3rd FSA Workshop on Space Plasma Physics
Gent (Belgium), September 27-29, 2006

Modulated EM wavepackets in *pair-ion* and *e-p-i* plasmas:

Recent results on the ordinary (O-) mode

Ioannis Kourakis

Universiteit Gent, Sterrenkundig Observatorium, Gent, Belgium

www.tp4.rub.de/∼ioannis

In collaboration with: F Verheest, M Hellberg, G S Lakhina.

www.tp4.rub.de/∼ioannis/conf/200609-FSAW-oral.pdf

Outline

- Introduction

  – *The physical mechanism: Amplitude modulation (AM)* – formulation, relevance with space and laboratory observations.

  – *The context: Pair-ion and e-p-i plasmas* – Prerequisites.

- General fluid model for EM waves in multi-component plasmas

  – The reductive perturbation *(multiple scales)* formalism for AM.

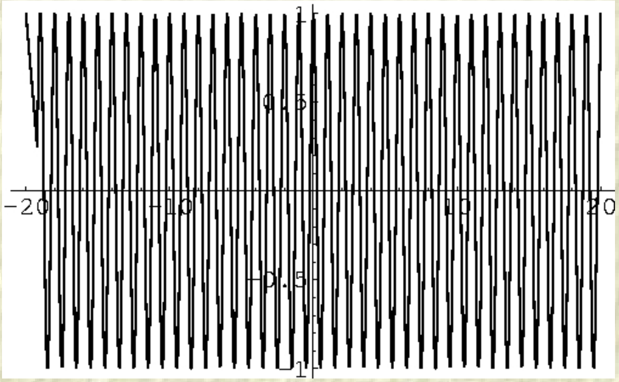
  – AM, *Modulational instability (MI) & envelope excitations.*

  – The ordinary (O-) mode for *e-i, e-p-i* and *pair plasmas (p.p.)*.

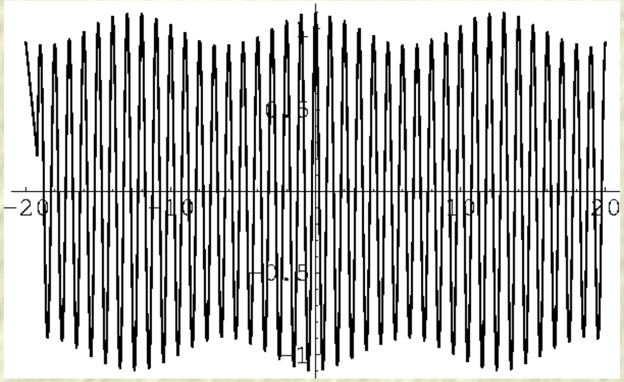
- Conclusions.

Intro.: The mechanism of *wave amplitude modulation*

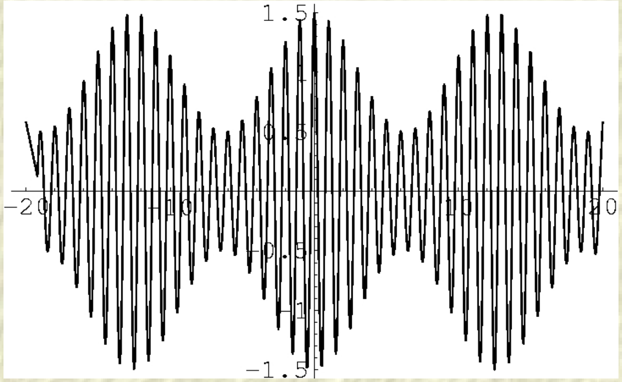
The *amplitude* of a harmonic wave may vary in space and time:



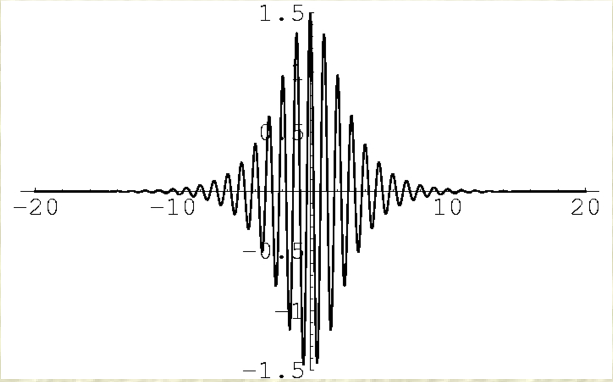
⟶



Amplitude modulation (AM) may lead to wave *collapse* (*modulational instability, MI*) or to the formation of *localized wavepackets*:



⟶ ?



Modulated structures occur widely in Nature, e.g. in oceans (freak waves, or rogue waves) ...

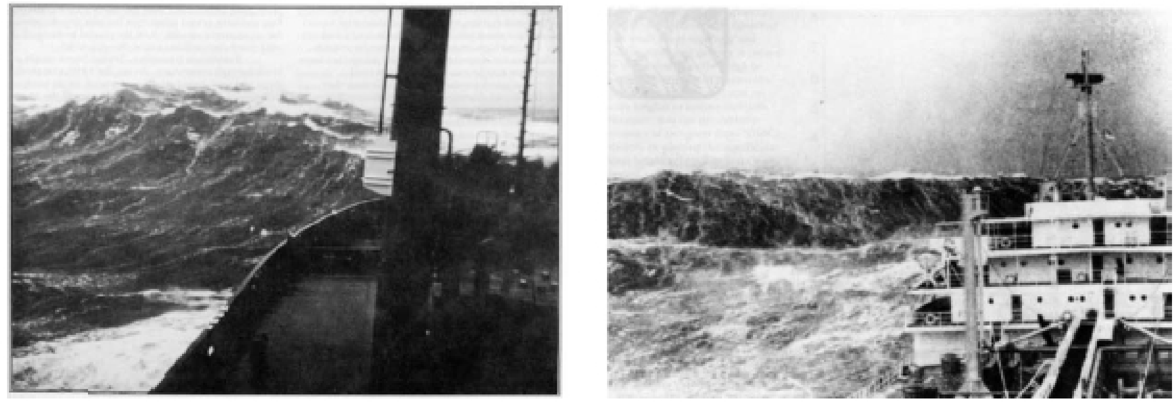


Fig. 2. Various photos of rogue waves.

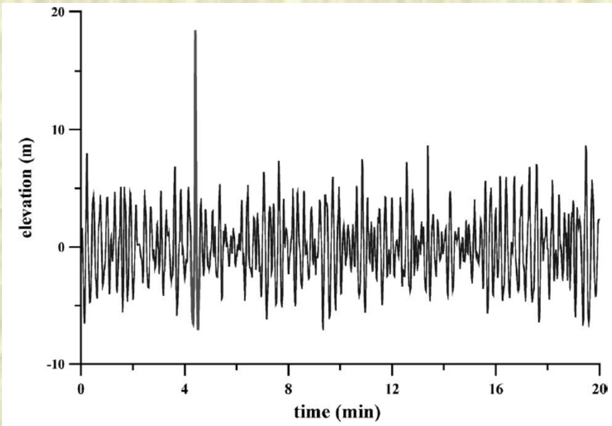


Fig. 3. Time record of the "New Year wave" in the North Sea.

(from: [Kharif & Pelinovsky, Eur. Journal of Mechanics B/Fluids **22**, 603 (2003)])

... during surface wave reconstitution in water basins, ...

