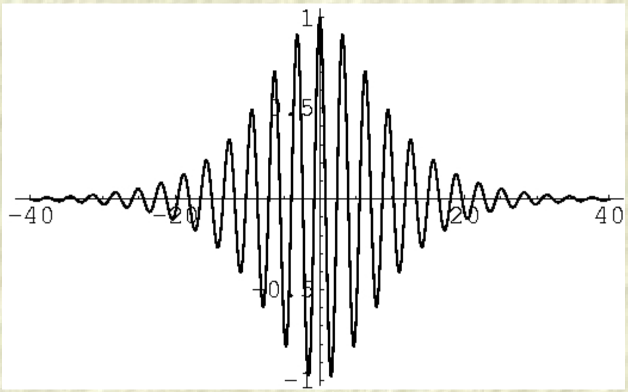
❑ The NLSE accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}$; the *total* electric potential is then: $\phi \approx \epsilon\, \rho\, \cos(\mathbf{kr} - \omega t + \Theta)$ where the amplitude ρ and phase correction Θ depend on ζ, τ.

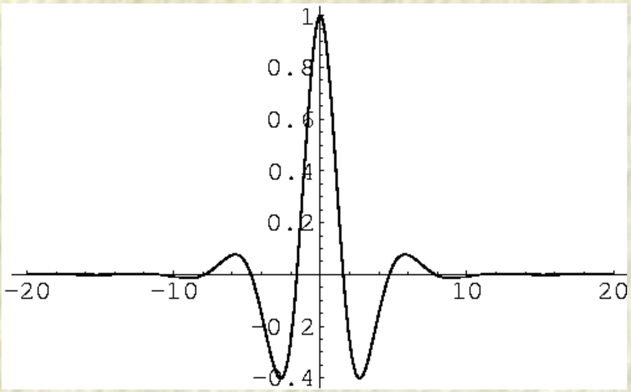
❑ Bright–type envelope soliton (pulse):

$$\rho = \rho_0\, \mathrm{sech}\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta\; - (\Omega + \frac{1}{2}v^2)\tau\right]. \tag{5}$$

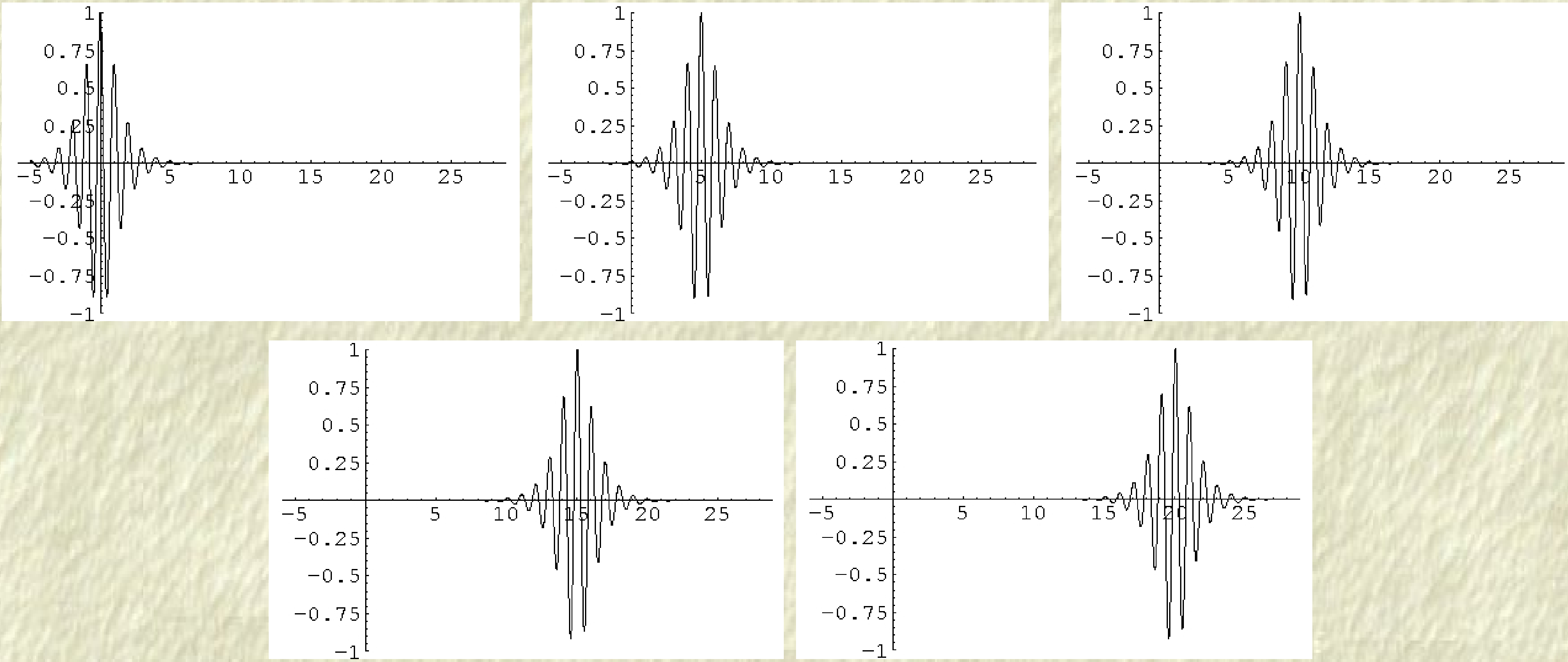
$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$

