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Localized envelope excitations in pair plasmas

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In collaboration with:
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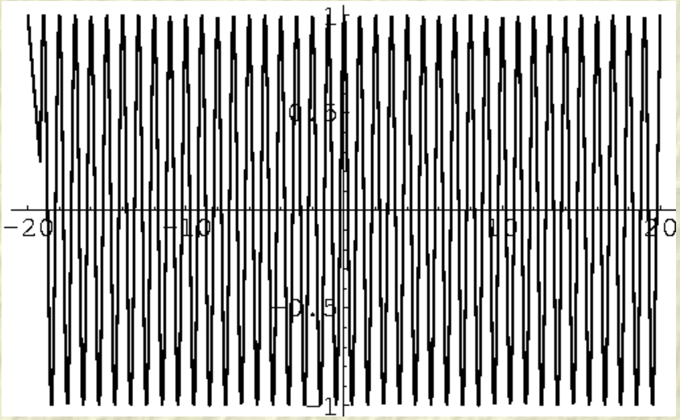
(Part B) F Verheest (U. Gent, Belgium), N Cramer (USW, Sydney, Australia).

Outline

- Introduction & prerequisites
  – Amplitude Modulation (AM), modulated excitations.
  – Pair plasmas, e-p plasmas, e-p-i plasmas.
- Part A: Fluid model for ES waves in pair plasmas and “doped” pair plasmas
  – Perturbation (multiple scales) method for AM.
  – Modulational instability (MI) analysis, envelope excitations.
- Part B: EM waves in pair plasmas
- Conclusions

Intro.: The mechanism of *wave amplitude modulation*

The *amplitude* of a harmonic wave may vary in space and time:



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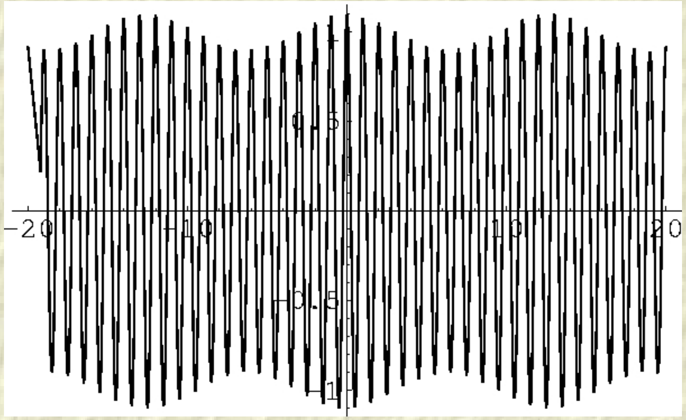

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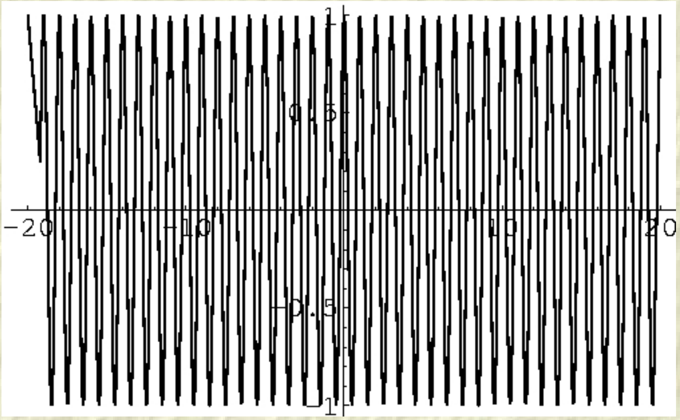

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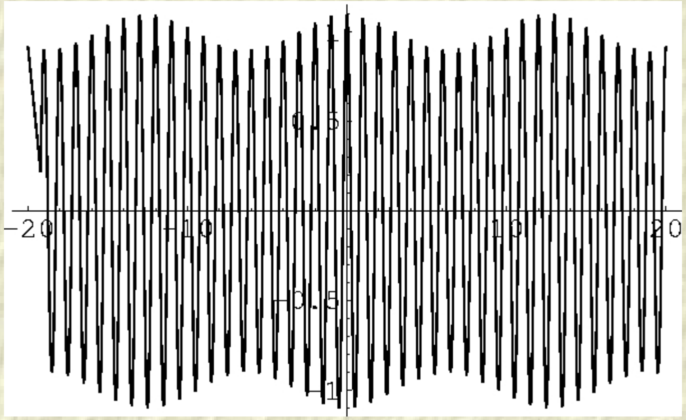
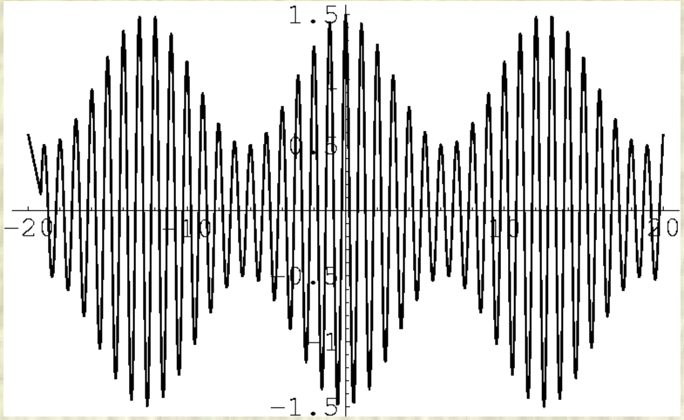


This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* (modulational instability) or to the formation of *envelope solitons*:



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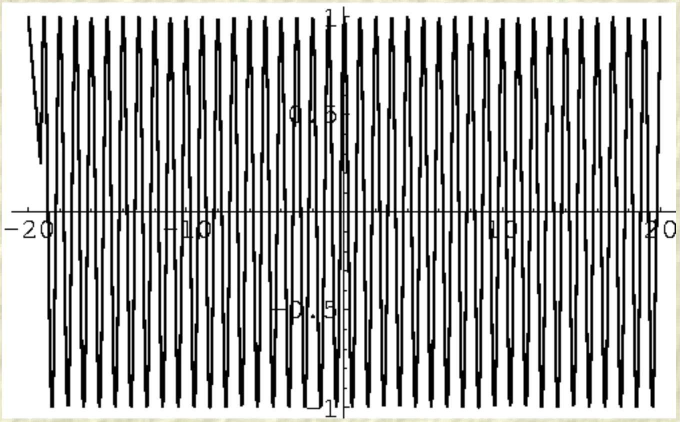

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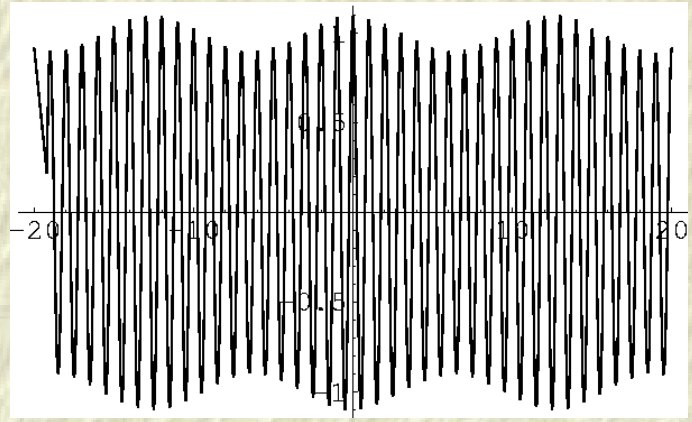

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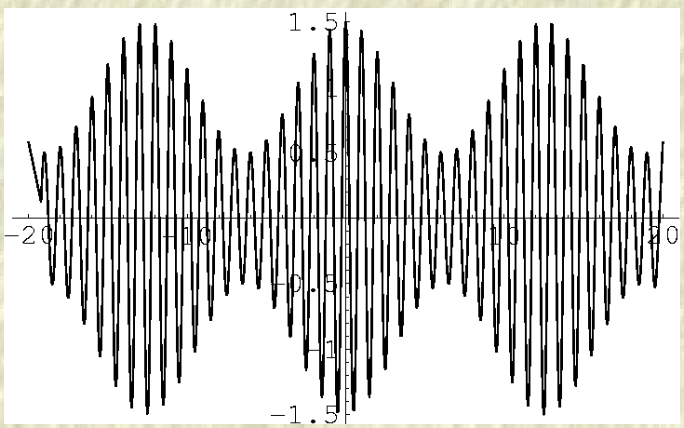

