Existence and stability of multibreathers in a dusty plasma crystal

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I. Introduction

Recent studies of collective processes in a dust-contaminated plasma (DP) [1] have revealed the formation of strongly coupled DP crystals by highly charged dust grains, typically in the sheath region above a horizontal negatively biased electrode in experiments (e.g. [1,2]. Typical low-frequency oscillations are known to occur [2] in these mesoscopic dust grain quasi-lattices in the longitudinal (in-plane, acoustic mode), horizontal transverse (in-plane) and vertical transverse (off-plane, inverse dispersive optic-like mode) directions.

In the present work a (quasi) one-dimensional DP crystal in considered. The transverse motion in this system is described by a Klein-Gordon like Hamiltonian with a quartic potential. Provided the potential (nonlinearity) constants by experiments [3,4] and using the results of [5,6], we prove that this system can support multi-site localised oscillations (multibreathers) [7].

II. Equation of motion

The vertical grain displacement obeys an equation in the form

$$\frac{d^2\delta z_n}{dt^2}+\nu\frac{d\delta z_n}{dt}+\omega_0^2\left(\delta z_{n+1}+\delta z_{n-1}+2\delta z_n\right)+\omega_g^2\delta z_n+\alpha\left(\delta z_n\right)^2+\beta\left(\delta z_n\right)^3=0$$

where $\delta z_n = z_n(t)-z_0$ denotes the small displacement of the n–th grain around the (levitated) equilibrium position z_0, in the transverse (z–) direction. The characteristic frequency results from the dust grain (electrostatic) interaction potential $\Phi(r)$, e.g. for a Debye-Hückel potential, one has $\omega_{0,D}^2 = q^2/\left(Mr_0^3\right)\left(1+r_0/\lambda_D\right)\exp\left(-r_0/\lambda_D\right)$ where λ_D denotes the effective DP Debye radius. The damping coefficient ν accounts for dissipation due to collisions between dust grains and neutral atoms. The gap frequency ω_g and the nonlinearity coefficients α, β are defined via the overall vertical force: $F(z) = F_{e/m} - Mg \approxeq -M[\omega_g^2(\delta z_n)+\alpha(\delta z_n)^2+ (\delta z_n)^3]+O[(\delta z_n)^4]$, which has been expanded around z_0 by formally taking into account the (anharmonicity of the) local form of the sheath electric and/or magnetic field(s), as well as, possibly, grain charge variation due to charging processes [8]. Recall that the electric/magnetic levitating force(s) $F_{e/m}$ balance(s) gravity at z_0. Notice the difference in structure from the usual nonlinear Klein-Gordon equation used to describe one-dimensional oscillator chains, TDLWs ('phonons') in this chain are stable only in the presence of the field force $F_{e/m}$.

For convenience, we may re-scale the time and vertical displacement variables over appropriate quantities, i.e. the characteristic (single grain) oscillation period ω_g^{-1} and the lattice constant r_0, respectively, viz. $t = \omega_g^{-1}$ and $z_n = r_0 q_n$ Eq. (1) is thus expressed as:

$$\frac{d^2 q_n}{d\tau^2}+\varepsilon\left(q_{n+1}+q_{n-1}-2q_n\right)+q_n+\alpha' q_n^2+\beta' q_n^3=0$$

where the (dimensionless) damping term, now expressed as $\left(\nu/\omega_g\right)dq_n/d\tau \equiv \nu'\dot{q}_n$, will be henceforth omitted in the left-hand side. The coupling parameter is now , and the nonlinearity coefficients are now: $a' = \alpha r_0/\omega_g^2$ and $\beta' = \beta r_0^2/\omega_g^2$

The experiment on anharmonic single grain oscillations by Ivlev et al. [4], carried out in Garching (Germany),provides $\alpha' \simeq -0.5$ and $\beta' \simeq 0.07$ (for a lattice spacing, typically, of the order of r_0=1mm). Note that the damping coefficient ν was as low as $\nu/2\pi \simeq 0.067\ sec^{-1}$, so that (with $\omega_g/2\pi \simeq 17$ sec−1) one has: $\nu' \simeq \nu/\omega_g \simeq 0.004$(the pressure in that experiment was kept as low as 0.5 Pa).