This is a propagating (and *oscillating*) localized pulse:

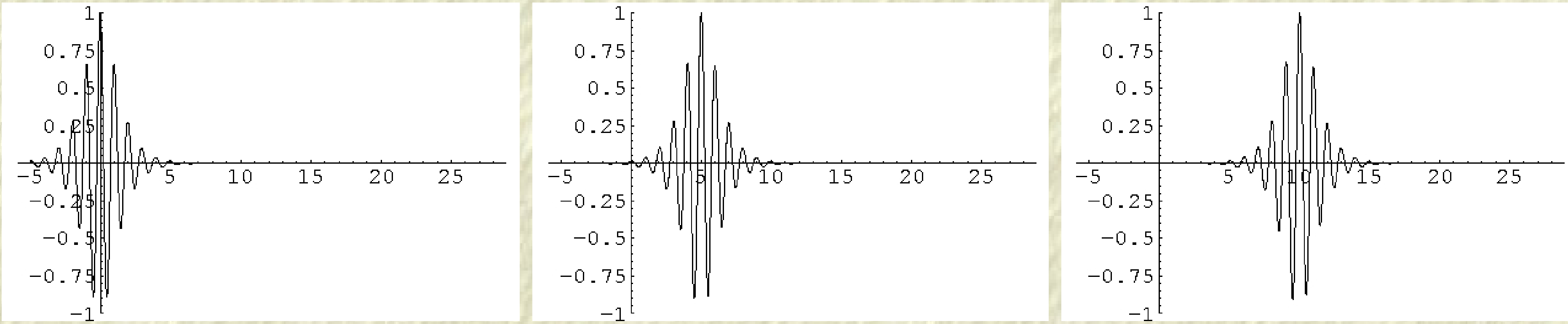




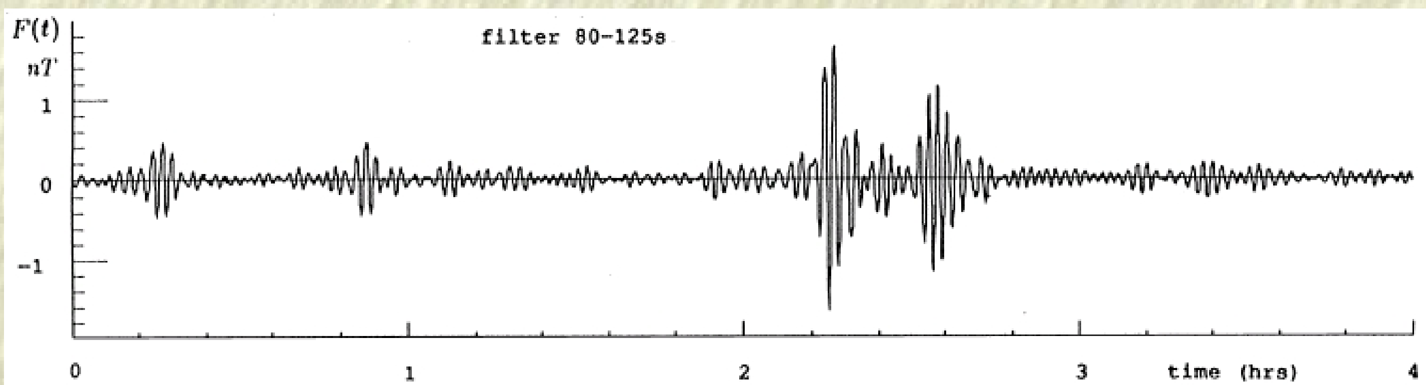
Propagation of a bright envelope soliton (pulse)



Propagation of a bright envelope soliton (pulse)