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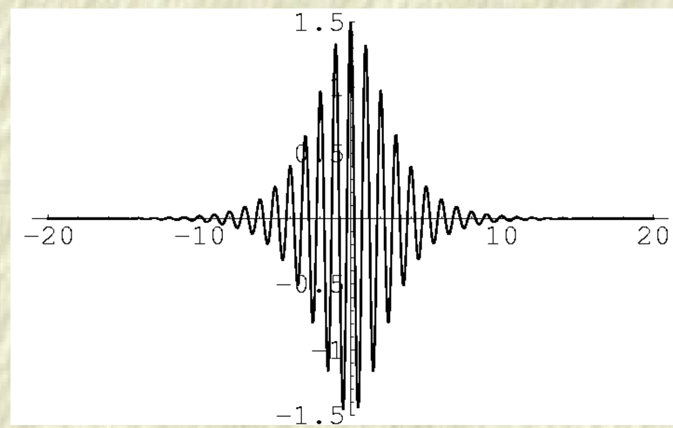
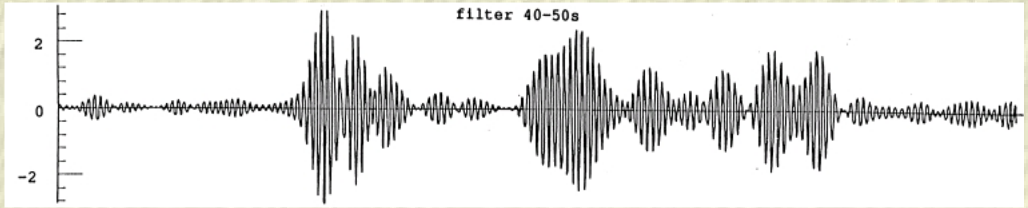
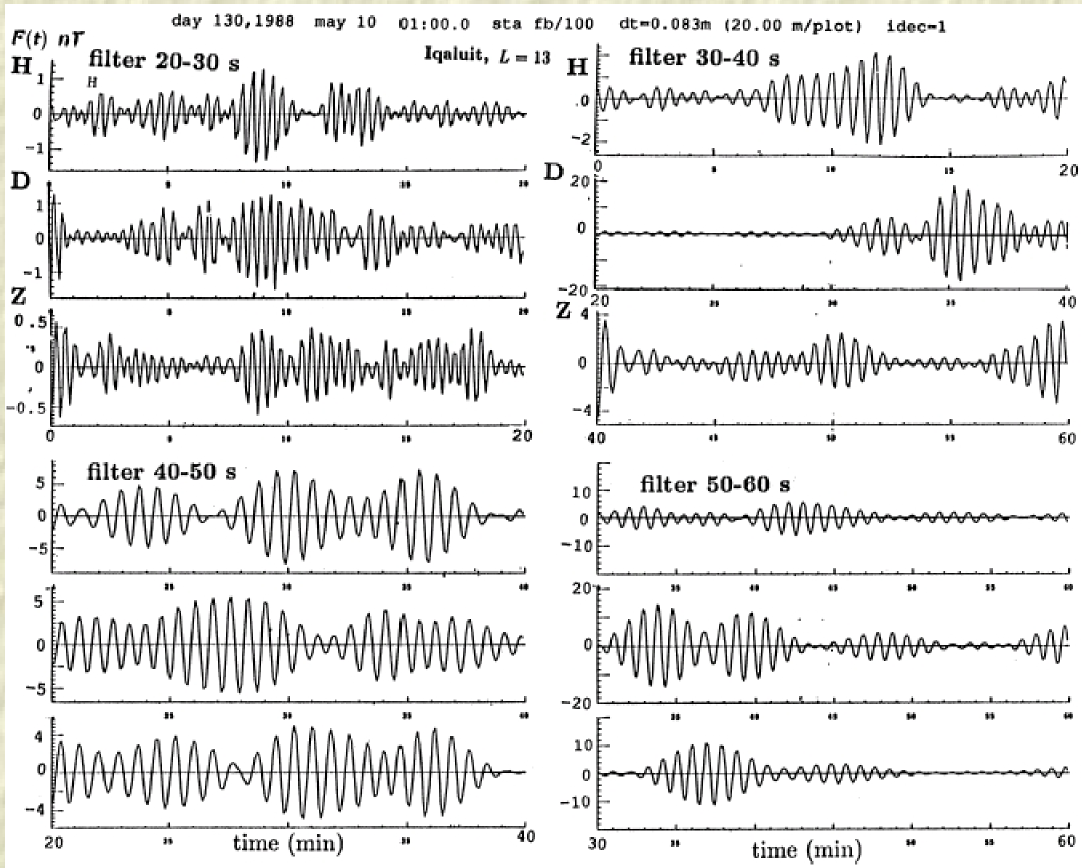


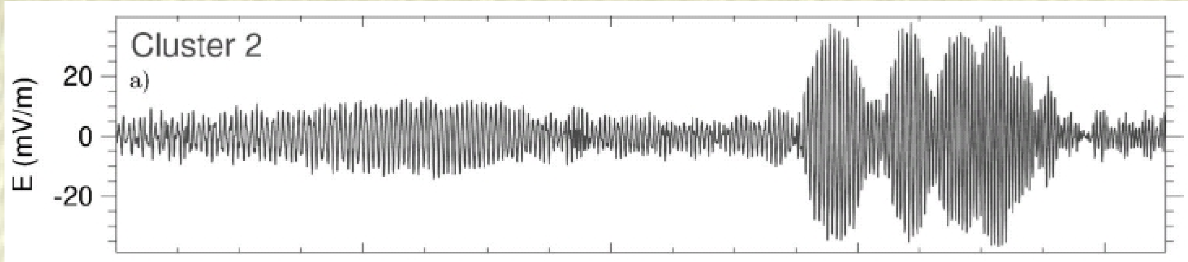
Modulated structures are encountered in various environments e.g., in the magnetosphere:





From: Ya. Alpert, Phys. Reports **339**, 323 (2001))

..., in satellite (e.g. CLUSTER, FAST, ...) observations:



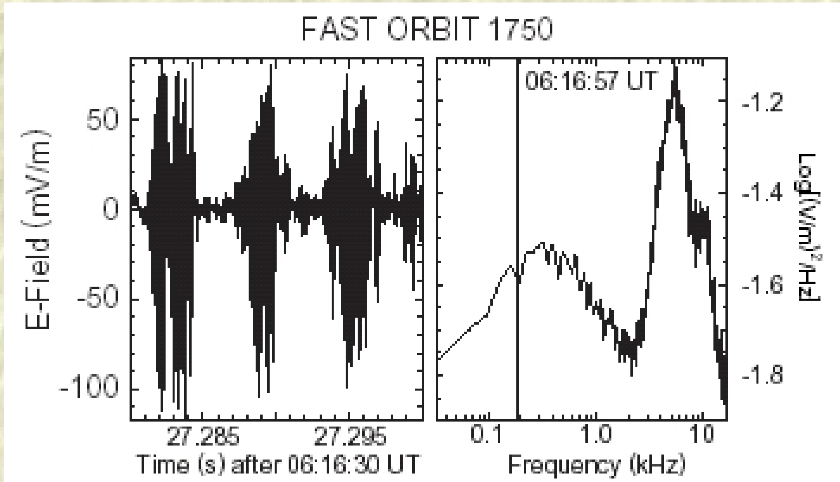


Figure 2. *Left*: Wave form of broadband noise at base of AKR source. The signal consists of highly coherent (nearly monochromatic frequency of trapped wave) wave packets. *Right*: Frequency spectrum of broadband noise showing the electron acoustic wave (at ~ 5 kHz) and total plasma frequency (at ~ 12 kHz) peaks. The broad LF maximum near 300 Hz belongs to the ion acoustic wave spectrum participating in the 3 ms modulation of the electron acoustic waves.

From: O. Santolik *et al.*, *JGR* **108**, 1278 (2003); R. Pottelette *et al.*, *GRL* **26** 2629 (1999).

..., in numerical simulations:

e.g. early (1972) numerical experiments of EM cyclotron waves:

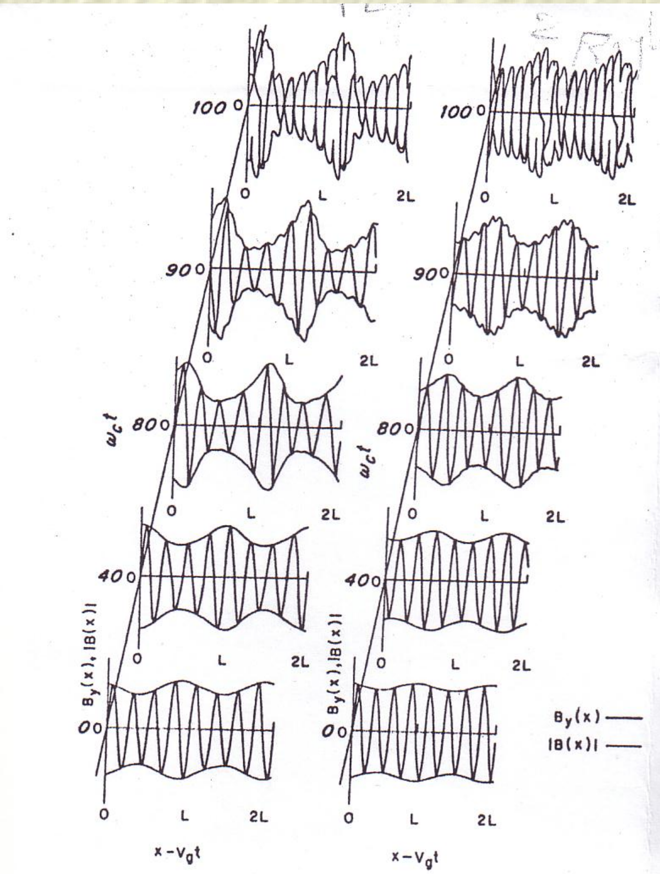


FIG. 2. Spatial variation of the y component of the wave magnetic field $B_y(x)$ (solid curve) and the magnitude of the total field $|B(x)|$ (thin curve) at different time of evolution for cases 1 (left) and 2 (right). Note the harmonic generation in $|B(x)|$ and the sudden collapse of the wave.

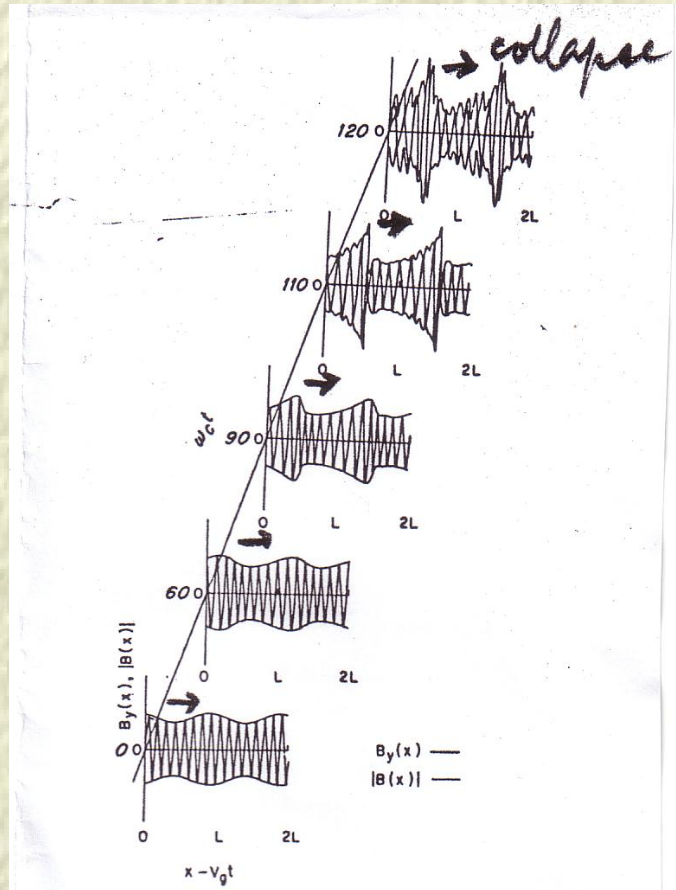


FIG. 3. $B_y(x)$ and $|B(x)|$ for case 4. Note the steepening of the wavefront which leads to the collapse of the wave.