Zafiu et al. [3], (Kiel, Germany) provides various possible values for the anharmonicity parameters, via successive experiments: $\alpha' \simeq +0.02/+0.016/-0.27$ and $\beta' \simeq -0.16/-0.17/-0.03$. Again, we consider lattice spacing of the order of r_0=1mm, damping very low (ν'=0.02).

The results of the experiment on linear TDLWs by Misawa et al. [6] allows for a rough estimation of the coupling strength ω_g 155 ≃ sec^{-1} and $\omega_0 \simeq 19.5\ sec^{-1}$, which give $\varepsilon \simeq 0.016$. Note that the effective damping term was kept as low as _ $\nu \simeq 0.239\ sec^{-1}$, i.e. $\nu' = \nu/\omega_g \simeq 0.00154$.

III. Existence of Multibreathers in Dusty Plasma Crystals

The above system can be produced by a Hamiltonian of the form

$$H = H_0+\varepsilon H_1 = \sum_{i=-\infty}^{\infty}\left(\frac{p_i^2}{2}+V_i\left(x_i\right)\right)+\frac{\varepsilon}{2}\sum_{i=-\infty}^{\infty}\left(x_{i+1}-x_i\right)^2$$

with $V(x)=\frac{x^2}{2}+\frac{a'}{3}x^3+\frac{\beta'}{4}x^4$ and **negative** ε. This potential could be considered as a generalisation of the potential used in [9] since it includes also a cubic term.

Consider at the anticontinuous limit ε =0,n+1 oscillators moving in periodic orbits satisfying $\omega_i = k_i\,\omega = 2\pi\,k_i/T$. Then, under rather general conditions (see e.g. [5]) this motion is continued for ε small enough to supply multibreather solutions.

The 3 sets of values of [3] provide similar results. We consider α'=−0.27 and β'=−0.03. The frequency ω is a monotone function of the amplitude and consequently of J (fig.1). For ε small enough, the stable breather is the one with the central oscillators moving in phase [6]. Consider a small amplitude breather, soon it is destroyed since the linear spectrum reaches +1 (fig.2). For larger amplitudes, the breather remains stable until the eigenvalues which correspond to the central oscillator coll des with the linear spectrum and complex instability occurs. Note that by increasing the value of ε the eigenvalues return to the unit circle. So, we have a "bubble" of instability and the breather is finally destroyed when the eigenvalues meet at –1 and a period doubling bifurcation occurs (fig.3). Since, we can choose the amplitude of the breather freely in the permitted area of motion, we can always find a breather solution that is stable for ε =0.016.

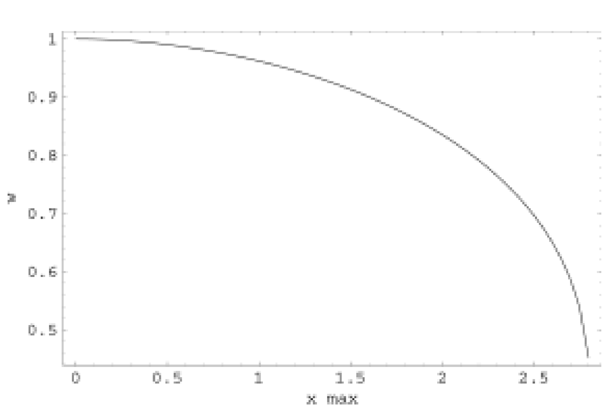

Fig.1: $\omega(J)$ for the first set of α' and β'