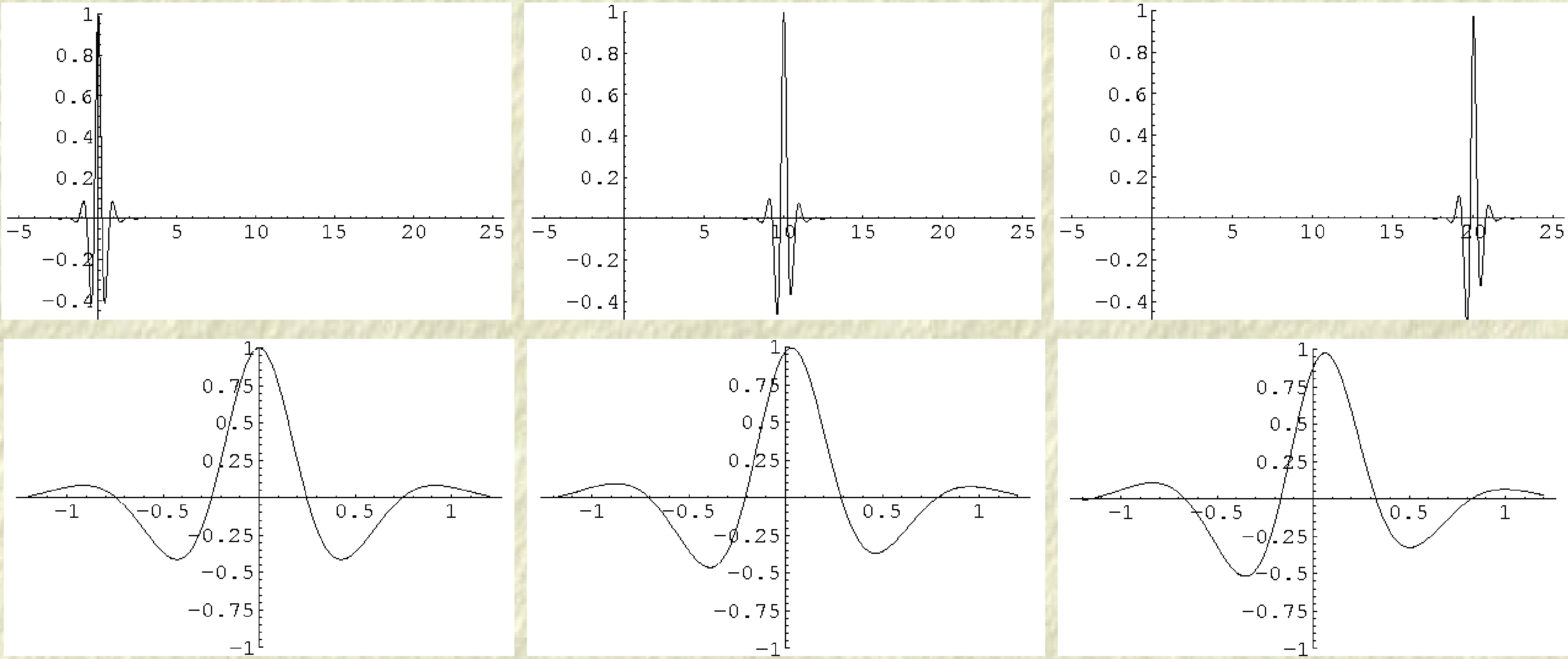


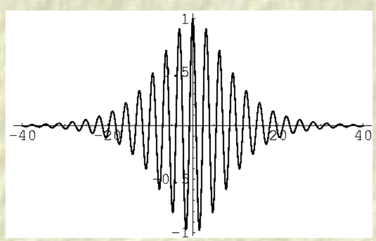
Cf. electrostatic plasma wave data from satellite observations:



(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

Propagation of a bright envelope soliton (continued...)





(⟶ *see video*)

Localized envelope excitations for $PQ < 0$

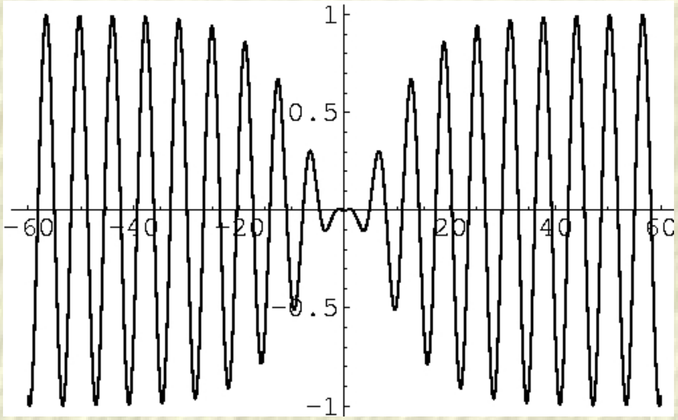
❑ Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1^2\right)\tau\right]$$

$$L' = \sqrt{2|\frac{P}{Q}|\frac{1}{\rho_1}} \qquad (6)$$

This is a *propagating localized* *hole* (zero density void):

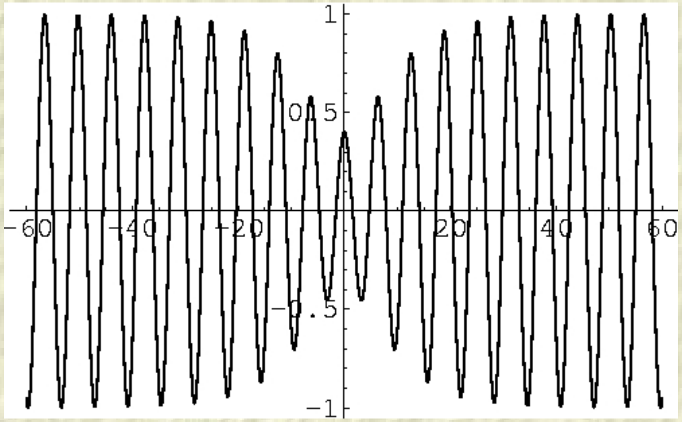


Localized envelope excitations for $PQ < 0$

❑ Grey–type envelope solution (*void soliton*):