From: A. Hasegawa, *PRA* **1**, 1746 (1970); *Phys. Fluids* **15**, 870 (1972).

..., in experiments on ES plasma waves:

Modulational instability in a multi-component plasma

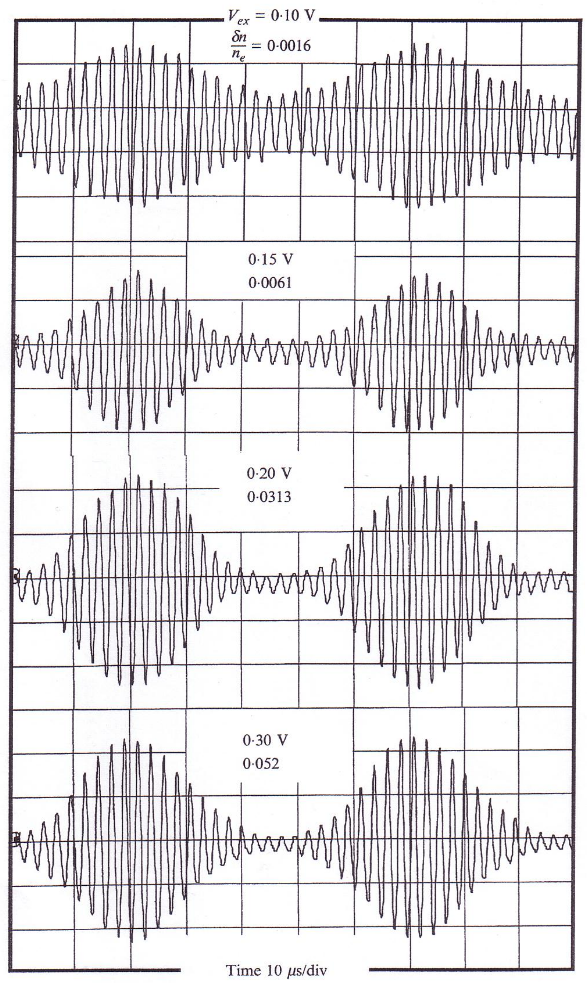


FIGURE 5. Oscilloscope traces of the detected signal for different excitation voltages. The probe was fixed at 14 cm from the grid. f_c = 400 kHz and f_m = 50 kHz.

From: Bailung and Nakamura, *J. Plasma Phys.* **50** (2), 231 (1993).

Pair-ion plasmas: prerequisites (1)

- Electron-ion plasmas:

  – *electrons* e^- (charge $-e$, mass m_e),

  – *ions* i^+ (charge $+Z_i e$, mass $m_i \gg m_e$),

  – ...

- Intrinsic features (*taken for granted* in "ordinary" e-i plasmas):

— *Distinct electron/ion frequency scales*, e.g.

$$\omega_{p,s} = \left(\frac{4\pi n_s q_s^2}{m_s}\right)^{1/2}, \qquad \omega_{c,s} = \frac{q_s B}{m_s c} \qquad (s = e, i)$$

hence $\omega_{p,e} \gg \omega_{p,i}$, $\omega_{c,e} \gg \omega_{c,i}$.

— Longevity (recombination neglected, total density conserved).

Pair-ion plasmas: prerequisites (2)

- Pair-ion plasmas:
  - *Positive ions* i^+ (charge $+Ze$, mass m),
  - *Negative ions* i^- (charge $-Ze$, mass m),
  - ... (heavier ions, in a multi-component eg. *e-p-i* composition).
- *No (pair-ion) frequency separation*: $\omega_{p,+} \approx \omega_{p,-} \quad \omega_{c,+} = \omega_{c,-}$.
- New Physics:

— *Novel ES/EM mode profile*

[Iwamoto PRE 1989, Stewart & Laing JPP 1992, Zank & Greaves PRE 1995, , Verheest & Cattaert 2005, 2006].

— No Faraday rotation.

Pair-ion plasmas: prerequisites (3)

- Magnetized *electron-positron (e-p)* and e-p-i plasmas exist(ed) in:
  - *pulsar magnetospheres* [Ginzburg 1971, Michel RMP 1982],
  - *bipolar outflows (jets) in active galactic nuclei (AGN)* [Miller 1987, Begelman RMP 1984]
  - *the center of our own galaxy* [Burns 1983],
  - the *early universe* [Hawking 1983],
  - *inertial confinement fusion schemes* [Liang *et al.* PRL 1998]
  - experiments [Greaves, Surko *et al.* PoP 1994, Zhao *et al.* PoP 1996].

- *Pair-ion plasmas (p.p.)* have been formed in laboratory,
  - recent *fullerene ion* ($C_{60}^{\pm}$) experiments [Oohara & Hatakeyama PRL 2003].

  $\rightarrow$ a very promising perspective (no recombination, unlike e-p)

Open questions

- Are ES/EM plasma waves propagating in p.p. modulationally stable?
- Can envelope excitations occur in pair plasmas?
- What if a third massive species were present? ($X^+X^-d^\pm$, $e^-p^+i^+$, ...)
- $\rightarrow X^+X^-d^\pm$: pair plasmas “doped” with dust defects;
- $\rightarrow e^-p^+i^+$: role of i^+ in e-p-i plasmas (w.r.t. high f oscillations)

Part A: (2+1)-fluid model for ES waves:

Fluid equations: (for $j = 1^+, 2^-$)

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j \; = \; -s_j \frac{Ze}{m} \nabla \phi - \frac{1}{m n_j} \nabla p_j$$

$$p_j = C n_j^\gamma \,, \qquad p_{j,0} = n_{j,0} k_B T_j \,, \qquad \gamma = 1 + 2/f \,, \qquad s_j = q_j/|q_j| = \pm 1$$

Poisson's equation

$$\nabla^2 \Phi \; = \; -4\pi \sum_s q_s \, n_s \; = \; 4\pi \, e \, (Z \, n_- - Z \, n_+ - s_3 Z_3 \, n_3)$$

Neutrality hypothesis *at equilibrium (only)*:

$$Z n_{+,0} - Z n_{-,0} + s_3 Z_3 \, n_3 = 0$$

$\rightarrow$ $3^\pm$: a massive (*immobile*) background species, eg. $3 = i^+$ in *epi* plasmas.

"Pure" p.p.: $n_3 = 0$, i.e. $n_{+,0} = n_{-,0}$, whereas $n_3 \neq 0$ in $e^- p^+ i^+$ or $X^+ X^- d^\pm$.

Perturbation method for modulated waves

– 1st step. Define *multiple scales* (*fast* and *slow*):

$$\mathbf{r}_0 = \mathbf{r}\,, \quad \mathbf{r}_1 = \epsilon\,\mathbf{r}\,, \quad \mathbf{r}_2 = \epsilon^2\,\mathbf{r}\,, \quad \ldots$$
$$T_0 = t\,, \quad T_1 = \epsilon\,t\,, \quad T_2 = \epsilon^2\,t\,, \quad \ldots$$

$\mathbf{r} = (x, y, z)$

– 2nd step. Expand near equilibrium:

$$n_j \approx n_{j,0} + \epsilon\,n_{j,1} + \epsilon^2\,n_{j,2} + \ldots$$
$$\mathbf{u}_j \approx \mathbf{0} + \epsilon\,\mathbf{u}_{j,1} + \epsilon^2\,\mathbf{u}_{j,2} + \ldots$$
$$\phi \approx 0 + \epsilon\,\phi_1 + \epsilon^2\,\phi_2 + \ldots$$

($\epsilon \ll 1$).

Perturbation method *(continued)*

– 3rd step. Project on Fourier space, i.e. consider $\forall m = 1, 2, ...$

$$S_m = \sum_{l=-m}^{m} \hat{S}_l^{(m)} \, e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \hat{S}_0^{(m)} + 2 \sum_{l=1}^{m} \hat{S}_l^{(m)} \cos l(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

– 4rth step. (for multi-dimensional propagation) *Modulation obliqueness*:

the slow amplitudes $\hat{\phi}_l^{(m)}$, etc. vary *only along* the x-axis:

$$\hat{S}_l^{(m)} = \hat{S}_l^{(m)}(X_j, T_j)\,, \qquad\qquad j = 1, 2, ...$$

while the fast carrier phase $\theta = \mathbf{k}\cdot\mathbf{r} - \omega t$ is now (in 2d):

$$k_x\, x + k_y\, y - \omega t = k\, r \cos\alpha - \omega t \quad .$$

First-order solution ($\sim \epsilon^1$)

- *Dispersion relation* $\omega = \omega(k)$, for $\omega \leftarrow \frac{\omega}{\omega_{p,-}}$, $\quad k \leftarrow k\lambda_{D,-} = \frac{k\,T_-^{1/2}}{m^{1/2}\,\omega_{p,-}}$:

$$\omega_1 \approx c_s k \,, \qquad \omega_2 \approx (\omega_0^2 + c_s^2 k^2)^{1/2} \,,$$

where

$$\omega_0^2 = (1+\beta)\,\omega_{p,-}^2 \,, \qquad c_s^2 = 3\,\beta\,\frac{1+\sigma\beta}{1+\beta}\,\frac{T_-}{m} \,.$$