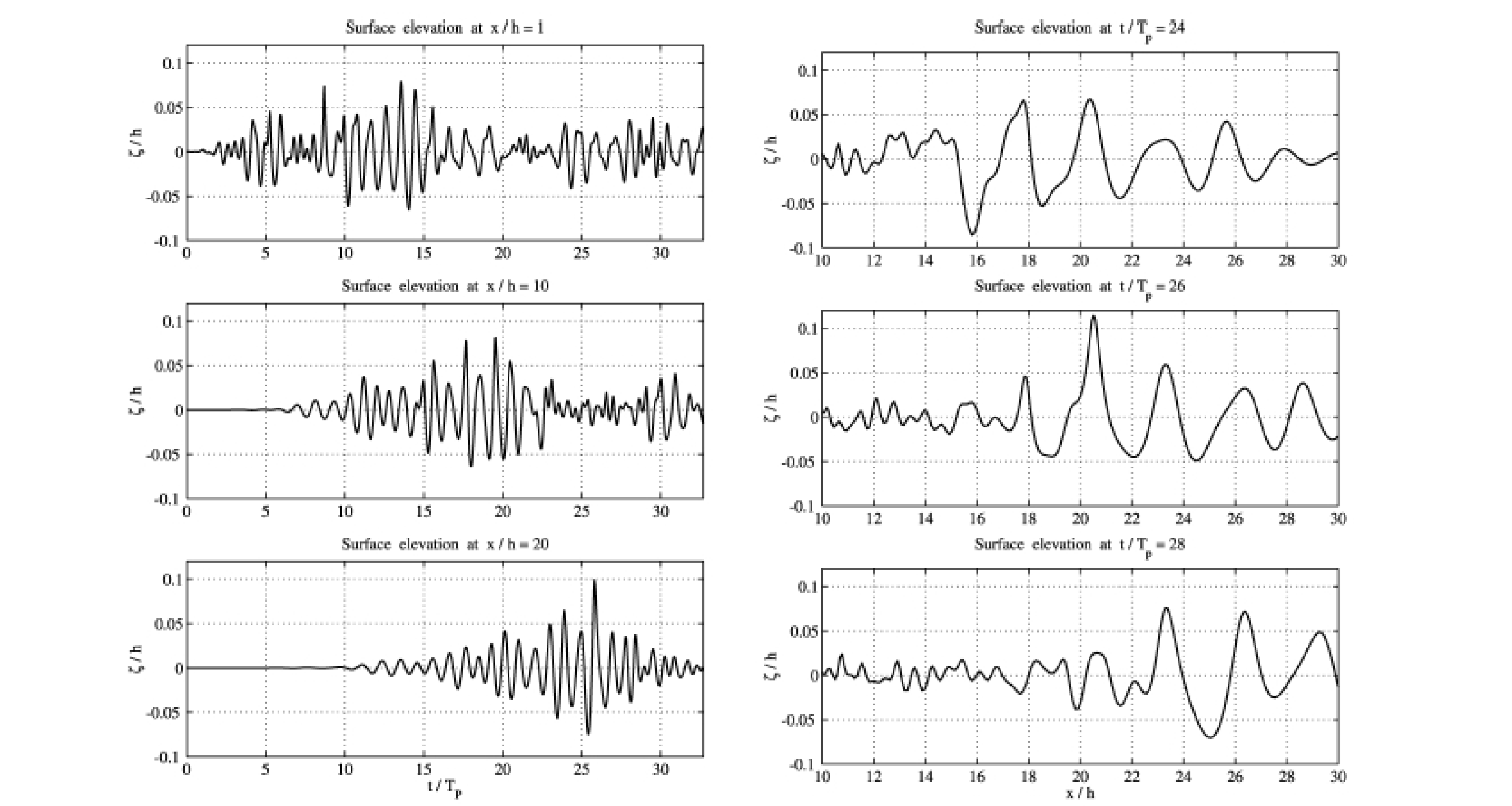
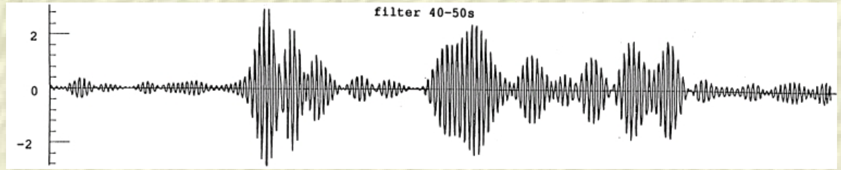
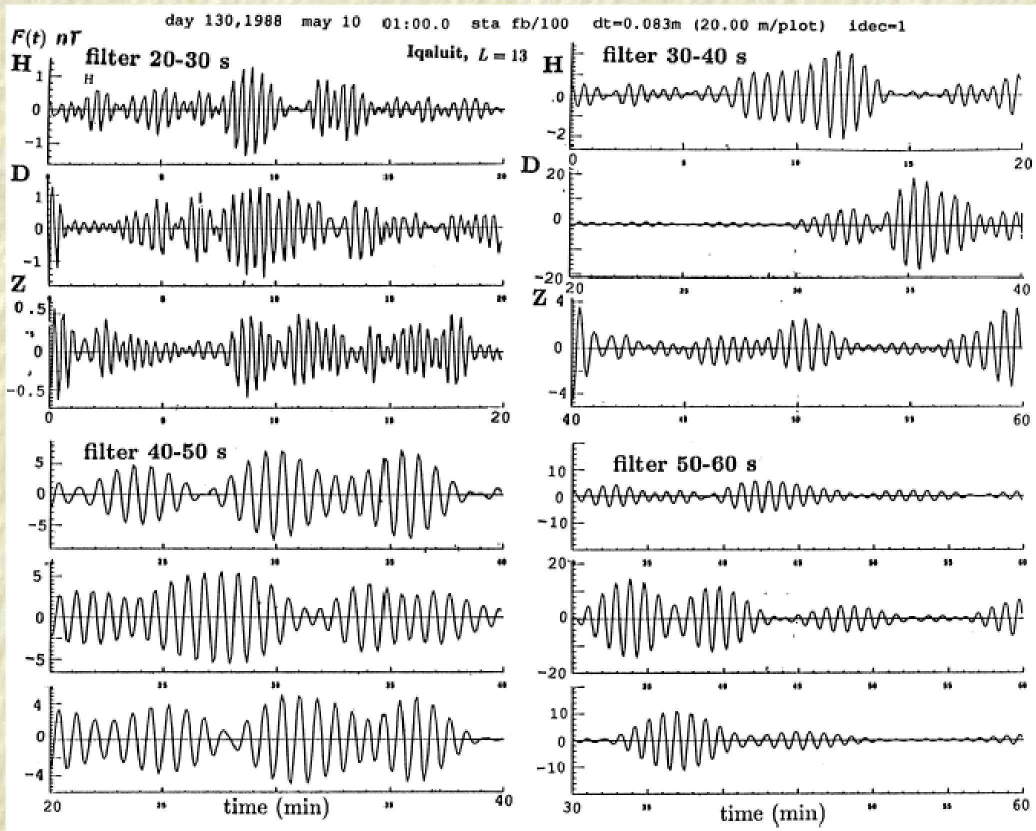


Fig. 16. Evolution of rogue wave sequence—registrations at $x = 5$ m, 50 m and 100 m (left) as well as wave profiles at $t = 75$ s, 81 s and 87 s (right hand side) (water depth $h = 5$ m, $T_p = 3.13$ s).

(from: [Klauss, Applied Ocean Research **24**, 147 (2002)])

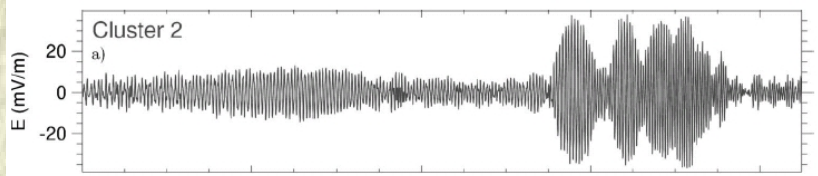
..., in EM field measurements in the magnetosphere, ...





(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

..., in satellite (e.g. CLUSTER, FAST, ...) observations:



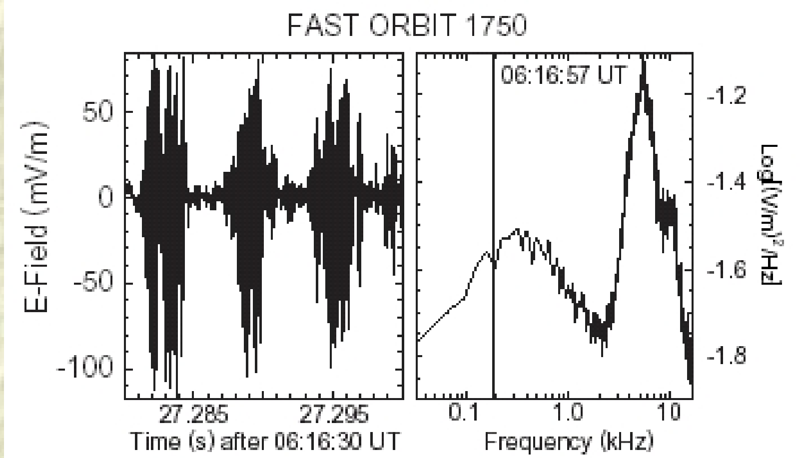


Figure 2. *Left*: Wave form of broadband noise at base of AKR source. The signal consists of highly coherent (nearly monochromatic frequency of trapped wave) wave packets. *Right*: Frequency spectrum of broadband noise showing the electron acoustic wave (at ~ 5 kHz) and total plasma frequency (at ~ 12 kHz) peaks. The broad LF maximum near 300 Hz belongs to the ion acoustic wave spectrum participating in the 3 ms modulation of the electron acoustic waves.