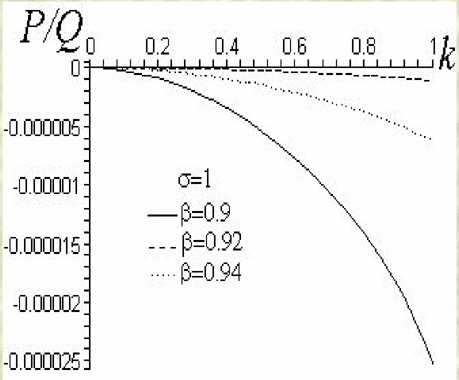
$$\rho = \pm\rho_2 \left[1 - a^2\,\mathrm{sech}^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$
$$\Theta = \dots$$
$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{a\rho_2} \qquad (7)$$

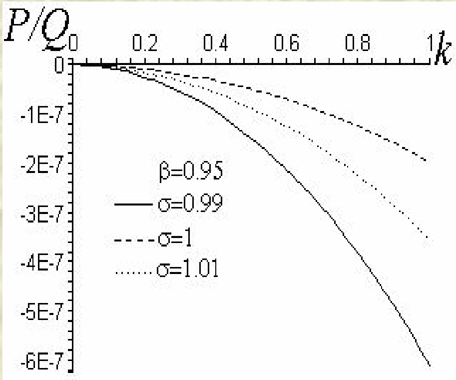
This is a propagating *(finite-density)* void:



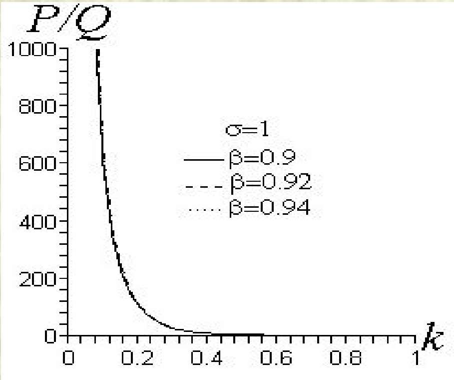
Stability profile (ESW): P/Q ratio versus reduced wavenumber $k\lambda_{D,-}$

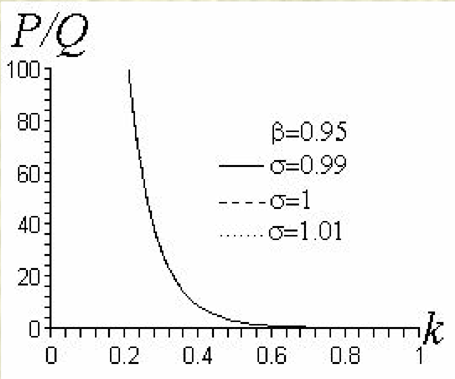
– *Lower (acoustic) mode*:





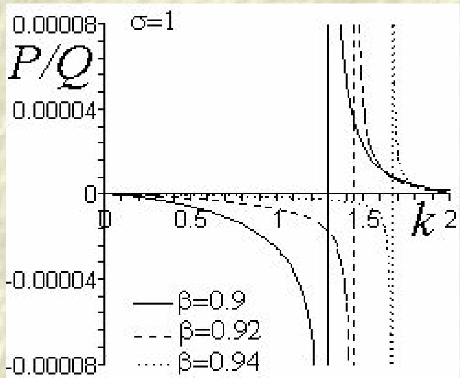
– *Upper (optic-type) mode*:

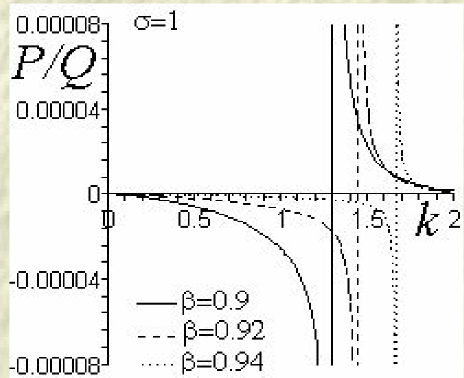




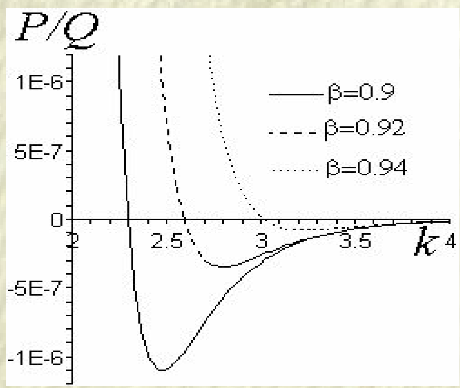
Stability profile (ESW): P/Q ratio versus reduced wavenumber $k\lambda_{D,-}$