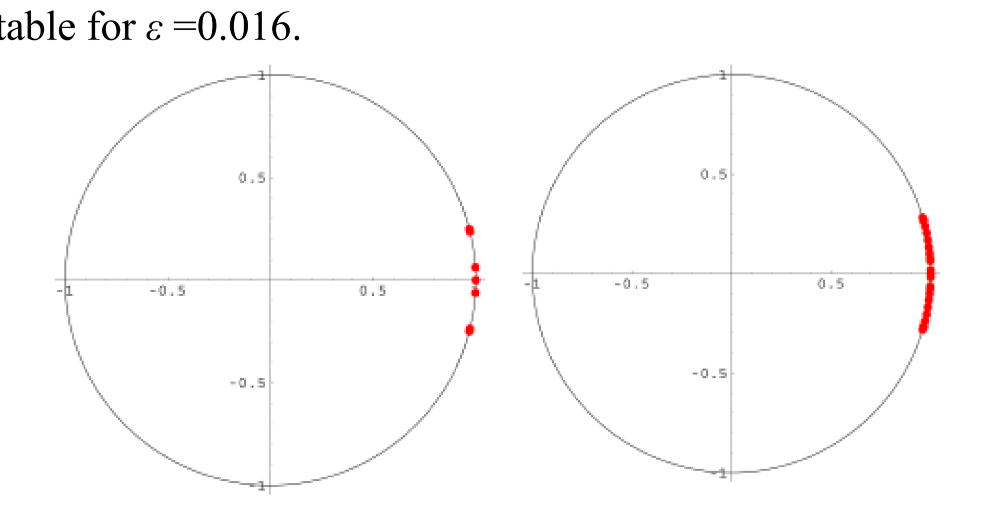

Fig.2: The destruction scenario of a small amplitude multibrreather

The time evolution of an 1:1 multibreather is shown in fig. 4.

On the hand, more exotic constructions, such as a 2:3 multibreather (fig.5), cannot occur since the choice of the amplitude of the breather is very limited and it destabilises very quickly (for very small values of ε).