(*) From: O. Santolik *et al.*, *JGR* **108**, 1278 (2003); R. Pottelette *et al.*, *GRL* **26** 2629 (1999).

Modulational instability (MI) was observed in simulations,

e.g. early numerical experiments of EM cyclotron waves:

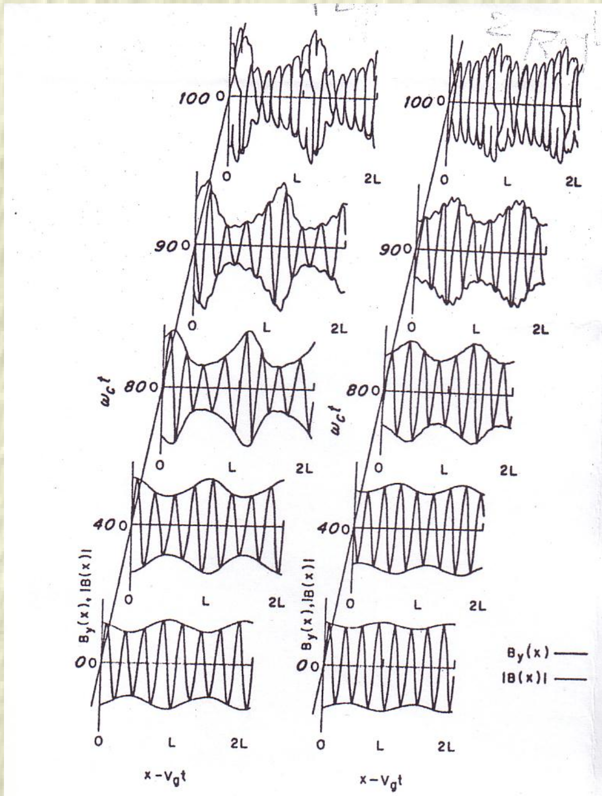


FIG. 2. Spatial variation of the y component of the wave magnetic field $B_y(x)$ (solid curve) and the magnitude of the total field $|B(x)|$ (thin curve) at different time of evolution for cases 1 (left) and 2 (right). Note the harmonic generation in $|B(x)|$ and the sudden collapse of the wave.

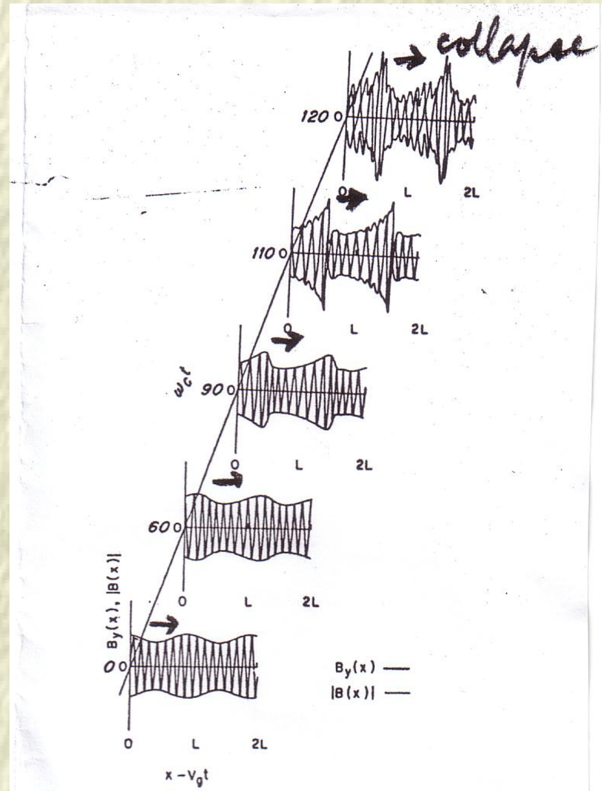


FIG. 3. $B_y(x)$ and $|B(x)|$ for case 4. Note the steepening of the wavefront which leads to the collapse of the wave.

[from: A. Hasegawa, *PRA* **1**, 1746 (1970); *Phys. Fluids* **15**, 870 (1972)].

Spontaneous MI **has been observed in experiments,:**

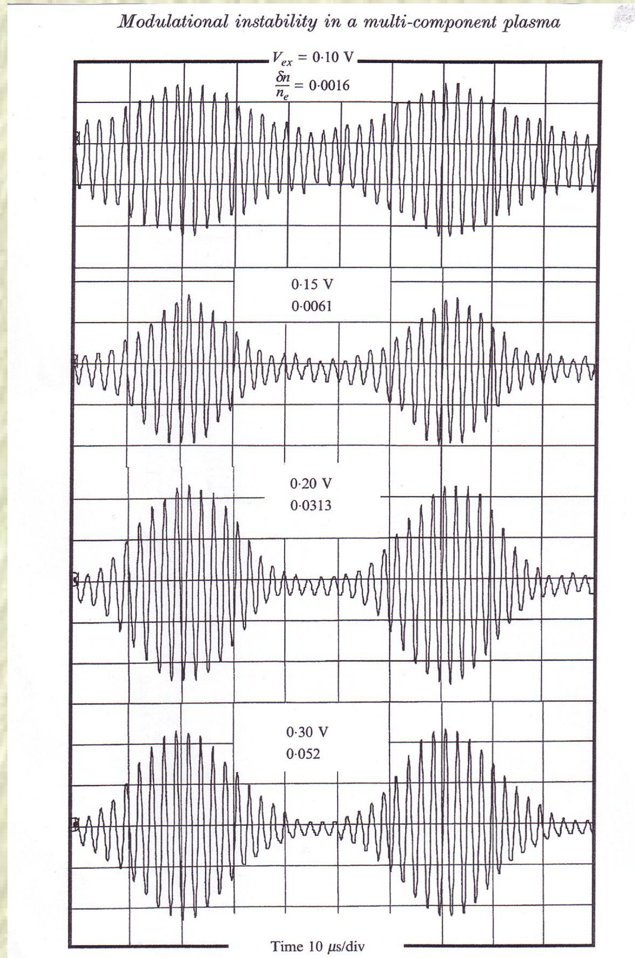


FIGURE 5. Oscilloscope traces of the detected signal for different excitation voltages. The probe was fixed at 14 cm from the grid. f_c = 400 kHz and f_m = 50 kHz.

e.g. on *ES plasma waves*

[from: Bailung and Nakamura, *J. Plasma Phys.* **50** (2), 231 (1993)].

Questions to be addressed in this brief presentation:

- The Formalism: How can one describe the (slow) evolution (*modulation*) of a wave *amplitude* in space and time?
- Can *Modulational Instability* (MI) of plasma “fluid” modes be predicted by a simple, tractable analytical model?
- Can *envelope modulated localized structures* (such as those observed in space and laboratory plasmas) be modeled by an exact theory?
- *Focus issue*: Modulated *electromagnetic (EM) waves* in *e-i, pair & e-p-i plasmas.*

Pair-ion plasmas: prerequisites (1)

- Electron-ion plasmas:

  – *electrons* e^- (charge $-e$, mass m_e),

  – *ions* i^+ (charge $+Z_i e$, mass $m_i \gg m_e$),

  – ...

- Intrinsic features (long *"taken for granted"*):

— *Distinct electron/ion frequency scales*, far apart, e.g.

$$\omega_{p,s} = \left(\frac{4\pi n_s q_s^2}{m_s} \right)^{1/2}, \qquad \omega_{c,s} = \frac{q_s B}{m_s c} \qquad (s = e, i)$$

hence $\omega_{p,e} \gg \omega_{p,i}$, $\omega_{c,e} \gg \omega_{c,i}$.

— Longevity (recombination neglected, no overall density variation).

Pair-ion plasmas: prerequisites (2)

- Pair-ion plasmas:
  - *Positive ions* i^+ (charge $+Ze$, mass m),
  - *Negative ions* i^- (charge $-Ze$, mass m),
  - ... (heavier ions, in a multi-component eg. *e-p-i* composition).
- *No (pair-ion) frequency separation*: $\omega_{p,+} = \omega_{p,-}\,, \qquad \omega_{c,+} = \omega_{c,-}\,.$
- Novel Physics:

— *New (linear) ES/EM mode dispersion profile*
[Iwamoto PRE 1989, Stewart & Laing JPP 1992, Zank & Greaves PRE 1995].

— No Faraday rotation.

