Existence of breathers and multibreathers in a two-dimensional dusty plasma crystal

V. Koukouloyannis*, I. Kourakis**

**Aristoteleion University of Thessaloniki, Theoretical Mechanics, 54124 Thessaloniki, Greece,*

***Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr Universität Bochum, D-44780 Bochum, Germany*

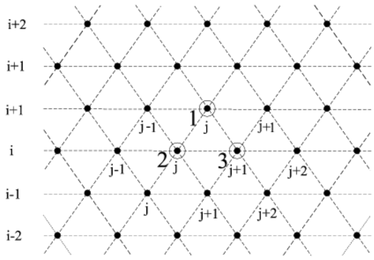
I. Introduction

Recent theoretical and experimental investigations of dust-contaminated plasmas (*dusty plasmas*, DP) [1] have established the existence of *strongly coupled* DP lattices (crystals). These crystalline configurations, consisting of highly charged massive dust grains, are typically formed in the sheath region above a horizontal negatively biased electrode in gas discharge experiments (e.g. [1, 2]). Typical low-frequency oscillations are known to occur [1, 2] in these *mesoscopic* dust grain quasi-lattices in the longitudinal (in-plane, acoustic mode), horizontal transverse (in-plane) and vertical transverse (off-plane, inverse dispersive optic-like mode) directions. A variety of 2D and 3D configurations are possible [1b], although the spontaneous occurrence of successive *hexagonal* 2D layers seems to be the most often encountered possibility. A 1D DP crystal has also been realized experimentally, by using appropriate substrate potentials. Such 1D lattices have been shown to host collective excitations, in the form of solitons, localized envelope wavepackets, as well as discrete breather-type excitations (see [8] and Refs. therein).

In the present work a hexagonal DP lattice in considered. Transverse motion in this system is described by a Klein-Gordon-like Hamiltonian in the presence of an asymmetric quartic potential. By adopting real values for the potential (nonlinearity) parameters, as provided by experiments [4, 5, 6], and using the results of [6, 7], we shall prove that 2D DP crystals may support single-site as well as multi-site localised oscillations (multibreathers) [9].

II. Equation of motion

Fig.1: In a 2D dusty plasma crystal the lattice is self-organised in a hexagonal lattice.



The vertical grain displacement obeys an equation in the form

$$\frac{d^2\delta z_{ij}}{dt^2}+\nu\frac{d\delta z_{ij}}{dt}+\omega_0^2\left(\delta z_{i-1,j}+\delta z_{i-1,j+1}+\delta z_{i,j-1}+\delta z_{i,j+1}+\delta z_{i+1,j-1}+\delta z_{i+1,j}-2\delta z_{ij}\right)+\omega_g^2\delta z_{ij}+\alpha\left(\delta z_{ij}\right)^2+\beta\left(\delta z_{ij}\right)^3=0$$

where $\delta z_{ij}=z_{ij}(t)-z_0$ denotes the small displacement of the *ij*−th grain around the (levitated) equilibrium position z_0, in the transverse (*z*−) direction. The characteristic frequency results from the dust grain (electrostatic) interaction potential Φ*(r),* e.g. for a Debye-Hückel potential, one has $\omega_{0,D}^2=q^2/\left(Mr_0^3\right)\left(1+r_0/\lambda_D\right)\exp\left(-r_0/\lambda_D\right)$ where λ_D denotes the effective DP Debye radius. The damping coefficient *ν* accounts for dissipation due to collisions between dust grains and neutral atoms. The gap frequency ω_g and the nonlinearity coefficients *α, β* are defined via the overall vertical force: $F(z) = F_{e/m} - Mg \simeq -M[\omega_g^2(\delta z)+\alpha(\delta z)^2+ (\delta z)^3]+O[(\delta z)^4]$, which has been expanded around z_0 by formally taking into account the (anharmonicity of the) local form of the sheath electric and/or magnetic field(s), as well as, possibly, grain charge variation due to charging processes [10]. Recall that the electric/magnetic levitating force(s) $F_{e/m}$ balance(s) gravity at z_0. Notice the difference in structure from the usual nonlinear Klein-Gordon equation used to describe one-dimensional oscillator chains, TDLWs ('phonons') in this chain are stable only in the presence of the field force $F_{e/m}$.