Density ratio ($\rightarrow 1$ in pure *p.p.*):

$$\beta = n_{+,0}/n_{-,0}$$

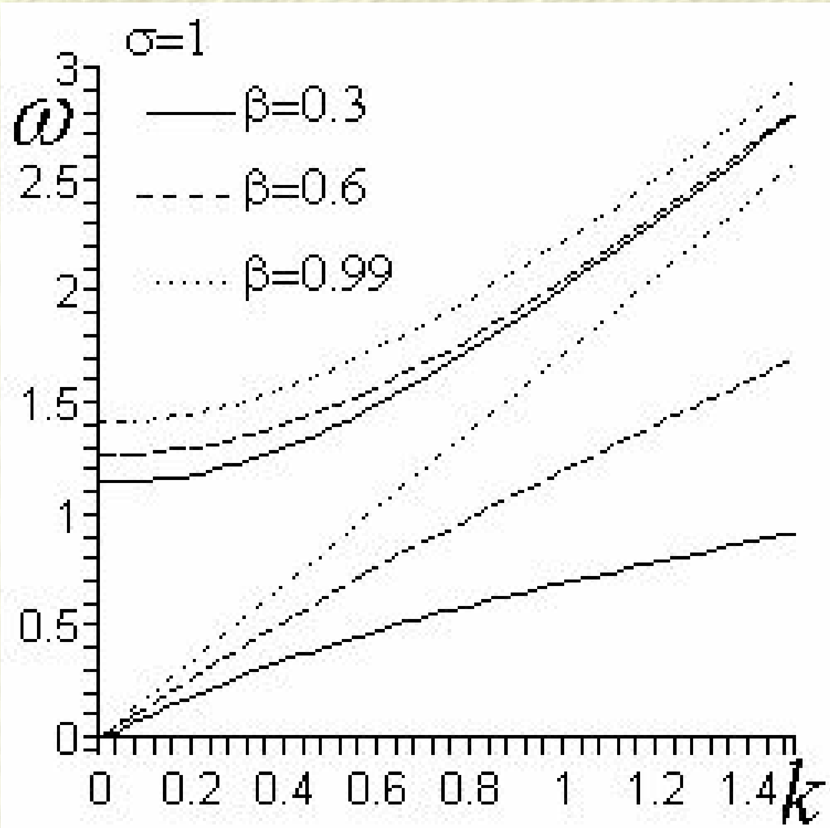
Temperature ratio:

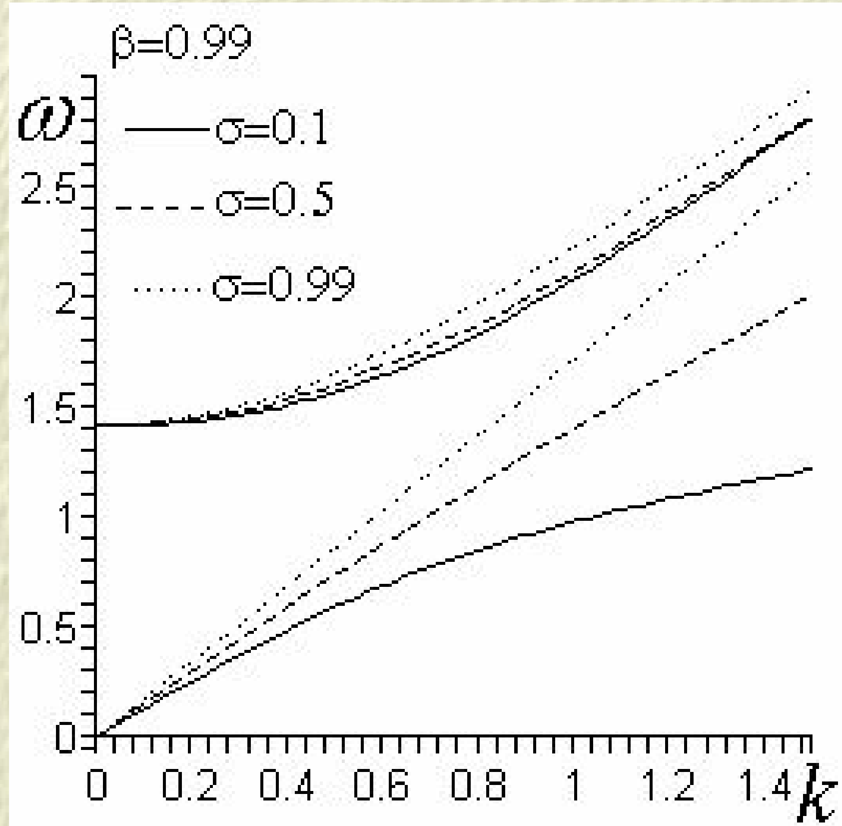
$$\sigma = T_+/T_-$$

- The *solution(s)* for the 1st–harmonic amplitudes (e.g. $\propto \phi_1^{(1)}$) read:

$$n_{+,1}^{(1)} = \frac{\beta k^2}{\omega^2 - 3\sigma\beta^2 k^2}\phi_1^{(1)} = \frac{\beta k}{\omega}\,u_{+,1}^{(1)} \,, \qquad n_{-,1}^{(1)} = -\frac{k^2}{\omega^2 - 3k^2}\phi_1^{(1)} = \frac{k}{\omega}\,u_{-,1}^{(1)}$$

Dispersion relation vs. parameters $\beta = n_{+,0}/n_{-,0}$, and $\sigma = T_+/T_-$





From: Esfandyari, Kourakis, Mehdipoor & Shukla, *JPA: Math. Phys.* **39**, 13817 (2006).

Second-order solution ($\sim \epsilon^2$)

- From $m = 2, l = 1$, we obtain the relation:

$$\frac{\partial \psi}{\partial T_1} + v_g \frac{\partial \psi}{\partial X_1} = 0 \tag{1}$$

where

– $\psi = \phi_1^{(1)}$ is the potential correction ($\sim \epsilon^1$);

– $v_g = \frac{\partial \omega(k)}{\partial k_x}$ is the group velocity along $\hat{x}$;

– the wave's envelope satisfies: $\psi = \psi(\epsilon(x - v_g t)) \equiv \psi(\zeta)$.

- The solution, up to $\sim \epsilon^2$, is of the form:

$$\phi \approx \epsilon \, \psi \, \cos \theta_c + \epsilon^2 \left[\phi_0^{(2)} + \phi_1^{(2)} \cos \theta_c + \phi_2^{(2)} \cos 2\theta_c \right] + \mathcal{O}(\epsilon^3) \, ,$$

(+ similar expressions for $n_{+/-}$ and $\mathbf{u}_{+/-}$)

- $\rightarrow$ *Harmonic generation!*.

Third-order solution ($\sim \epsilon^3$)

- Compatibility equation (from $m = 3, l = 1$), in the form of:

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0\,.$$

i.e. a *Nonlinear Schrödinger–type Equation* (NLSE) .

- Variables: $\zeta = \epsilon(x - v_g t)$ and $\tau = \epsilon^2\, t$;

- *Dispersion coefficient* P:

$$P = \frac{1}{2}\,\frac{\partial^2 \omega}{\partial k_x^2} = \frac{1}{2}\left[\omega''(k)\cos^2\alpha + \omega'(k)\,\frac{\sin^2\alpha}{k}\right]; \qquad (2)$$

- *Nonlinearity coefficient* Q: ... $\rightarrow$*(omitted)*

  $=$ A *(lengthy!)* function of k, angle α and plasma parameters.

NLSE Story 1: Modulational (in)stability analysis

- The NLSE admits the *harmonic wave solution*:

$$\psi \,=\, \hat{\psi}\, e^{iQ|\hat{\psi}|^2 \tau} \,+\, \mathrm{c.c.}$$

- *Perturb* the amplitude by setting: $\hat{\psi} \,=\, \hat{\psi}_0 \,+\, \epsilon\, \hat{\psi}_{1,0} \cos{(\tilde{k}\zeta - \tilde{\omega}\tau)}$

- We obtain the *(perturbation)* dispersion relation:

$$\tilde{\omega}^2 \,=\, P^2\, \tilde{k}^2 \left(\tilde{k}^2 \,-\, 2\frac{Q}{P} |\hat{\psi}_{1,0}|^2 \right).$$

- If $PQ < 0$: the amplitude ψ is *stable* to external perturbations;

- If $PQ > 0$: the amplitude ψ is *unstable* for $\tilde{k} < \sqrt{2\frac{Q}{P}}|\psi_{1,0}|$.

NLSE Story 2: Localized envelope excitations (envelope solitons)

- The NLSE:

$$i\,\frac{\partial\psi}{\partial\tau} + P\,\frac{\partial^2\psi}{\partial\zeta^2} + Q\,|\psi|^2\,\psi = 0$$

accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}$;

The *total* electric potential is then:

$$\phi \approx \epsilon\,\rho\,\cos(\mathbf{kr} - \omega t + \Theta)$$

where the amplitude ρ and phase correction Θ depend on ζ, τ.

- If $PQ > 0$: *Bright* solitons (envelope pulses);

- If $PQ < 0$: *Dark (black/grey)* solitons (envelope holes).