$\rightarrow$ *Talks by F. Verheest and H. Saleem.*

Pair-ion plasmas: prerequisites (3)

- Magnetized *electron-positron (e-p)* and e-p-i plasmas exist:
  - in *pulsar magnetospheres* [Ginzburg 1971, Michel RMP 1982],
  - in *bipolar outflows (jets) in active galactic nuclei (AGN)* [Miller 1987, Begelman RMP 1984]
  - at *the center of our own galaxy* [Burns 1983],
  - in the *early universe* [Hawking 1983],
  - in *inertial confinement fusion schemes* [Liang *et al.* PRL 1998]
  - in (very sophisticated, yet short-lived) experiments [Greaves, Surko *et al.* PoP 1994, Zhao *et al.* PoP 1996].
- *Pair-ion plasmas (p.p.)* have been formed in laboratory,
  - in recent *fullerene ion* ($C_{60}^{\pm}$) experiments [Oohara & Hatakeyama PRL 2003].

Part B: Two-fluid model for *oblique* EM plasma waves

Fluid Eqs. (for $j = 1^+, 2^-$):

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j \; = \; \frac{q_j}{m_j} \left(\mathbf{E} + \frac{1}{c} \mathbf{u}_j \times \mathbf{B} \right)$$

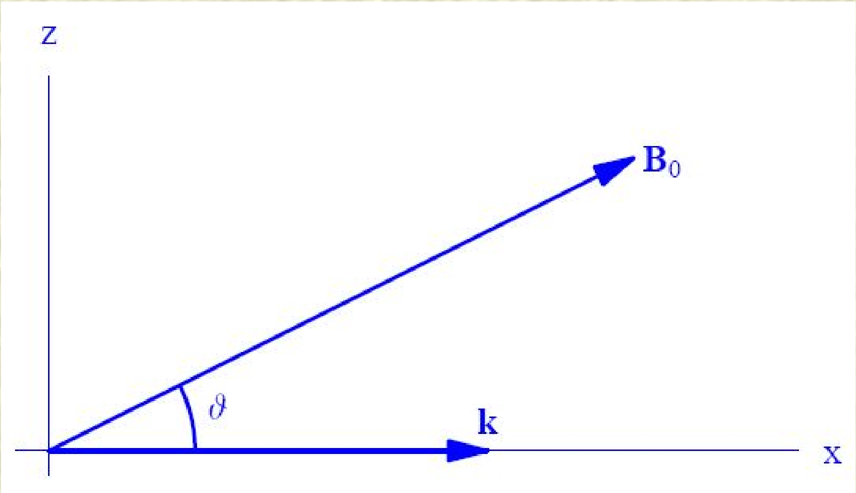
Maxwell's laws:

$$\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \, , \qquad \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c} \sum_j n_j q_j \, , \qquad \nabla \cdot \mathbf{B} = 0$$

+ a convenient frame:

$$\mathbf{k} = (k, 0, 0)$$

$$\mathbf{B}_0 = (B_0 \cos\theta, 0, B_0 \sin\theta)$$



Fluid model for EM waves in pair-ion or e-p-i plasmas (continued)

- 2 species $1^+, 2^-$, + a "fixed" third species $3^\pm$ ($q_3 = \pm Z_3 e$); $n_3 = n_{3,0} =$cst.

- Consider $q_1 = -q_2 = +Ze$ and $m_1 = m_2 = m$, hence $\omega_{c,+} = \omega_{c,-} = \Omega$.

- Distinguish:

  * *"Pure"* pair-ion plasmas:

$$n_{+,0} - n_{-,0} = 0\,, \qquad \text{thus} \qquad \omega_{p,+} = \omega_{p,-}\,;$$

  * Three-component, i.e. e-p-i, or "doped" (e.g. dusty) pair plasmas:

$$n_{+,0} - n_{-,0} \pm Z_3 n_3 = 0\,, \qquad \text{i.e.} \qquad \omega_{p,+} \neq \omega_{p,-}\,.$$

- No neutrality assumption (off equilibrium): $n_+ \neq n_-$ (possibly).

Reductive perturbation (multiple scales) technique

– *1st step.* Define *multiple scales* (*fast* and *slow*) i.e.

$$\mathbf{R}_0 = \mathbf{r}\,, \quad \mathbf{R}_1 = \epsilon\,\mathbf{r}\,, \quad \mathbf{R}_2 = \epsilon^2\,\mathbf{r}\,, \quad ...$$

$$T_0 = t\,, \quad T_1 = \epsilon\,t\,, \quad T_2 = \epsilon^2\,t\,, \quad ...$$

$$\mathbf{r} = (x, y, z)\,, \qquad \mathbf{R} = (X, Y, Z) \qquad \epsilon \ll 1$$

– *2nd step.* Expand near equilibrium:

$$n_j \approx n_{j,0} + \epsilon\,n_{j,1} + \epsilon^2\,n_{j,2} + ...$$

$$\mathbf{u}_j \approx \mathbf{0} + \epsilon\,\mathbf{u}_{j,1} + \epsilon^2\,\mathbf{u}_{j,2} + ...$$

$$\mathbf{B} \approx \mathbf{B}_0 + \epsilon\,\mathbf{B}_1 + \epsilon^2\,\mathbf{B}_2 + ...$$

$$\mathbf{E} \approx \mathbf{0} + \epsilon\,\mathbf{E}_1 + \epsilon^2\,\mathbf{E}_2 + ...$$

Reductive perturbation technique *(continued)*

– *3rd step.* Project on Fourier space, i.e. consider $\forall n = 1, 2, ...$

$$S_n = \sum_{l=-n}^{n} S_l^{(n)} \, e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = S_0^{(n)} + 2\sum_{l=1}^{n} S_l^{(n)} \cos l(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

for $S_l^{(n)} \in (n_{j,l}^{(n)}, \mathbf{u}_{j,l}^{(n)}, \mathbf{E}_l^{(n)}, \mathbf{B}_l^{(n)})$, where the slow amplitudes vary $\sim \hat{x}$

$$S_l^{(n)} = S_l^{(n)}(X_j, T_j)\,, \qquad\qquad j = 1, 2, ...$$

i.e. *essentially* ($\phi_c = \mathbf{k}\cdot\mathbf{r} - \omega t$):

$$S_1 = S_0^{(1)} + S_1^{(1)} \sin\phi_c\,, \qquad S_2 = S_0^{(2)} + S_1^{(2)} \sin\phi_c + S_2^{(2)} \sin 2\phi_c \text{ etc.}$$

– *4rth step.* Collect terms in n, l (*) and solve for the respective amplitudes.

(*) $n = 0, 1, 2, ...,\quad l = 0, \pm1, \pm2, ..., \pm n$

First-order ($\sim \epsilon^1$): linear dynamics – oblique propagation (in p.p.)

- *Dispersion relation* $\forall\theta$: $D(\omega, k; \theta) = d_0(\omega, k) + d_1(\omega, k)\sin^2\theta = 0$

$$\begin{aligned} d_0(\omega, k) &\equiv D(\omega, k; \theta = 0) \\ &= (\omega^2 - \omega_{p,eff}^2) \\ &\quad \times\left\{\left[(\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega^2\omega_{p,eff}^2\right]^2 - \omega^2\Omega^2(\omega_{p,1}^2 - \omega_{p,2}^2)^2\right\} \\ &= (\omega^2 - \omega_{p,eff}^2) \\ &\quad \times\left\{(\omega + \Omega)\left[-(\omega^2 - c^2k^2)(\omega - \Omega) + \omega\omega_{p,1}^2\right] + \omega(\omega - \Omega)\omega_{p,2}^2\right\} \\ &\quad \times\left\{(\omega - \Omega)\left[-(\omega^2 - c^2k^2)(\omega + \Omega) + \omega\omega_{p,1}^2\right] + \omega(\omega + \Omega)\omega_{p,2}^2\right\}, \end{aligned}$$

$$d_1(\omega, k; \theta) = -c^2k^2\Omega^2\left\{c^2k^2\omega_{p,eff}^2(\omega^2 - \Omega^2) + \omega^2[4\omega_{p,1}^2\omega_{p,2}^2 - (\omega^2 - \Omega^2)\omega_{p,eff}^2]\right\},$$

Notation: $\omega_{p,eff}^2 = \omega_{p,1}^2 + \omega_{p,2}^2$; Ω is the (common) cyclotron frequency.

First-order ($\sim \epsilon^1$): linear dynamics (2) – propagation $\perp \mathbf{B_0}$ (in p.p.)