For convenience, we may re-scale the time and vertical displacement variables over appropriate quantities, i.e. the characteristic (single grain) oscillation period ω_g^{-1} and the lattice constant r_0, respectively, viz. $t=\omega_g^{-1}$ and $\delta z_{ij}=r_0 q_{ij}$ Eq. (1) is thus expressed as:

$$\frac{d^2q_{ij}}{d\tau^2}+\varepsilon\left(q_{i-1,j}+q_{i-1,j+1}+q_{i,j-1}+q_{i,j+1}+q_{i+1,j-1}+q_{i+1,j+1}-2q_{ij}\right)+q_{ij}+\alpha' q_{ij}^2+\beta' q_{ij}^3=0$$

where the (dimensionless) damping term, now expressed as $\left(\nu/\omega_g\right)dq_n/d\tau\equiv\nu'\dot{q}_n$, will be henceforth omitted in the left-hand side. The coupling parameter is now , and the nonlinearity coefficients are now: $a'=\alpha r_0/\omega_g^2$ and $\beta'=\beta r_0^2/\omega_g^2$

The experiment on anharmonic single grain oscillations by Ivlev et al. [3], carried out in Garching (Germany),provides $\alpha' \simeq -0.5$ and $\beta' \simeq 0.07$ (for a lattice spacing, typically, of the order of r_0=1mm). Note that the damping coefficient *ν* was as low as $\nu/2\pi \simeq 0.067$ sec^{-1}, so that (with $\omega_g/2\pi \simeq 17$ sec−1) one has: $\nu' = \nu/\omega_g \simeq 0.00$ (the pressure in that experiment was kept as low as 0.5 Pa).

Zafiu et al. [4], (Kiel, Germany) provides various possible values for the anharmonicity parameters, via successive experiments: $\alpha' \simeq +0.02/+0.016/-0.27$ and $\beta' \simeq -0.16/-0.17/-0.03$. Again, we consider lattice spacing of the order of r_0=1mm, damping very low (ν'=0.02).

The results of the experiment on linear TDLWs by Misawa et al. [6] allows for a rough estimation of the coupling strength $\omega_g \simeq 155$ sec^{-1} and $\omega_0 \simeq 19.5$ sec^{-1}, which give $\varepsilon \simeq 0.016$. Note that the effective damping term was kept as low as $\nu \simeq 0.239$ sec^{-1}, i.e. $\nu' = \nu/\omega_g \simeq 0.00154$.

III. Existence of Multibreathers in Dusty Plasma Crystals

The above system can be produced by a Hamiltonian of the form

$$H=H_0+\varepsilon H_1=\sum_{i,j=-\infty}^{\infty}\frac{p_{ij}^2}{2}+V(x_{ij})+\frac{\varepsilon}{2}\sum_{i,j=-\infty}^{\infty}[(x_{ij}-x_{i-1,j})^2+(x_{ij}-x_{i-1,j+1})^2+(x_{ij}-x_{i,j-1})^2+(x_{ij}-x_{i,j+1})^2+(x_{ij}-x_{i+1,j-1})^2+(x_{ij}-x_{i+1,j})^2]$$

The theoretical prediction of the existence of multi-site breathers in hexagonal DP crystals follows closely the generic proof suggested in [6]. Apart from single-site breathers, three distinct types of multibreathers were found, depending on the phase difference, between the oscillators. We define $\phi_1=\vartheta_2-\vartheta_1,\ \phi_2=\vartheta_3-\vartheta_1$ and consequently $\phi_3=\vartheta_3-\vartheta_1=\phi_2-\phi_1$. The key point is to show that these structures remain stable up to $\varepsilon = 0.016$, which is the experimental value for the coupling.