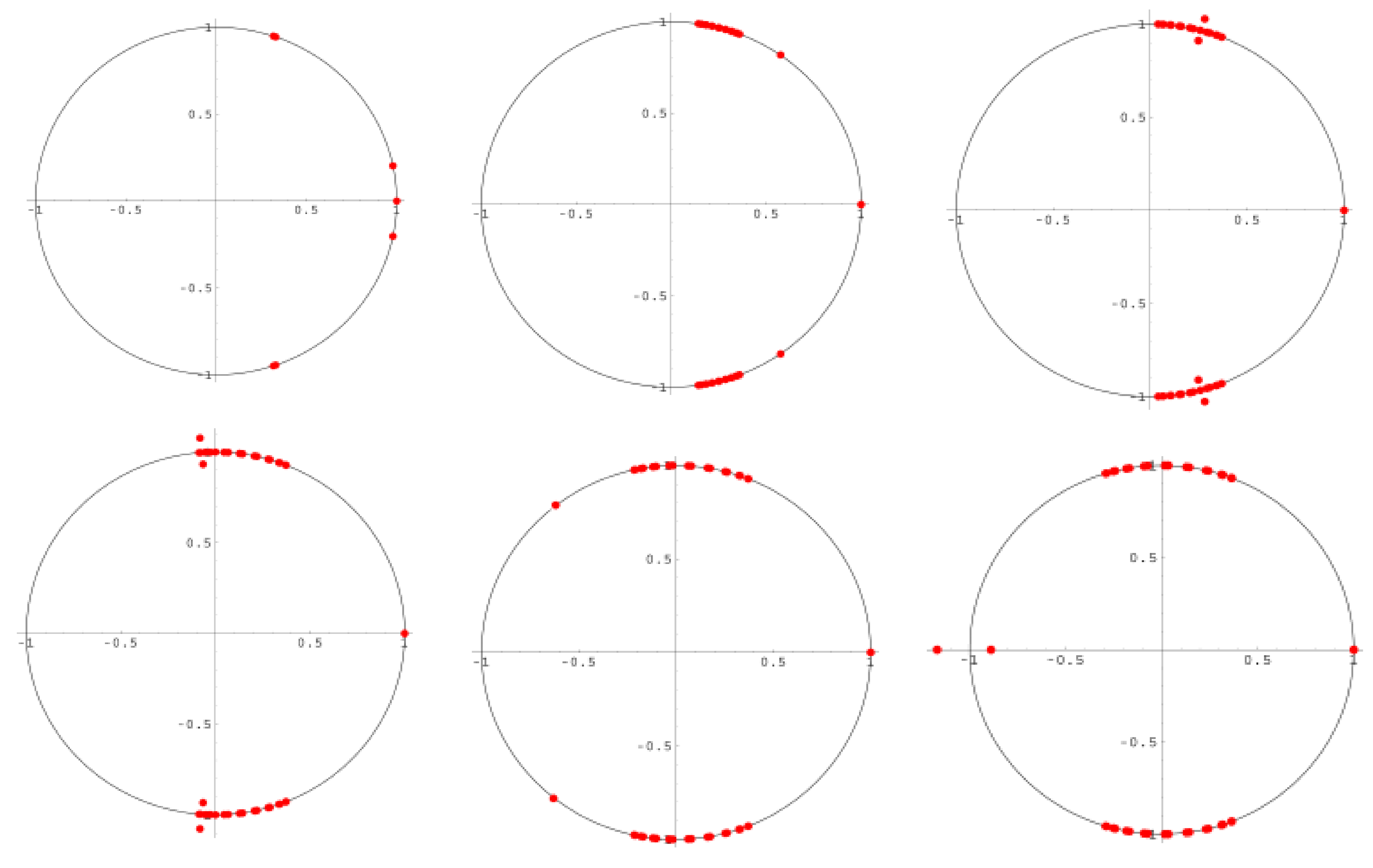

Fig.3: The destabilisation scenario for large amplitude multibrethers

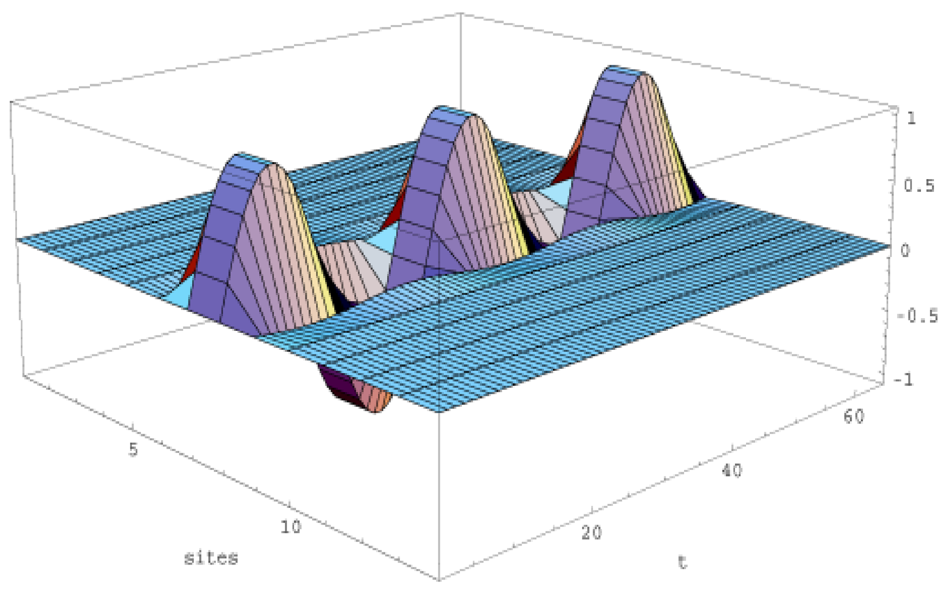

Fig.4: Time evolution of a 1:1 multibreather

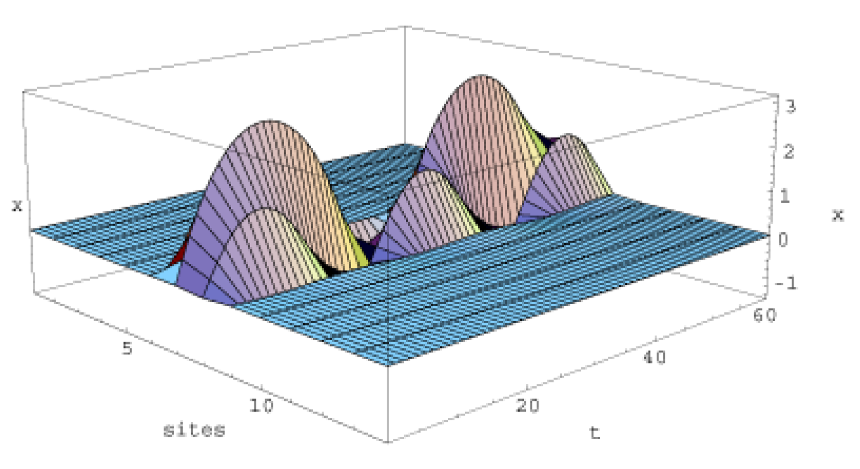

Fig.5: Time evolution of a 2:3 multibreather. This one cannot be supported my the system

By using the parameter values given in [4] we get every multibreather mentioned above. In addition, this system supports and some more exotic cases. Note that the curve $\omega(J)$ is note monotone in this case (fig.6). So,, we can have the same frequency for two values of the amplitude, which gives birth to 1:1 asymmetric multibreathers (fig.7). This kind of breathers, for suitable amplitude, remain stable for values up to ε =0.02, so they can be supported by the system.