- *Dispersion relation* for $\theta = \pi/2$: $D(\omega, k; \frac{\pi}{2}) = d_{\perp,1}(\omega, k)\, d_{\perp,2}(\omega, k) = 0$

$$
\begin{aligned}
d_{\perp,1}(\omega, k) \quad = \quad & -\omega^6 + \omega^4[c^2\, k^2 + 2(\Omega^2 + \omega_{p,eff}^2)] \\
& -\omega^2[(\Omega^2 + \omega_{p,eff}^2)^2 - c^2k^2(2\Omega^2 + \omega_{p,eff}^2)] \\
& +\Omega^2[c^2k^2\, (\Omega^2 + \omega_{p,eff}^2) + (\omega_{p,1}^2 - \omega_{p,2}^2)^2]
\end{aligned}
$$

$$
d_{\perp,2}(\omega, k) = \omega^2 - \omega_{p,eff}^2 - c^2k^2
$$

O-mode: a robust perpendicular mode, whose dispersion characteristics do not depend on the ambient magnetic field ; same form for *e-i* plasmas.

Cf. (for $\mathbf{B_0} = \mathbf{0}$) G S Lakhina & B Buti, Astrophys. Space Sci. **79**, 25 (1981).

Solution (upto $\sim \epsilon^2$) for the state variables

$$n_j = n_{j,0} + \epsilon\, c_j^{(11)} B'_y\, e^{i\phi_c} + \epsilon^2 [c_j^{(22)} B'^2_y e^{i2\phi_c} + n_j^{(20)}]$$

$$\mathbf{u_j} = \mathbf{0} + \epsilon\, c_{j,z}^{(11)} B'_y e^{i\phi_c}\, \hat{z} + \epsilon^2 \left\{ c_{j,z}^{(21)} \frac{\partial B'_y}{\partial X_1} e^{i\phi_c} \hat{z} + B'^2_y e^{i2\phi_c} [c_{j,x}^{(22)} \hat{x} + c_{j,y}^{(22)} \hat{y}] + \mathbf{u_j}^{(20)} \right\}$$

$$\mathbf{E} = \mathbf{0} + \epsilon\, c_{el,z}^{(11)} B'_y e^{i\phi_c} \hat{z} + \epsilon^2 \left\{ c_{el,z}^{(21)} \frac{\partial B'_y}{\partial X_1} e^{i\phi_c} \hat{z} + B'^2_y e^{i2\phi_c} [c_{el,x}^{(22)} \hat{x} + c_{el,y}^{(22)} \hat{y}] + \mathbf{E}^{(20)} \right\}$$

$$\mathbf{B} = B_0 \hat{z} + \epsilon\, B'_y e^{i\phi_c} \hat{y} + \epsilon^2 \left[c_{B,y}^{(21)} \frac{\partial B'_y}{\partial X_1} e^{i\phi_c} \hat{y} + c_{B,z}^{(22)} B'^2_y e^{i2\phi_c} \hat{z} + \mathbf{B}^{(20)} \right]$$

$$+\mathcal{O}(\epsilon^3) \text{ everywhere}$$

$(j = 1, 2 \equiv +, -)$ where $B'_y = B_y^{(11)}/B_0$ and $\phi_c = kx - \omega t$;
$S_i^{(20)}$ are *arbitrary* state variable corrections satisfying

$$u_{1,x}^{(20)} = -u_{2,x}^{(20)} = cE'^{(20)}_y, \qquad u_{1,y}^{(20)} = -u_{2,y}^{(20)} = -cE'^{(20)}_x .$$

Solution/2 — *harmonic amplitudes*

$$u_{j,z}^{(11)} = (-1)^j \, i\frac{\Omega_j}{k} B'_y \,, \qquad E'^{(11)}_z = -\frac{\omega}{ck} B'_y \qquad (j = 1, 2)\,,$$

$$n_j^{(11)} = u_{j,x}^{(11)} = u_{j,y}^{(11)} = E_x^{(11)} = E_y^{(11)} = 0\,,$$

$$u_{j,z}^{(21)} = (-1)^j \, \frac{c^2 \omega_{p,eff}^2 \Omega_j}{\omega^2 k^2} \frac{\partial B'_y}{\partial X_1}\,,$$

$$E_z^{(21)} = i\,\frac{\omega}{ck^2} \frac{\partial B'_y}{\partial X_1}\,, \qquad B_y^{(21)} = -i\,\frac{\omega^2 + \omega_{p,eff}^2}{\omega^2 k} \frac{\partial B'_y}{\partial X_1}\,,$$

$$n_j^{(21)} = u_{j,x/y}^{(21)} = E_{x/y}^{(21)} = B_{x/z}^{(21)} = 0\,,$$

$$u_{j,x}^{(22)} = \frac{\omega n_j^{(22)}}{k n_{j,0}} = \frac{D_{j,x}^{(22)}}{D_0^{(22)}} B'^{\,2}_y\,, \qquad u_{j,y}^{(22)} = \frac{D_{j,y}^{(22)}}{D_0^{(22)}} B'^{\,2}_y\,, \qquad u_{j,z}^{(22)} = 0$$

$$E_x^{(22)} = \frac{D_{el,x}^{(22)}}{D_0^{(22)}} B'^{\,2}_y\,, \qquad E_y^{(22)} = \frac{\omega}{ck} B_z^{(22)} = \frac{D_{el,y}^{(22)}}{D_0^{(22)}} B'^{\,2}_y\,, \qquad E_z^{(22)} = B_y^{(22)} = 0\,,$$

where $j = 1, 2 \equiv +, -$; $D_{*,\dagger}^{(nl)}$ are given by ... (→ *next slide*)

Solution/3: n=l=2 coefficients for e-i and e-p-i (if $\Omega_1 = \Omega_2 = \Omega$) plasmas

$$
\begin{aligned}
D_{1,x}^{(22)} &= 6c^2 k\omega\Omega_1\omega_{p,eff}^2[4\omega^2\Omega_1 - \Omega_1\Omega_2^2 - \omega_{p,2}^2(\Omega_1+\Omega_2)]\,,\\
D_{2,x}^{(22)} &= 6c^2 k\omega\Omega_2\omega_{p,eff}^2[4\omega^2\Omega_2 - \Omega_1^2\Omega_2 - \omega_{p,1}^2(\Omega_1+\Omega_2)]\,,\\
D_{1,y}^{(22)} &= ic^2k\Omega_1\{-4\omega_{p,eff}^2\Omega_1^2(4\omega^2-\Omega_2^2)\\
&\quad -(\Omega_1+\Omega_2)\omega_{p,2}^2[4c^2k^2\Omega_1+\omega^2(-8\Omega_1+4\Omega_2)-\Omega_2\omega_{p,1}^2] - \Omega_1(\Omega_1+\Omega_2)\omega_{p,2}^4\}\,,\\
D_{2,y}^{(22)} &= -ic^2k\Omega_2\{-4\omega_{p,eff}^2\Omega_2^2(4\omega^2-\Omega_1^2)\\
&\quad -(\Omega_1+\Omega_2)\omega_{p,1}^2[4c^2k^2\Omega_2+\omega^2(-8\Omega_2+4\Omega_1)-\Omega_1\omega_{p,2}^2] - \Omega_2(\Omega_1+\Omega_2)\omega_{p,1}^4\}\,,\\
D_{el,x}^{(22)} &= -3ick\omega_{p,eff}^2[\Omega_1\omega_{p,1}^2(4\omega^2-\Omega_2^2) - \Omega_2\omega_{p,2}^2(4\omega^2-\Omega_1^2)]\,,\\
D_{el,y}^{(22)} &= 2ck\omega[-4\omega^2(\Omega_1^2\omega_{p,1}^2+\Omega_2^2\omega_{p,2}^2) + \Omega_1^2\Omega_2^2\omega_{p,eff}^2 + (\Omega_1+\Omega_2)^2\omega_{p,1}^2\omega_{p,2}^2]\,,\\
D_0^{(22)} &= c^2k^2\{-64\omega^6 + 16(4\omega^2 - 2\omega_{p,eff}^2 + \Omega_1^2 + \Omega_2^2)\omega^4\\
&\quad -4[4c^2k^2(\Omega_1^2+\Omega_2^2+\omega_{p,eff}^2) + \Omega_1^2\Omega_2^2 + \omega_{p,eff}^4 + 2(\Omega_2^2\omega_{p,1}^2+\Omega_1^2\omega_{p,2}^2)]\omega^2\\
&\quad -(\Omega_2\omega_{p,1}^2 - \Omega_1\omega_{p,2}^2)^2 + 4c^2k^2(\Omega_2^2\omega_{p,1}^2+\Omega_1^2\omega_{p,2}^2+\Omega_1^2\Omega_2^2)\}\,.
\end{aligned}
$$