Localized envelope excitations (solitons) for $PQ > 0$

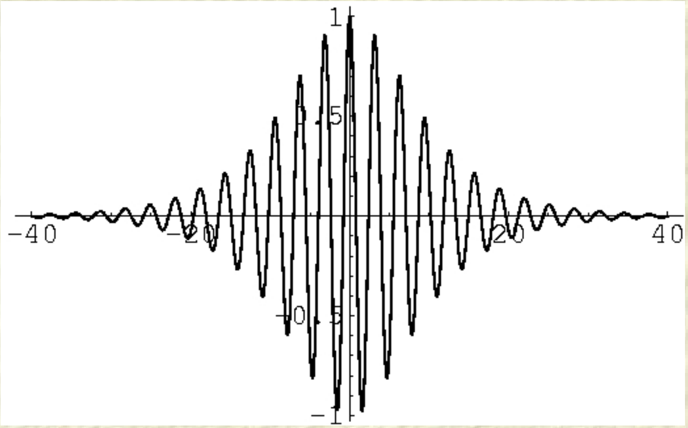
- Bright–type envelope soliton (pulse):

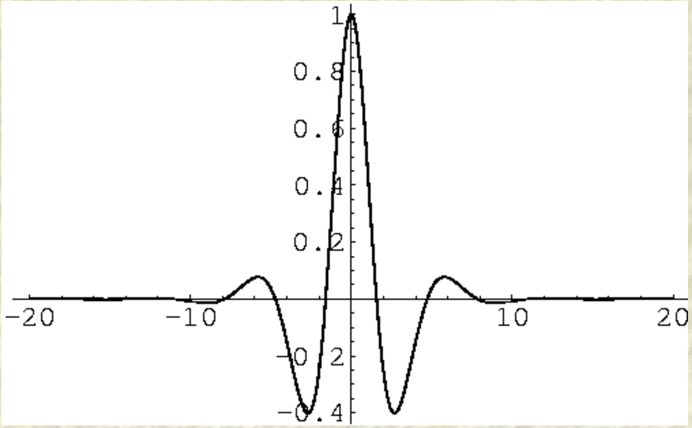
$$\rho = \rho_0 \,\mathrm{sech}\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta - (\Omega + \frac{1}{2}v^2)\tau\right].$$

where

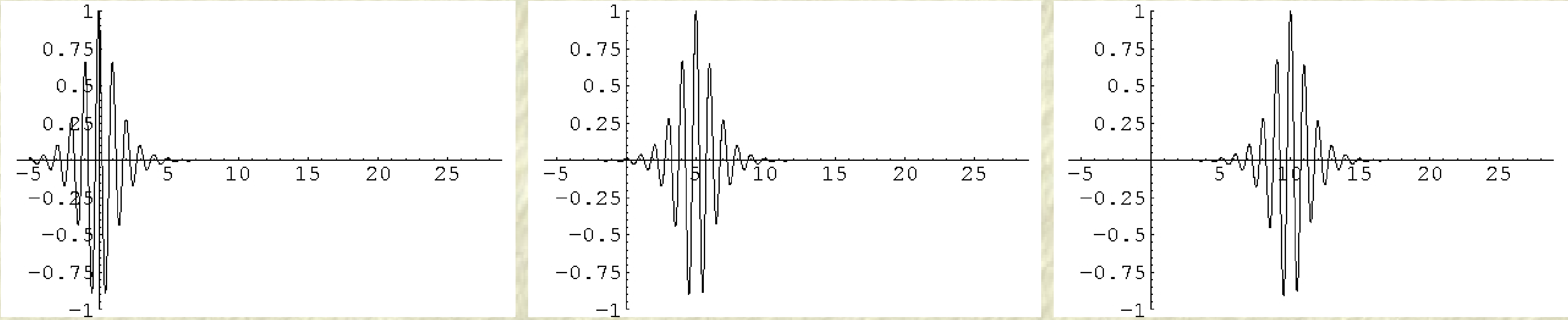
$$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}.$$

This is a propagating (and simultaneously *oscillating*) localized pulse envelope :

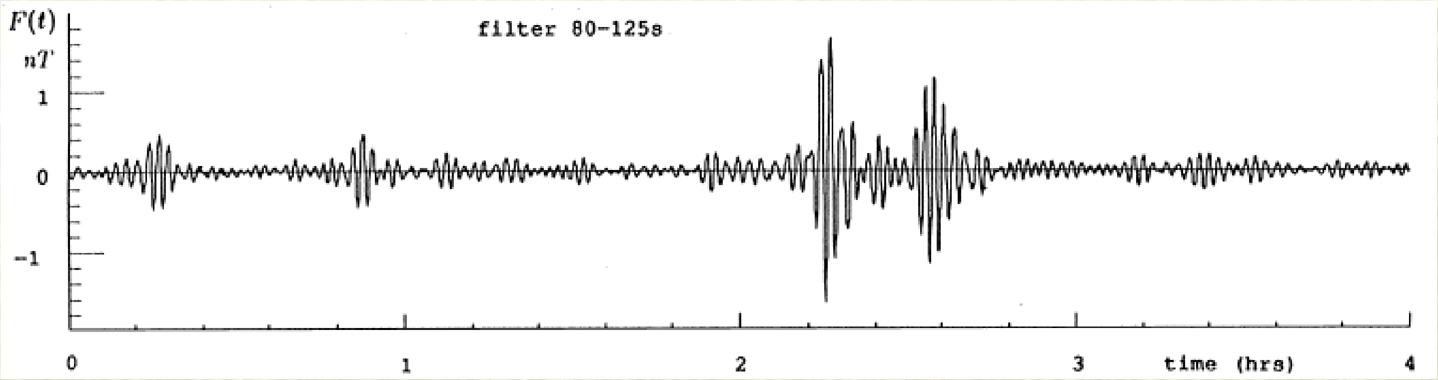




Propagation of a bright envelope soliton (pulse)

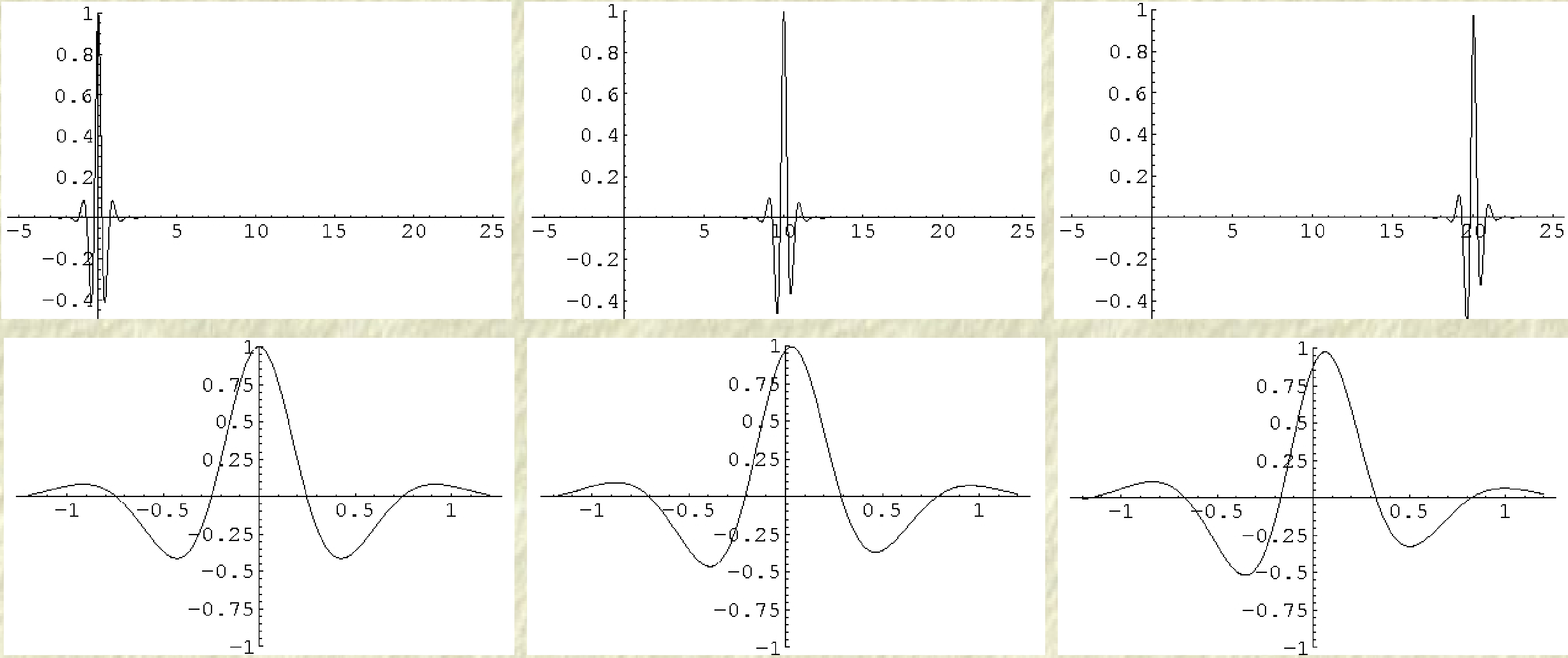


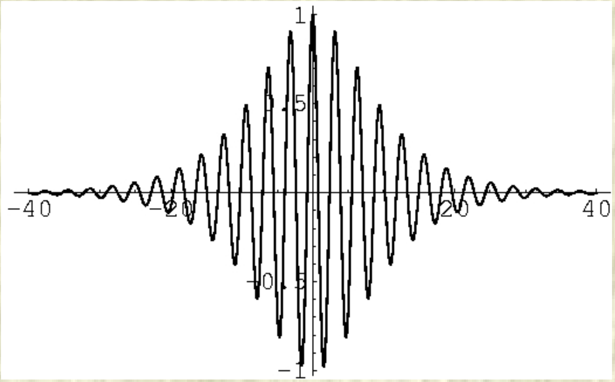
Cf. electrostatic plasma wave data from satellite observations:



(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

Propagation of a bright envelope soliton (continued...)