Single-site breathers

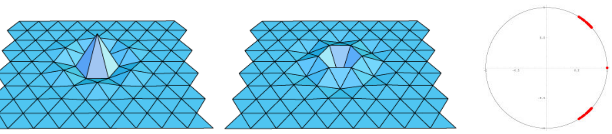


Fig2. Two snapshots of a single site breather and the corresponding Floquet multipliers

The case of the single breathers is quite simple, since we only have to avoid the phonon band as it can be shown by fig.2. We can acquire linearly stable breathers up to ε=0.016 for all the choices of nonlinearity parameters.

Multi-site breathers

In-phase breathers : $\phi_1=\phi_2=0$

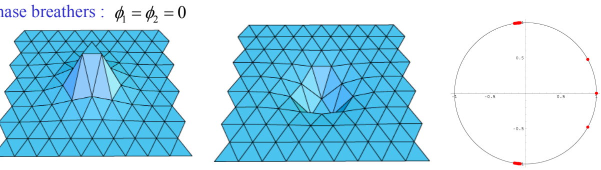


Fig.3: An in-phase 3-site breather nad the corresponding Floquet multipluiers.

The multipliers of the central oscillators move along the unit circle since $\varepsilon\frac{\partial\omega}{\partial J}>0$ so the breather is linearly stable and remains this way until these multipliers reache the linear (phonon) spectrum. We can achieve this for ε=0.016 for the two sets of parameters of [4] and the set of [3]. Note that the eigenvalues which correspond to the central oscillators are double but they don't leave the unit circle due to symplectic signature reasons.

Out of phase breathers : $\phi_1=\pi,\ \phi_2=0$ or $\phi_1=0,\ \phi_2=\pi$

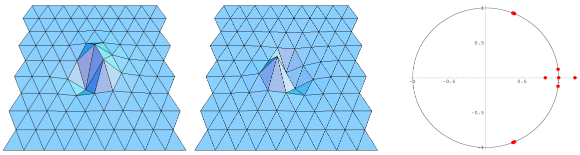


Fig.4: The anti-phase 3-site breather destabilises even for ε=0.001

One pair of multipliers corresponding to the central oscilltors, leave the unit circle for $\varepsilon>0$ in the case of the anti-phase 3-site breather, as it is shown in [6].

Vortex breathers : $\phi_1=2\pi/3,\ \phi_2=4\pi/3$ or $\phi_1=4\pi/3,\ \phi_2=2\pi/3$

In this case, the resulting breathers are vortex breathers. In the first case the breather is moving anti-clockwise (in the fig. below: a-b-c-d) while in the second case the breather is moving clockwise (a-d-c-b)

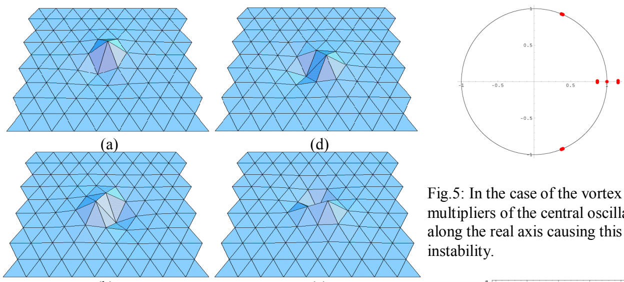


Fig.5: In the case of the vortex breather the multipliers of the central oscillators move both along the real axis causing this way a strong instability.

Note that the system proposed in [4] has a graph of $\omega(J)$ which is shown in fig.6. If we consider initial conditions further than the turning point of the graph, the stability is reversed, so we can acquire vortex breathers.

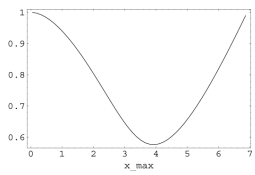


Fig.6: $\omega(J)$ of the system proposed in [4].

IV. Conclusions – Future Work

We have shown that a two-dimensional hexagonal dusty plasma crystal can support single-site breathers, and in-phase 3-site breathers, while the rest of the theoretically predicted are highly unstable. An intriguing result refers to the system of [4]. In this system for high amplitudes of oscillation the expected 3-site breathers are are essentially vortex breathers. These results will hopefully be confirmed by appropriate experiments.

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