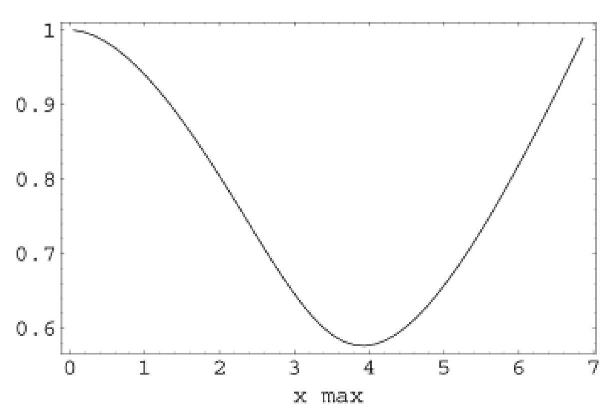

Fig.6: $\omega(J)$ of the system proposed in [4].

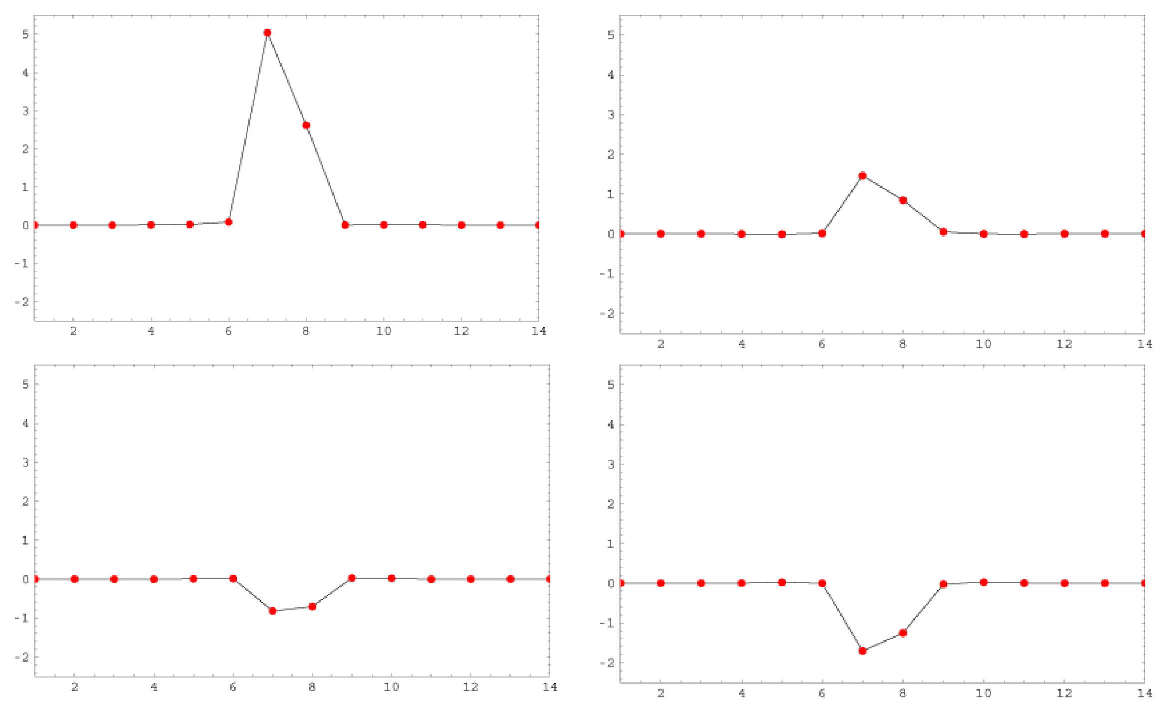

Fig.7: Time evolution of a 1:1 asymmetric multibreather.

IV. Conclusions – Future Work

We have shown that the quasi one-dimensional dusty plasma crystals can support some multibreather motions, while some others are excluded due to the nature of the system although they are theoretically predicted [5]. We plan to confirm these results using experiments. The intriguing in this work is that we finally obtained a macroscopic system to check our theoretical results. After that we shall expand our work to a 2D dusty plasma crystal which self-organises to a triangular lattice.

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