(→ *see movie*)

Localized envelope excitations for $PQ < 0$

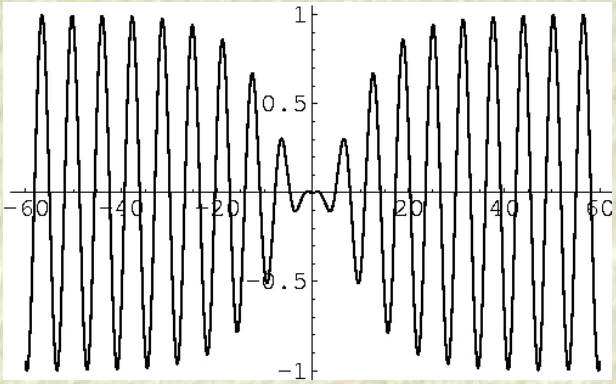
- Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \operatorname{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1^2\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|\frac{1}{\rho_1}}$$

This is a *propagating localized* envelope hole (a void):



Localized envelope excitations for $PQ < 0$

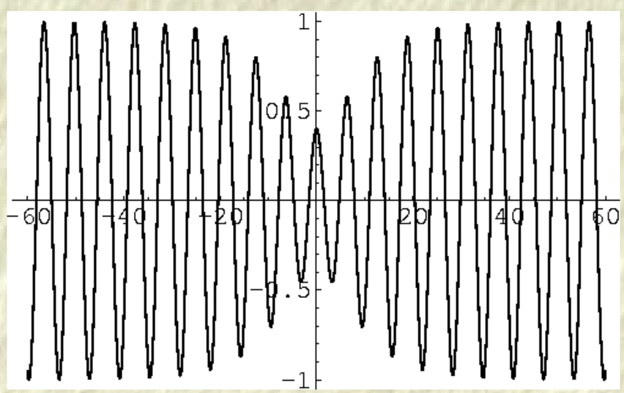
- Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - a^2 \,\mathrm{sech}^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

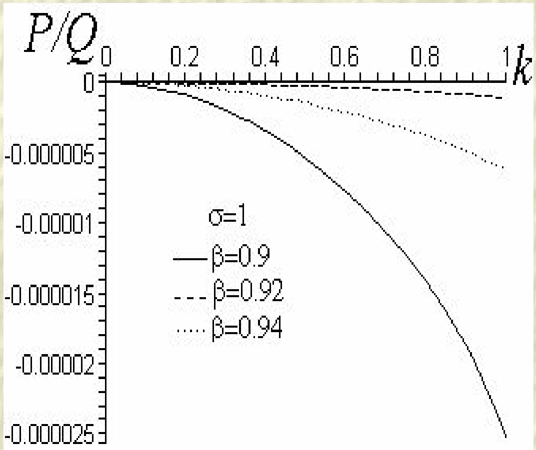
$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{a\rho_2}$$

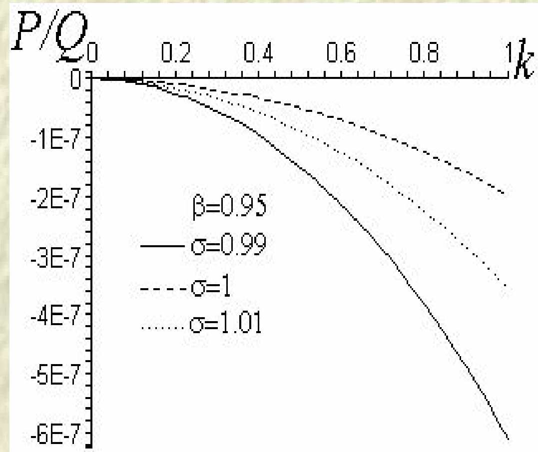
This is a *propagating localized* envelope hole (a void, yet not vanishing at the center):



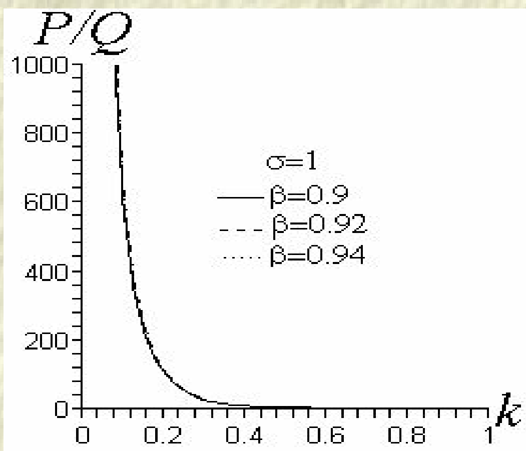
Stability profile (ESW): P/Q ratio versus reduced wavenumber $k\lambda_{D,-}$

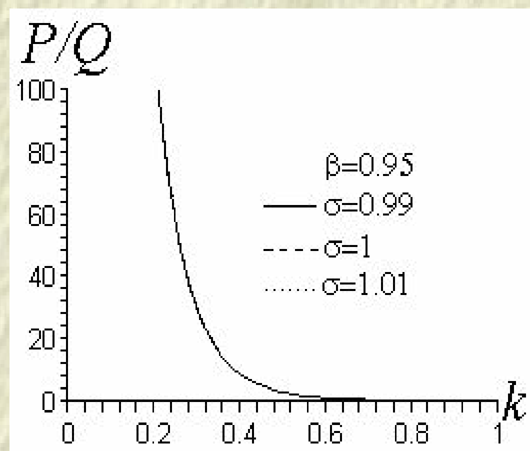
– *Lower (acoustic) mode*:





– *Upper (optic-type) mode*:





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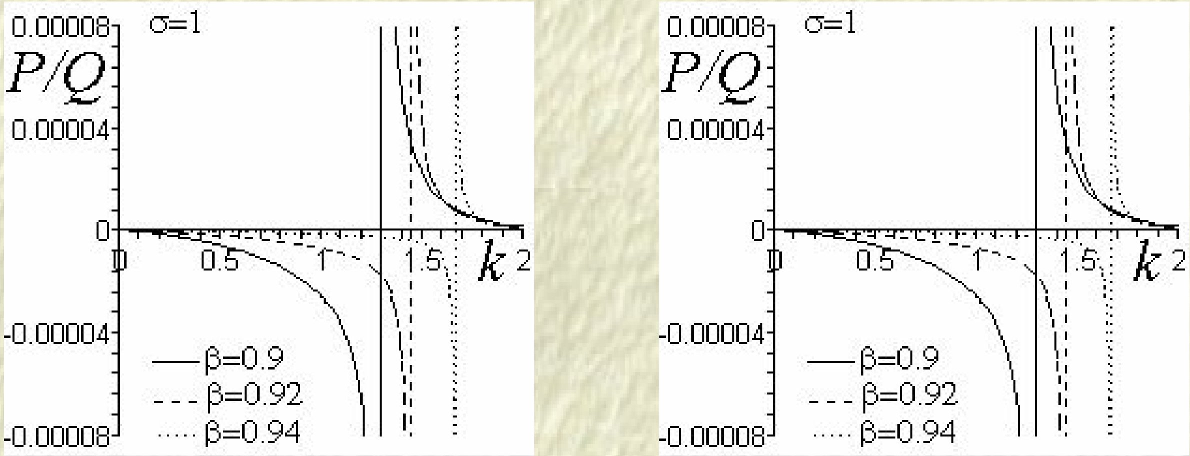

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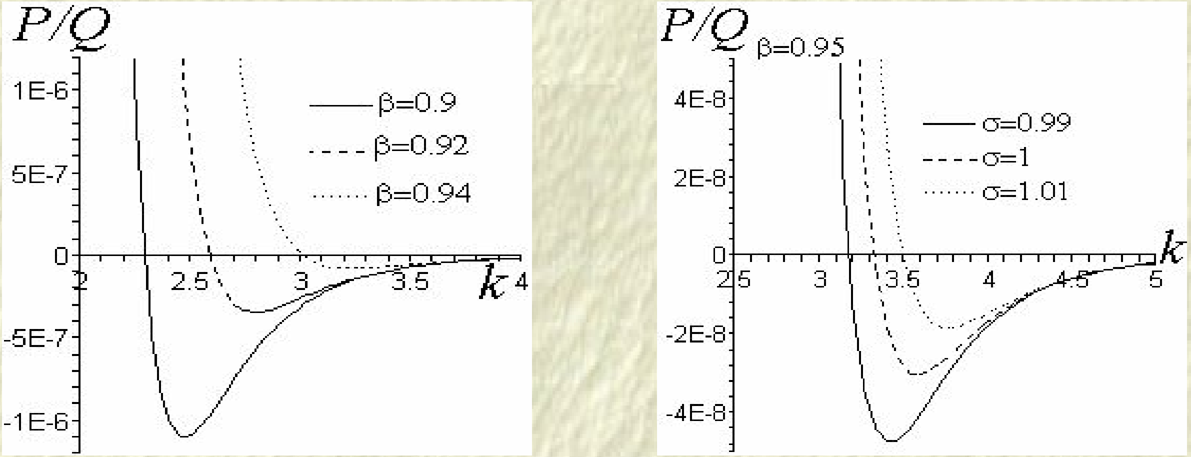


Part B: (2+1)-fluid model for *oblique* EM waves in p.p. or e-p-i plasmas

Fluid equations (for $j = 1^+, 2^-$):

$(q_1 = -q_2 = +Ze)$

$(m_1 = m_2 = m)$

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = \frac{q_j}{m_j} \left(\mathbf{E} + \frac{1}{c} \mathbf{u}_j \times \mathbf{B} \right)$$

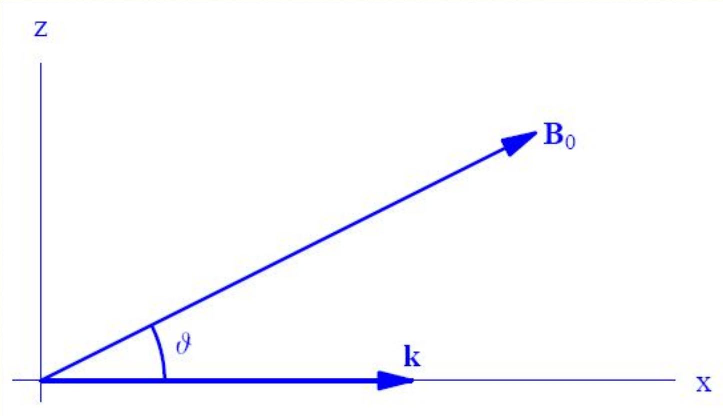
Maxwell's laws:

$$\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \,, \qquad \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c} \sum_j n_j q_j \,, \qquad \nabla \cdot \mathbf{B} = 0$$

+ a convenient frame:

$\mathbf{k} = (k, 0, 0)$

$\mathbf{B}_0 = (B_0 \cos\theta, 0, B_0 \sin\theta)$



First-order ($\sim \epsilon^1$): linear dynamics

- *Dispersion relation* $\forall\theta$: $D(\omega, k; \theta) = d_0(\omega, k) + d_1(\omega, k) \sin^2\theta = 0$

$$
\begin{aligned}
d_0(\omega, k) &\equiv D(\omega, k; \theta = 0) \\
&= (\omega^2 - \omega^2_{p,eff}) \\
&\quad \times \left\{ \left[(\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega^2 \omega^2_{p,eff} \right]^2 - \omega^2 \Omega^2 (\omega^2_{p,1} - \omega^2_{p,2})^2 \right\} \\
&= (\omega^2 - \omega^2_{p,eff}) \\
&\quad \times \left\{ (\omega + \Omega) \left[-(\omega^2 - c^2k^2)(\omega - \Omega) + \omega\omega^2_{p,1} \right] + \omega(\omega - \Omega)\omega^2_{p,2} \right\} \\
&\quad \times \left\{ (\omega - \Omega) \left[-(\omega^2 - c^2k^2)(\omega + \Omega) + \omega\omega^2_{p,1} \right] + \omega(\omega + \Omega)\omega^2_{p,2} \right\},
\end{aligned}
$$

$$
d_1(\omega, k; \theta) = -c^2k^2\Omega^2 \left\{ c^2k^2\omega^2_{p,eff}(\omega^2 - \Omega^2) + \omega^2 [4\omega^2_{p,1}\omega^2_{p,2} - (\omega^2 - \Omega^2)\omega^2_{p,eff}] \right\},
$$

Notation: $\omega^2_{p,eff} = \omega^2_{p,1} + \omega^2_{p,2}$; Ω is the (common) cyclotron frequency.