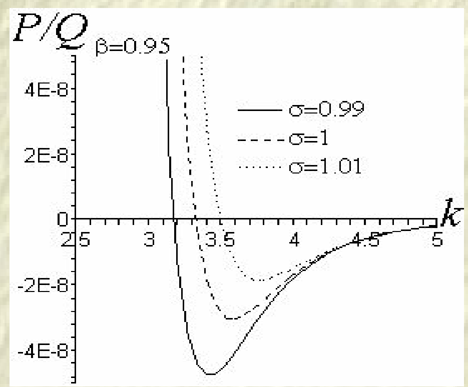
– *Lower (acoustic) mode*:





– *Upper (optic-type) mode*:





Part B: Two-fluid model for *oblique* EM waves in p.p. or e-p-i plasma

Fluid Eqs. (for $j = 1^+, 2^-$):

$(q_1 = -q_2 = +Ze)$

$(m_1 = m_2 = m)$

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = \frac{q_j}{m_j} \left(\mathbf{E} + \frac{1}{c} \mathbf{u}_j \times \mathbf{B} \right)$$

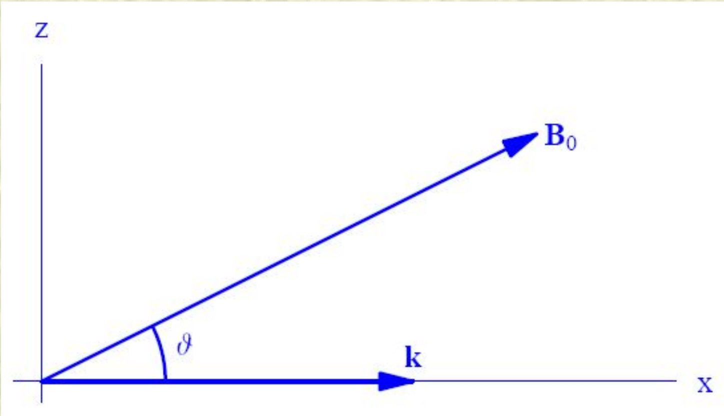
Maxwell's laws:

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \,, \qquad \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c} \sum_j n_j q_j \,, \qquad \nabla \cdot \mathbf{B} = 0$$

+ a convenient frame:

$\mathbf{k} = (k, 0, 0)$

$\mathbf{B}_0 = (B_0 \cos\theta, 0, B_0 \sin\theta)$



First-order ($\sim \epsilon^1$): linear dynamics

- *Dispersion relation*: $D(\omega, k; \theta) = d_0(\omega, k) + d_1(\omega, k) \sin^2\theta = 0$

$$
\begin{aligned}
d_0(\omega, k) &\equiv D(\omega, k; \theta = 0) \\
&= (\omega^2 - \omega_{p,eff}^2) \\
&\quad \times \left\{ \left[(\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega^2 \omega_{p,eff}^2 \right]^2 - \omega^2 \Omega^2 (\omega_{p,1}^2 - \omega_{p,2}^2)^2 \right\} \\
&= (\omega^2 - \omega_{p,eff}^2) \\
&\quad \times \left\{ (\omega + \Omega) \left[-(\omega^2 - c^2k^2)(\omega - \Omega) + \omega \omega_{p,1}^2 \right] + \omega(\omega - \Omega)\omega_{p,2}^2 \right\} \\
&\quad \times \left\{ (\omega - \Omega) \left[-(\omega^2 - c^2k^2)(\omega + \Omega) + \omega \omega_{p,1}^2 \right] + \omega(\omega + \Omega)\omega_{p,2}^2 \right\} ,
\end{aligned}
$$

$$
d_1(\omega, k; \theta) = -c^2k^2\Omega^2 \left\{ c^2k^2\omega_{p,eff}^2(\omega^2 - \Omega^2) + \omega^2 [4\omega_{p,1}^2\omega_{p,2}^2 - (\omega^2 - \Omega^2)\omega_{p,eff}^2] \right\} ,
$$

Notation: $\omega_{p,eff}^2 = \omega_{p,1}^2 + \omega_{p,2}^2$; Ω is the (common) cyclotron frequency.

First-order solution ($\sim \epsilon^1$)

$$
\begin{aligned}
n_j^{(11)} &= n_{j,0}\frac{k}{\omega}u_{j,x}^{(11)} = c_{j,n,y}^{(11)}B'_y + c_{j,n,z}^{(11)}B'_z\,, \\
u_{j,i}^{(11)} &= c_{j,i,y}^{(11)}B'_y + c_{j,i,z}^{(11)}B'_z\,. \\
E_i^{(11)} &= c_{el,i,y}^{(11)}B'_y + c_{el,i,z}^{(11)}B'_z \qquad (\text{for } j = 1,2 \text{ and } i = x,y,z) \\
B_x^{(nl)} &= \text{cst}\,.
\end{aligned}
$$

where

$$
\begin{aligned}
c_{j,x,y}^{(11)} &= i(-1)^{j+1}\frac{\omega^2\Omega^3\sin\theta\cos\theta}{k\,[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta]} \\
c_{j,x,z}^{(11)} &= \frac{\Omega^2\sin\theta}{k\,(\omega^2-\Omega^2)\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}\times \\
&\quad \{-\omega^3(\omega^2-\Omega^2-\omega_{p,eff}^2) \\
&\qquad +i\Omega\omega_{p,eff}^2\cos\theta\left[(-1)^{j+1}\omega^2+\Omega\cos\theta(i\omega+(-1)^j\Omega\cos\theta)\right]\}
\end{aligned}
$$