Solution/4: coefficients for pure pair plasmas

For $\omega_{p,1} = \omega_{p,2}$:

$$
\begin{aligned}
D_{1,x}^{(22)} &= D_{2,x}^{(22)} = 6c^2 k\omega\Omega^2\omega_p^2(4\omega^2 - \Omega^2 - 2\omega_p^2)\,, \\
D_{1,y}^{(22)} &= -D_{2,y}^{(22)} = -i8c^2 k\Omega^3\omega_p^2(4\omega^2 - \Omega^2 - 2\omega_p^2)\,, \\
D_{el,x}^{(22)} &= 0\,, \\
D_{el,y}^{(22)} &= -4ck\omega\Omega^2\omega_p^2(4\omega^2 - \Omega^2 - 2\omega_p^2)\,, \\
D_0^{(22)} &= 4c^2k^2\{-16\omega^6 + 8(2\omega^2 - 2\omega_p^2 + \Omega^2)\omega^4 \\
&\quad -[8c^2k^2(\Omega^2 + \omega_p^2) + \Omega^4 + 16\omega_p^4 + 4\Omega^2\omega_p^2]\omega^2 + c^2k^2(2\Omega^2\omega_p^2 + \Omega^4)\}\,.
\end{aligned}
$$

* Neutrality ($n_1 - n_2 = 0$) preserved upto $\sim \epsilon^2$ for pure p.p. (*only*).

* No electric field $\parallel \hat{x}$!

Second-order compatibility condition ($\sim \epsilon^2$)

- From $n = 2, l = 1$, we obtain a compatibility condition in the form:

$$\frac{\partial B_y}{\partial T_1} + v_g \frac{\partial B_y}{\partial X_1} = 0$$

- $v_g = d\omega(k)/dk = c^2 k/\omega$ is the group velocity;

- The magnetic field correction (amplitude) satisfies:

$$B_y = B_y(X_1 - v_g T_1, T_{n\geq 2})\,.$$

Nonlinear Schrödinger equation for the amplitude $B_y^{(11)}$

$$i\left(\frac{\partial B_y^{(11)}}{\partial T_2} + v_g \frac{\partial B_y^{(11)}}{\partial X_2}\right) + P\,\frac{\partial^2 B_y^{(11)}}{\partial X_1^2} + Q\,|B_y^{(11)}|^2\,B_y^{(11)} = 0\,.$$

i.e.

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0\,.$$

where

- $\psi \equiv B_y^{(11)}(\zeta, \tau)$.
- $\zeta = X_1 - v_g T_1, \quad \tau = T_2$.
- Dispersion coefficient: $P = \frac{1}{2}\omega''(k) = c^2\omega_{p,eff}^2/(2\omega^3)$.
- Nonlinearity coefficient: $Q = ...$ ($\rightarrow$ *next slide*)

Nonlinearity coefficient for *e-i* and *e-p-i** plasmas
(* for $\Omega_1 = \Omega_2$, $\omega_{p,1} \neq \omega_{p,2}$)

$$Q = Q_1 + Q_2$$

$$\begin{aligned}
Q_1 &= Q_A/Q_B \\
Q_A &= 3\omega_{p,eff}^2 \{-4\omega^2(\Omega_1^2\omega_{p,1}^2 + Z\Omega_2^2\omega_{p,2}^2) + \Omega_1\omega_{p,1}^2[\Omega_1\Omega_2^2 + (\Omega_1 + \Omega_2)\omega_{p,2}^2] \\
&\qquad + Z\Omega_2\omega_{p,2}^2[\Omega_2\Omega_1^2 + (\Omega_1 + \Omega_2)\omega_{p,1}^2]\} \\
Q_B &= \omega(c_4\omega^4 + c_2\omega^2 + c_0) \\
c_4 &= 48\omega_{p,eff}^2 \\
c_2 &= -4[3\omega_{p,eff}^4 + 3\omega_{p,eff}^2(\Omega_1^2 + \Omega_2^2) + \omega_{p,1}^2\Omega_1^2 + \omega_{p,2}^2\Omega_2^2] \\
c_0 &= 3(\omega_{p,1}^4\Omega_2^2 + \omega_{p,2}^4\Omega_1^2) + 2\Omega_1\Omega_2\omega_{p,1}^2\omega_{p,2}^2 + 4(\Omega_1^2 + \Omega_2^2)\omega_{p,1}^2\omega_{p,2}^2 + 4\Omega_1^2\Omega_2^2\omega_{p,eff}^2 \\
Q_2 &= -\frac{1}{2\omega}\left(\frac{n_1^{(20)}}{n_{1,0}}\omega_{p,1}^2 + \frac{n_2^{(20)}}{n_{2,0}}\omega_{p,2}^2\right).
\end{aligned}$$

Nonlinearity coefficient for *pure* pair plasmas

For $\Omega_1 = \Omega_2$ *and* $\omega_{p,1} = \omega_{p,2} = \omega_p$, the NLSE coefficients simplify to:

$$P = \frac{c^2 \omega_p^2}{\omega^3}\,, \qquad\qquad Q = \frac{3\Omega^2 \omega_p^2}{2\omega(\Omega^2 - 3\omega^2)} - \frac{1}{2\omega}\frac{\omega_p^2}{n_0}(n_1^{(20)} + n_2^{(20)})\,.$$

Neglecting $n_j^{(20)}$:

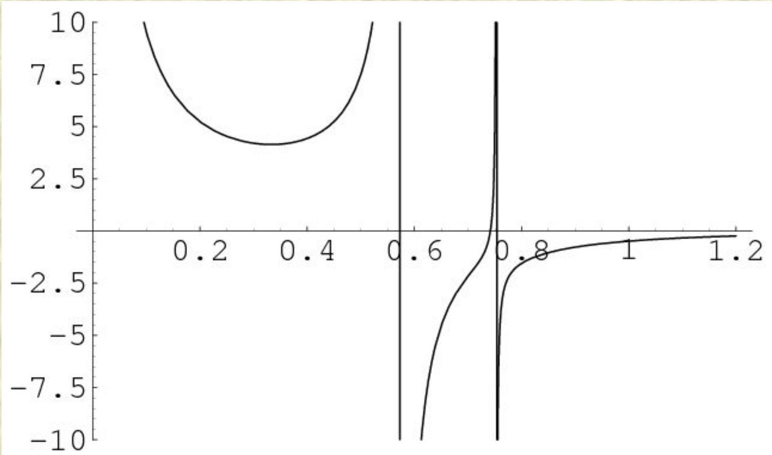
- *Anomalous dispersion* ($PQ > 0$) for $\omega < \Omega/\sqrt{3}$:

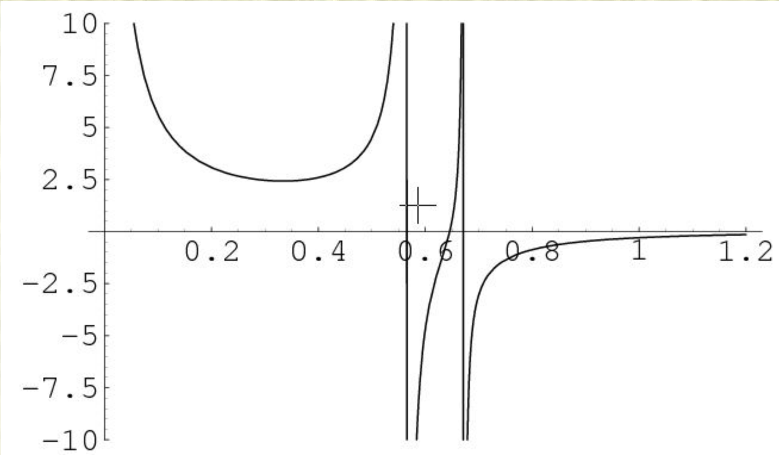
  $\rightarrow$ MI, *bright* envelope solitons;

- *Normal dispersion* ($PQ < 0$) for $\omega > \Omega/\sqrt{3}$:

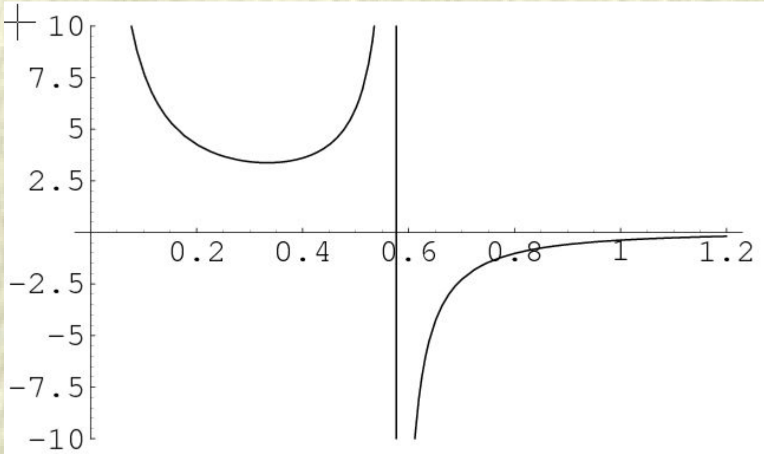
  $\rightarrow$ *No* MI, *dark* solitons.