First-order solution ($\sim \epsilon^1$)

$$
\begin{aligned}
n_j^{(11)} &= n_{j,0}\frac{k}{\omega}u_{j,x}^{(11)} = c_{j,n,y}^{(11)}B_y' + c_{j,n,z}^{(11)}B_z' \,,\\
u_{j,i}^{(11)} &= c_{j,i,y}^{(11)}B_y' + c_{j,i,z}^{(11)}B_z' \,.\\
E_i^{(11)} &= c_{el,i,y}^{(11)}B_y' + c_{el,i,z}^{(11)}B_z' \qquad (\text{for } j=1,2 \text{ and } i=x,y,z)\\
B_x^{(nl)} &= \text{cst}\,.
\end{aligned}
$$

where

$$
\begin{aligned}
c_{j,x,y}^{(11)} &= i(-1)^{j+1}\frac{\omega^2\Omega^3\sin\theta\cos\theta}{k\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}\\
c_{j,x,z}^{(11)} &= \frac{\Omega^2\sin\theta}{k\,(\omega^2-\Omega^2)\left[\omega^2(\omega^2-\Omega^2-\omega_{p,eff}^2)+\Omega^2\omega_{p,eff}^2\cos^2\theta\right]}\times\\
&\quad\left\{-\omega^3(\omega^2-\Omega^2-\omega_{p,eff}^2)\right.\\
&\qquad\left.+i\Omega\omega_{p,eff}^2\cos\theta\left[(-1)^{j+1}\omega^2+\Omega\cos\theta(i\omega+(-1)^j\Omega\cos\theta)\right]\right\}
\end{aligned}
$$

($j, j' = 1, 2$ and $j' \neq j$) (*continued* →)

First-order solution ($\sim \epsilon^1$) *(continued)*

$$c^{(11)}_{j,y,y} = \frac{\omega\Omega^2(\omega^2 - \omega^2_{p,eff})\cos\theta}{k\,[\omega^2(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos^2\theta]}$$

$$c^{(11)}_{j,y,z} = \frac{\Omega\omega}{k\,(\omega^2 - \Omega^2)}\left[i(-1)^{j+1}\omega + \frac{\Omega^3\omega^2_{p,eff}\cos\theta\sin^2\theta}{\omega^2(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos^2\theta}\right]$$

$$c^{(11)}_{j,z,y} = i\,(-1)^j\,\frac{\omega^2\Omega(\omega^2 - \omega^2_{p,eff} - \Omega^2\sin^2\theta)}{k\,[\omega^2(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos^2\theta]}$$

$$c^{(11)}_{j,z,z} = \frac{\Omega^2\left\{\omega^3(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos\theta(\omega\cos\theta + i(-1)^j\Omega\sin^2\theta)\right\}\cos\theta}{k\,(\omega^2 - \Omega^2)\left[\omega^2(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos^2\theta\right]}\,,$$

$$c^{(11)}_{el,x,y} = c^{(11)}_{el,x,z} = \frac{\omega\Omega^2\omega^2_{p,eff}\sin\theta\cos\theta}{ck[\omega^2(\omega^2 - \Omega^2 - \omega^2_{p,eff}) + \Omega^2\omega^2_{p,eff}\cos^2\theta]}\,,$$

$$c^{(11)}_{el,y,y} = c^{(11)}_{el,z,z} = 0$$

$$c^{(11)}_{el,y,z} = -c^{(11)}_{el,z,y} = \frac{\omega}{ck}\,.$$

Second-order solution ($\sim \epsilon^2$)

- From $m = 2, l = 1$, we obtain a compatibility condition in the form:

$$\frac{\partial \tilde{B}_\perp}{\partial T_1} + v_g \frac{\partial \tilde{B}_\perp}{\partial X_1} = 0 \qquad (3)$$

 – $\tilde{B}_\perp = B_z^{(11)} + C B_y^{(11)}$ is the magnetic field (envelope) correction;

 – $v_g = \frac{d\omega(k)}{dk} = -\frac{\partial D/\partial k}{\partial D/\partial \omega}$ is the group velocity;

 – the magnetic field correction (amplitude) satisfies:

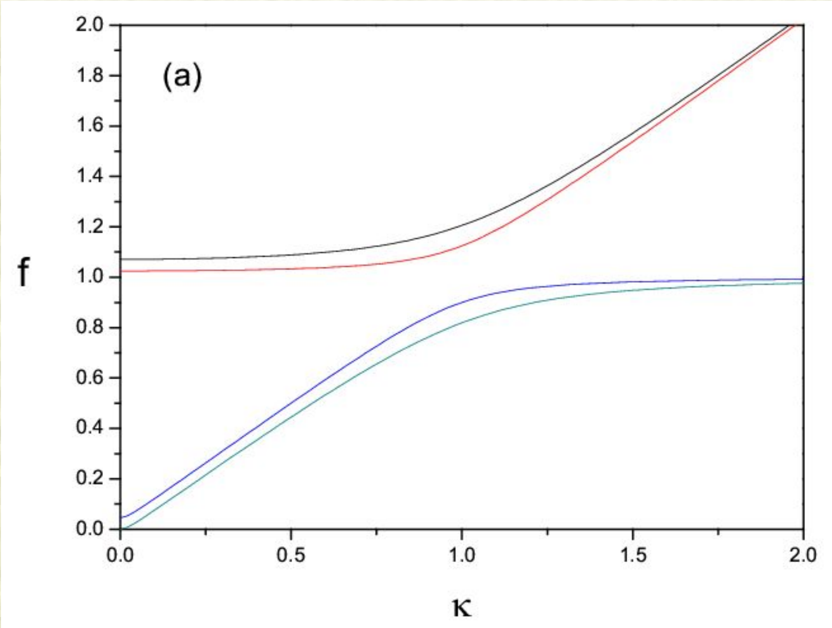
$$B_{y/z} = B_{y/z}(X_1 - v_g T_1) \equiv B_{y/z}(\zeta)\,.$$

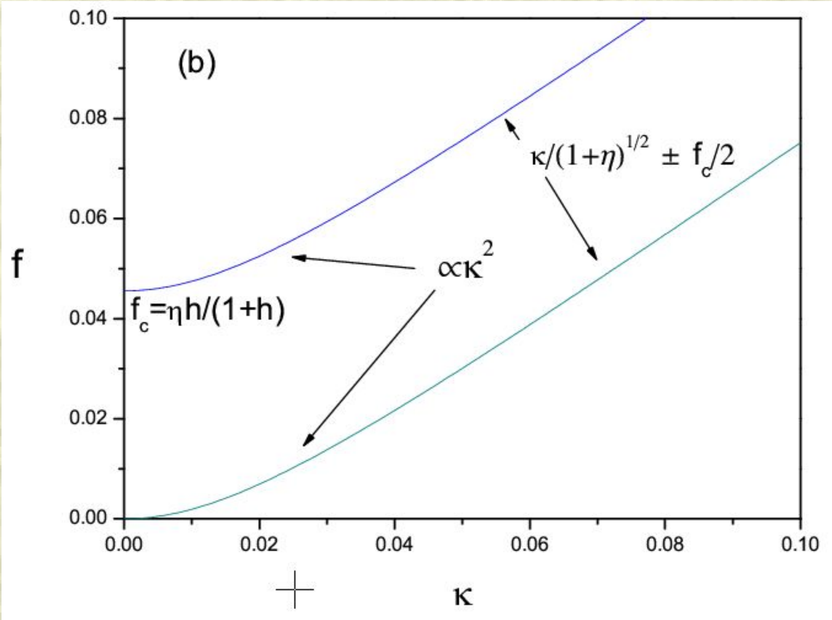
 – C is a (complex) phase shift factor; $C \to \pm i$ for $\theta \to 0$.

- *Second and zeroth harmonic generation!* (expressions omitted).

Parallel **($\parallel \mathrm{B}_0$) EM wave propagation: $f = \omega/\Omega$ vs. $\kappa = ck/\Omega$ (for $n_{+,0} \neq n_{-,0}$)**

$$D_{\parallel}(\omega, k) = (\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega^2\omega_{p,eff}^2 \pm \omega\Omega(\omega_{p,1}^2 - \omega_{p,2}^2) = 0$$





Here $\eta = (n_{+,0} - n_{-,0})/(n_{+,0} + n_{-,0}) = 0.5$, $\quad h = \omega_{p,eff}^2/\Omega^2 = 0.1.$

From: N. Cramer, ICPP (2006).