($j, j' = 1, 2$ and $j' \neq j$) *(continued* $\rightarrow$)

First-order solution ($\sim \epsilon^1$) *(continued)*

$$c_{j,y,y}^{(11)} = \frac{\omega\Omega^2(\omega^2-\omega_{p,eff}^2)\cos\theta}{k\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}$$

$$c_{j,y,z}^{(11)} = \frac{\Omega\omega}{k\,(\omega^2-\Omega^2)}\left[i(-1)^{j+1}\omega+\frac{\Omega^3\omega_{p,eff}^2\cos\theta\sin^2\theta}{\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta}\right]$$

$$c_{j,z,y}^{(11)} = i\,(-1)^j\,\frac{\omega^2\Omega(\omega^2-\omega_{p,eff}^2-\Omega^2\sin^2\theta)}{k\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}$$

$$c_{j,z,z}^{(11)} = \frac{\Omega^2\left\{\omega^3(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos\theta(\omega\cos\theta+i(-1)^j\Omega\sin^2\theta)\right\}\cos\theta}{k\,(\omega^2-\Omega^2)\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}\,,$$

$$c_{el,x,y}^{(11)} = c_{el,x,z}^{(11)} = \frac{\omega\Omega^2\omega_{p,eff}^2\sin\theta\cos\theta}{ck\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}\,,$$

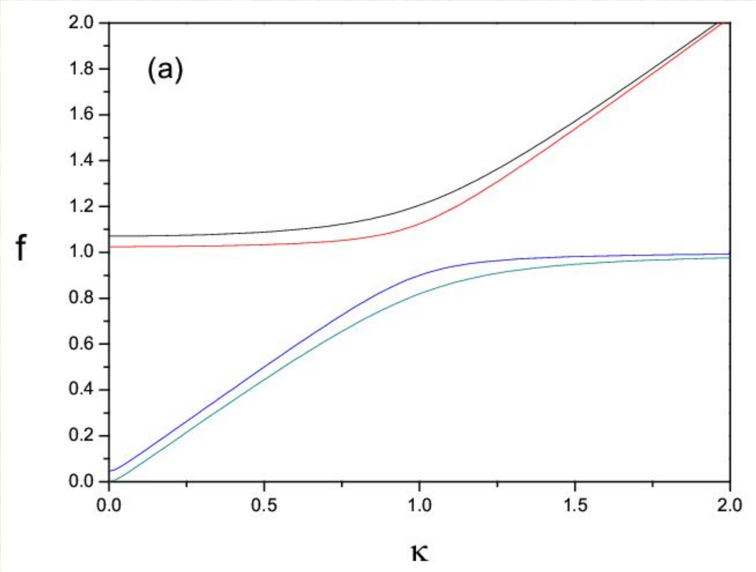
$$c_{el,y,y}^{(11)} = c_{el,z,z}^{(11)} = 0$$

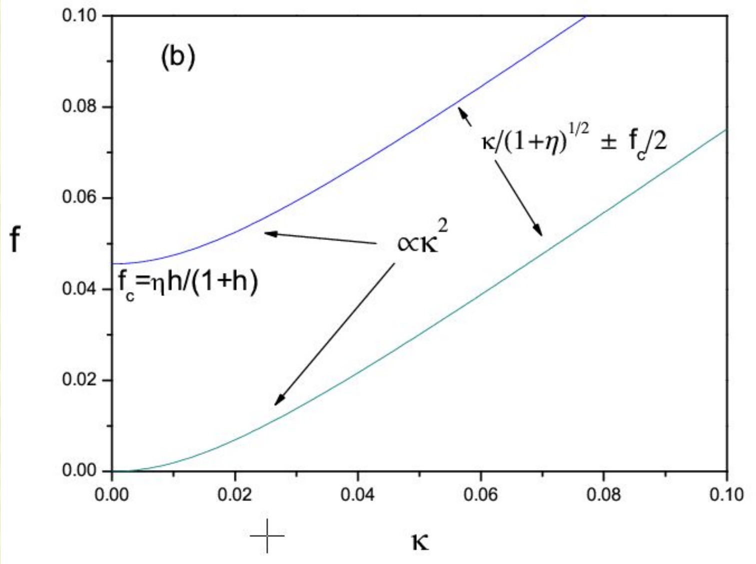
$$c_{el,y,z}^{(11)} = -c_{el,z,y}^{(11)} = \frac{\omega}{ck}\,.$$

$\| -$ Dispersion relation: $f = \omega/\Omega$ vs. $\kappa = ck/\Omega$ & effect of $n_{+,0} \neq n_{-,0}$

$$D_{\|}(\omega, k) = (\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega^2\omega_{p,eff}^2 \pm \omega\Omega(\omega_{p,1}^2 - \omega_{p,2}^2) = 0$$

$$D_{\|,p.p.}(\omega, k) = (\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - 4\omega^2\omega_p^2 = 0$$





Here $\eta = (n_{+,0} - n_{-,0})/(n_{+,0} + n_{-,0}) = 0.5$, $h = \omega_{p,eff}^2/\Omega^2 = 0.1$.