Modulational instability profile **in the presence of the 3rd species**





For pure pair plasmas:



Localized envelope excitations (solitons) for $PQ > 0$

- The NLSE accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}\,, i.e.$

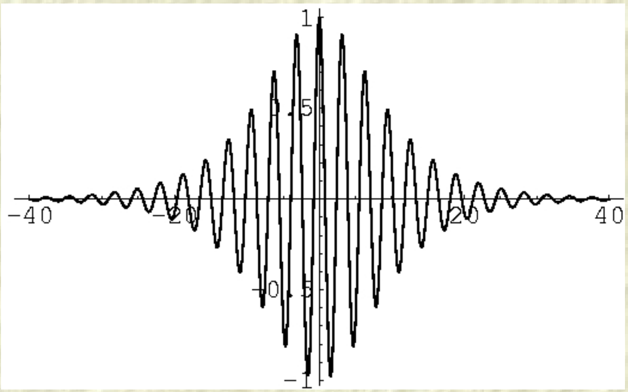
$$\mathbf{B} \approx B_0 \hat{z} + \epsilon\, \rho\, \cos(\mathbf{kr} - \omega t + \Theta)\, \hat{y} + \mathcal{O}(\epsilon^2)\,.$$

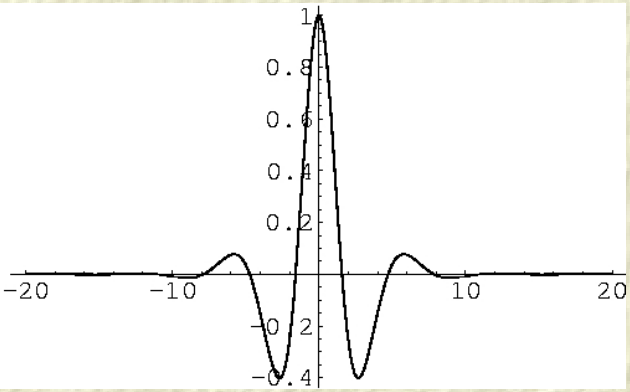
- Bright–type envelope soliton (pulse):

$$\rho = \rho_0\, \mathrm{sech}\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta\, - (\Omega + \frac{1}{2}v^2)\tau\right].$$

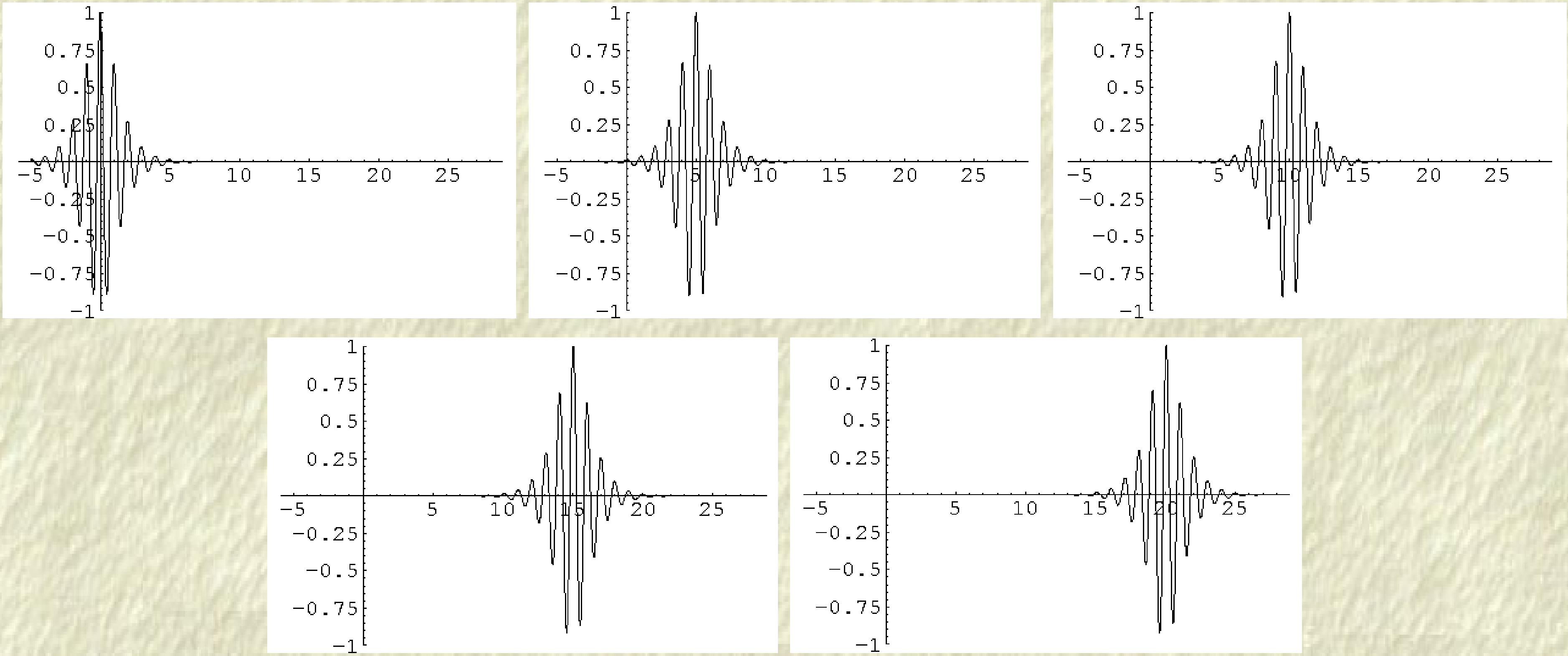
$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$

This is a propagating (and *oscillating*) localized pulse:

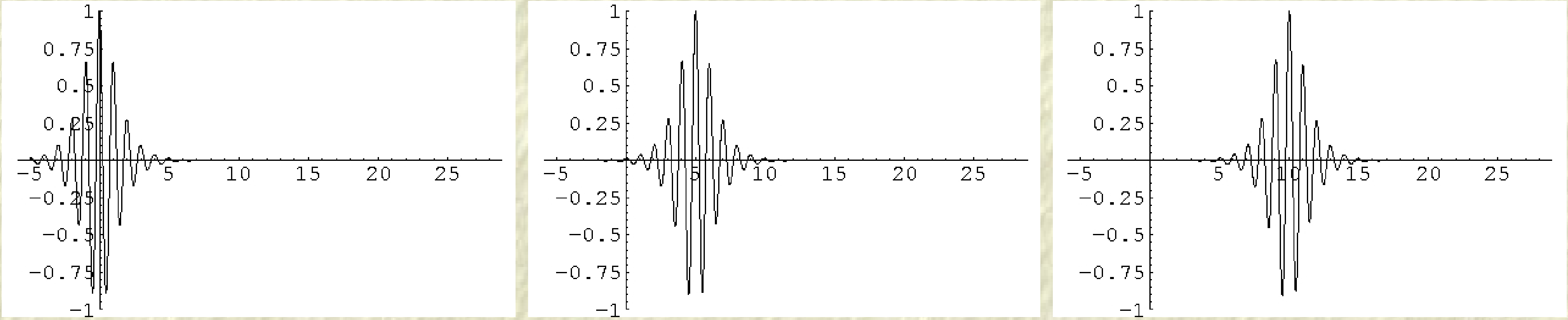




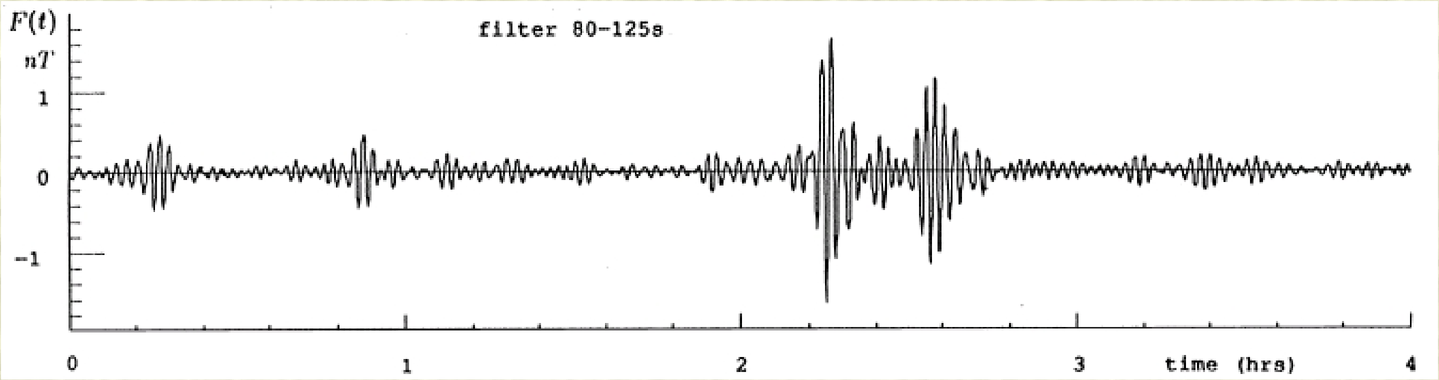
Propagation of a bright envelope soliton (pulse)