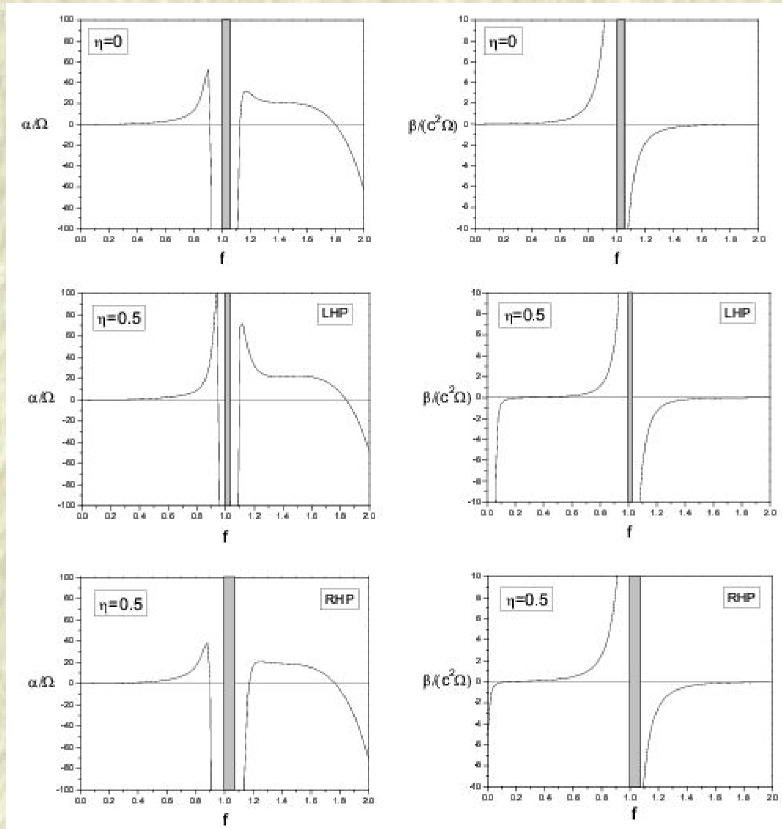
2 linearly polarized modes*($\eta = 0$, p.p.) vs. 4 circularly polarized modes ($\eta \neq 0$)

* [Iwamoto PRE 1989, Stewart & Laing JPP 1992, Zank & Greaves PRE 1995, Verheest and Cattaert 2006, 2007].

Nonlinear Schrödinger equation for the amplitudes $B_{y,z}^{(11)}$

$$i\,\frac{\partial \tilde{B}_\perp}{\partial \tau} + \beta\,\frac{\partial^2 \tilde{B}_\perp}{\partial \zeta^2} + \alpha\,|\tilde{B}_\perp|^2\,\tilde{B}_\perp = 0\,.$$

Influence of the 3rd species on EM wave stability:



From: Cramer, ICPP (2006).

Perpendicular ($\perp \mathbf{B}_0$) EM wave propagation

- *Dispersion relation* for $\theta = \pi/2$: $D(\omega, k; \frac{\pi}{2}) = d_{\perp,1}(\omega, k)\, d_{\perp,2}(\omega, k) = 0$

$$\begin{aligned} d_{\perp,1}(\omega, k) \quad &= \quad -\omega^6 + \omega^4[c^2\, k^2 + 2(\Omega^2 + \omega_{p,eff}^2)] \\ &\quad -\omega^2[(\Omega^2 + \omega_{p,eff}^2)^2 - c^2k^2(2\Omega^2 + \omega_{p,eff}^2)] \\ &\quad +\Omega^2[c^2k^2\, (\Omega^2 + \omega_{p,eff}^2) + (\omega_{p,1}^2 - \omega_{p,2}^2)^2] \end{aligned}$$

$$d_{\perp,2}(\omega, k) = \omega^2 - \omega_{p,eff}^2 - c^2k^2$$

O-mode: a robust perpendicular mode, whose dispersion characteristics do not depend on the ambient magnetic field ; same form for *e-i* plasmas.

Cf. (for $\mathbf{B_0} = \mathbf{0}$) G S Lakhina & B Buti, Astrophys. Space Sci. **79**, 25 (1981).

O-mode results (Kourakis, Verheest & Cramer, PoP 14, 022306, 2007)

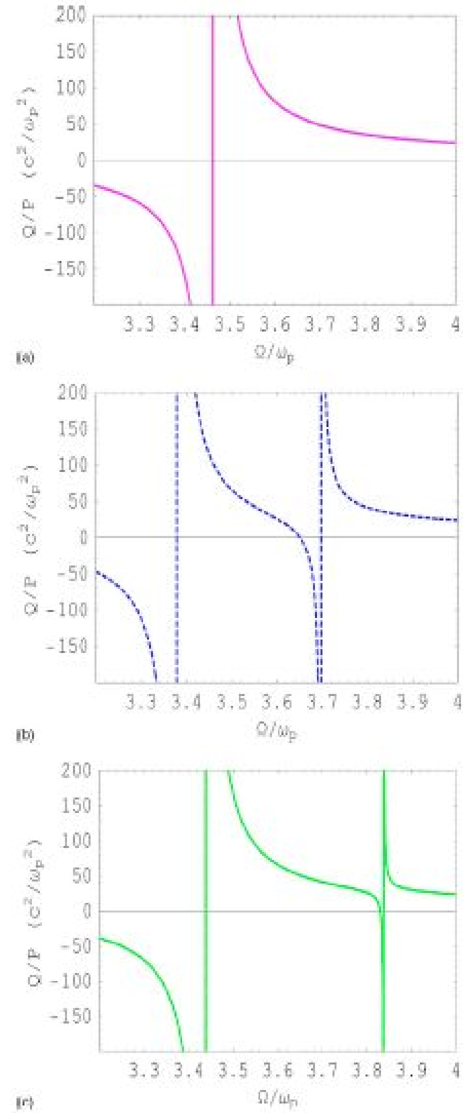


FIG. 5. Coefficient ratio Q/P for: (a) "pure" pair plasmas ($r=1$, i.e., $n_{+,0}=n_{-,0}$); (b) pair plasmas doped with positive background ions ($r=0.5$, i.e. $n_{-,0}=2n_{+,0}$); (c) pair plasmas doped with negative background ions ($r=2$, i.e., $n_{+,0}=2n_{-,0}$). Q/P values (scaled by ω_p^2/c^2), as provided by Eqs. (19), (24), and (14), vs cyclotron frequency Ω/ω_p. Here we have considered a carrier frequency near the plasma frequency cutoff, i.e., $\omega=\omega_{p,1}\sqrt{2}$.

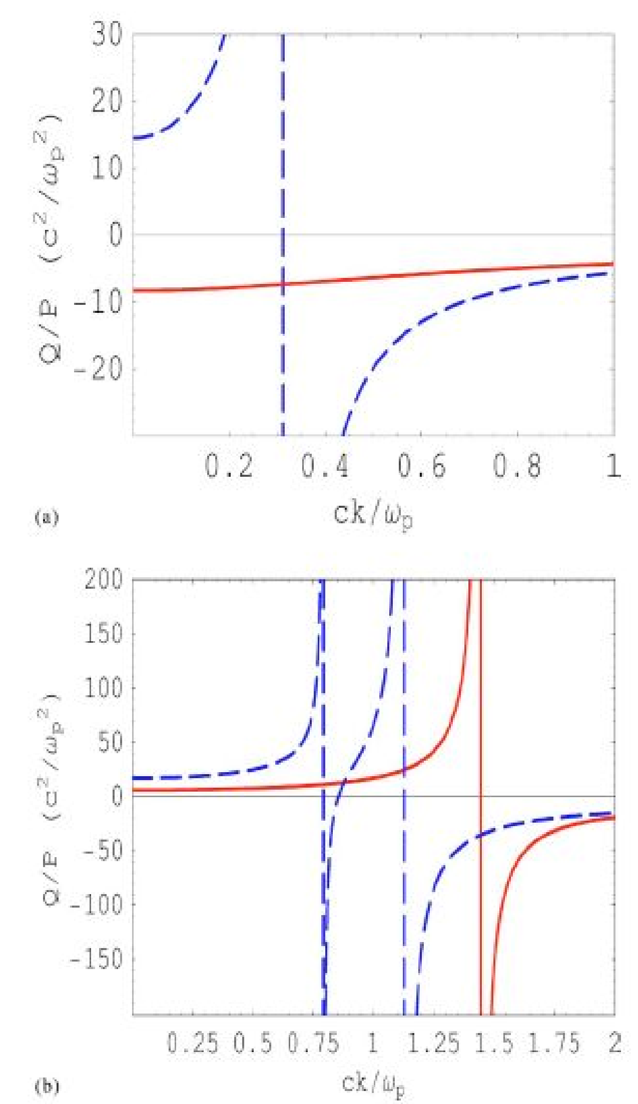


FIG. 6. Coefficient ratio Q/P (scaled by ω_p^2/c^2) vs wave number ck/ω_p, for: (a) $\Omega/\omega_{p,1}=2.1$ and $r=1$ (pure p.p., straight line) or $r=2$ (negative third ion species, dashed curve); (b) $\Omega/\omega_{p,1}=3.5$ and $r=1$ (pure p.p., straight line) or $r=0.5$ (positive third ion species, dashed curve).

Conclusions

- Modulated ES and EM waves may undergo spontaneous *modulational instability*; this may drive nonlinear evolution towards ...

- ... *energy localization*, via the formation of *envelope localized structures* (envelope solitons);

- Modulated *envelope solitons* bear specific "signature" (features like e.g. amplitude-width relation) which allow for a critical verification of the theory via observations and/or experiments.

- The stability profile of ES/EM modes in p.p. are modified if a third, massive species is present.

- Inherent drawback of a fluid theory: *Landau damping* overseen,
  $\longrightarrow$ to be considered *a posteriori*.

- Future extensions of the theory : relativistic effects, 2D geometry, more exotic localized envelope solutions (*dromions*?), ...

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Material from:

Kourakis, Verheest & Cramer, PoP **14**, 022306 (2007);

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A. Esfandyari-Kalejahi, I. Kourakis, M. Mehdipoor and P.K. Shukla, J. Phys. A: Math. Gen., **39**, 13817 (2006).

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