[from: Cramer, ICPP 2006]; Kourakis, Verheest & Cramer, in preparation.

Second-order solution ($\sim \epsilon^2$)

- From $m = 2, l = 1$, we obtain a compatibility condition in the form:

$$\frac{\partial \tilde{B}_\perp}{\partial T_1} + v_g \frac{\partial \tilde{B}_\perp}{\partial X_1} = 0 \qquad (8)$$

– $\tilde{B}_\perp = B_z^{(11)} + C B_y^{(11)}$ is the magnetic field (envelope) correction;

– $v_g = \frac{d\omega(k)}{dk} = -\frac{\partial D/\partial k}{\partial D/\partial \omega}$ is the group velocity;

– the magnetic field correction (amplitude) satisfies:

$$B_{y/z} = B_{y/z}(X_1 - v_g T_1) \equiv B_{y/z}(\zeta)\,.$$

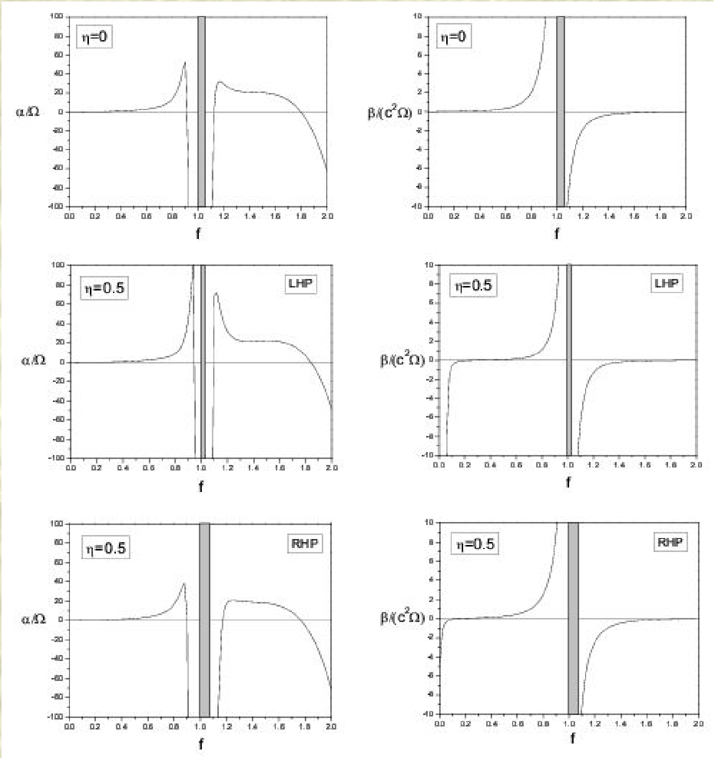
– C is a (complex) phase shift factor; $C \to \pm i$ for $\theta \to 0$.

- *Second and zeroth harmonic generation!* (expressions omitted).