Propagation of a bright envelope soliton (pulse)

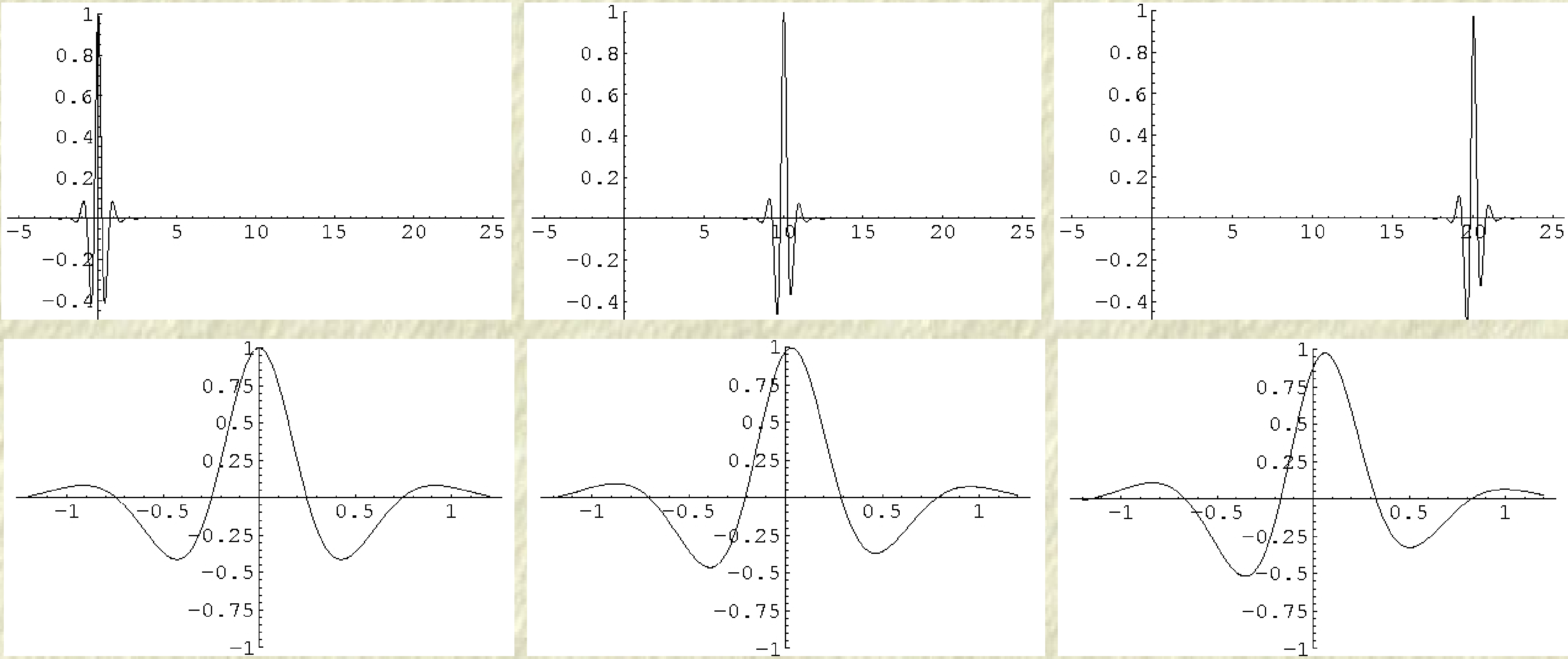


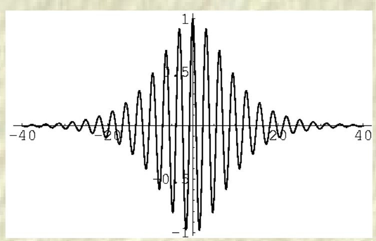
Cf. electrostatic plasma wave data from satellite observations:



(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

Propagation of a bright envelope soliton (continued...)





(⟶ *see video*)

www.tp4.rub.de/∼ioannis/conf/200609-FSAW-oral.pdf

Localized envelope excitations for $PQ < 0$

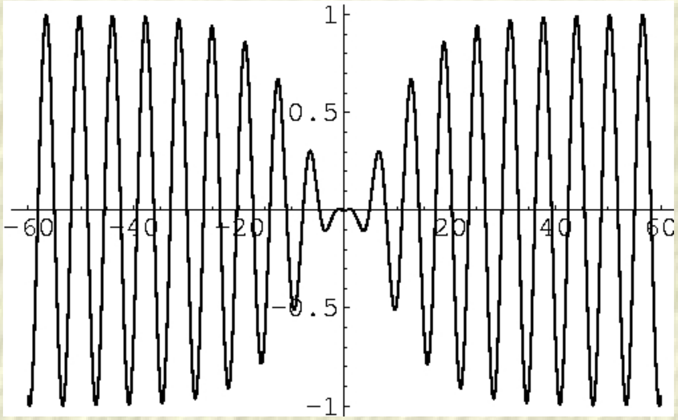
- Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1^2\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{\rho_1}}$$

This is a *propagating localized hole* (zero density void):



Localized envelope excitations for $PQ < 0$

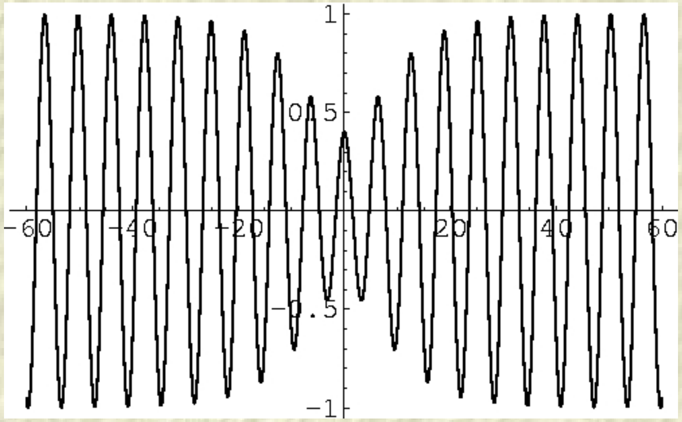
- Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - a^2 \,\mathrm{sech}^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\,\frac{1}{a\rho_2}$$

This is a propagating *(finite-density)* void:



Conclusions

- *Modulated EM wave packets*, abundantly observed in Space and in the lab, may be considered as the outcome of *energy localization* via *modulational instability*. This is an omnipresent nonlinear mechanism, which is related with *harmonic generation* and *envelope structure* formation.

- Localized EM structures may be efficiently modeled as *NLS solitons*. These bear "signatures" (i.e. specific features, e.g. amplitude-width relation) which may be traced in observation data.

- Either in *e-i* plasmas, in *e-p-i*, or in *pair plasmas*, the reductive perturbation is a powerful tool for the study of modulated EM waves.

- Investigation (here limited to the O-mode) to be extended to other modes.

- Future extensions of the theory : relativistic effects, 2D geometry, more realistic localized envelope solutions (*dromions*?), ...

Thank You !

Acknowledgements / interactions related to this research:

F Verheest, T Cattaert (U. Gent, Belgium)
M Hellberg (U. of KwaZulu-Natal, Durban, South Africa)
G S Lakhina (Inst. Geomagnetism, Bombay, India)
N Cramer (U. Sydney, Australia)

Slides available at: `www.tp4.rub.de/∼ioannis`

Ioannis.Kourakis@Ugent.be , ioannis@tp4.rub.de