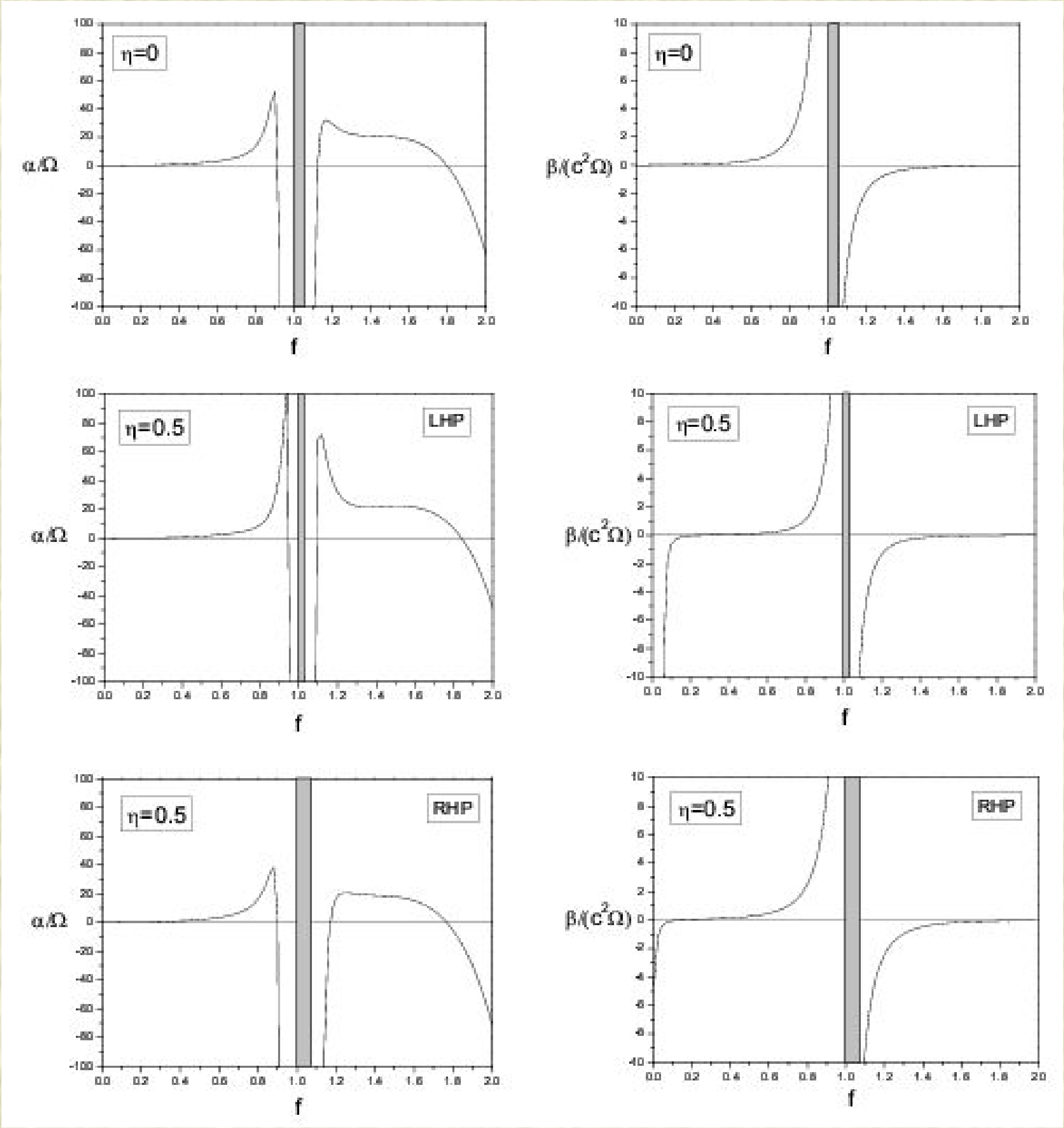
(Coupled) Nonlinear Schrödinger equation(s) for the amplitudes $B_{y,z}^{(11)}$

e.g. for $\theta = 0$: $$i\,\frac{\partial \tilde{B}_\perp}{\partial \tau} + \beta\,\frac{\partial^2 \tilde{B}_\perp}{\partial \zeta^2} + \alpha\,|\tilde{B}_\perp|^2\,\tilde{B}_\perp = 0\,.$$

Influence of the 3rd species on EM wave stability:



[from: Cramer, ICPP 2006]; Kourakis, Verheest & Cramer, in preparation.


η=0
η=0
η=0.5
LHP
η=0.5
LHP
η=0.5
RHP
η=0.5
RHP
α/Ω
β/(c²Ω)
f

Conclusions (1/2)

- *Amplitude Modulation* (due to carrier self-interaction) is a generic manifestation of nonlinearity on oscillatory mode dynamics.
- Modulated ES and EM waves may undergo spontaneous *modulational instability*; this may drive nonlinear evolution towards ...
- ... *energy localization*, via the formation of *envelope localized structures* (envelope solitons);
- *Modulated (ES, mostly) plasma wave packets* observed in Space and in the lab, may be efficiently modelled this way.
- *NLS solitons* bear specific "signature" (features like e.g. amplitude-width relation) which allow for a verification of the theory via observations.

(cont.) →

Conclusions *(cont.)*

- Among ES modes in pair plasmas, (i) the (Langmuir-like) upper mode is modulationally unstable ($\rightarrow$ *bright* envelope solitons), (ii) the acoustic branch is stable ($\rightarrow$ envelope *holes*), yet heavily damped (for $T_+ = T_-$).

- EM modes in p.p. are modified if a third, massive species is present; e.g., parallel p.p. modes (1 acoustic + 1 upper O-mode, both modulationally unstable for low k) split into four distinct modes, featuring *3 new frequency gaps*; two of these are stabilized, due to the 3rd species.

- Future extensions of the theory : relativistic effects, 2D geometry, more exotic localized envelope solutions (*dromions*?), ...

- Inherent drawback of a fluid theory: *Landau damping* overseen,
  $\rightarrow$ to be considered *a posteriori*.

Thank You !

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Material from:

I. Kourakis, F. Verheest and N. Cramer, *in preparation* (2006);

I. Kourakis, A. Esfandyari-Kalejahi, M. Mehdipoor and P.K. Shukla, Phys. Plasmas, 13 (5), 052117/1-9 (2006);

A. Esfandyari-Kalejahi, I. Kourakis, M. Mehdipoor and P.K. Shukla, J. Phys. A: Math. Gen., submitted (2006).

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