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Nonlinear Excitations in Dusty Plasma (Debye) Crystals

A new test-bed for nonlinear theories

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www.tp4.rub.de/∼ioannis/conf/200703-MPIPkS-oral.pdf

Outline

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites.
   (i) Focus: 1d dust crystals in lab.
   (ii) Nonlinearity in 1d DP crystals: Origin and modelling.

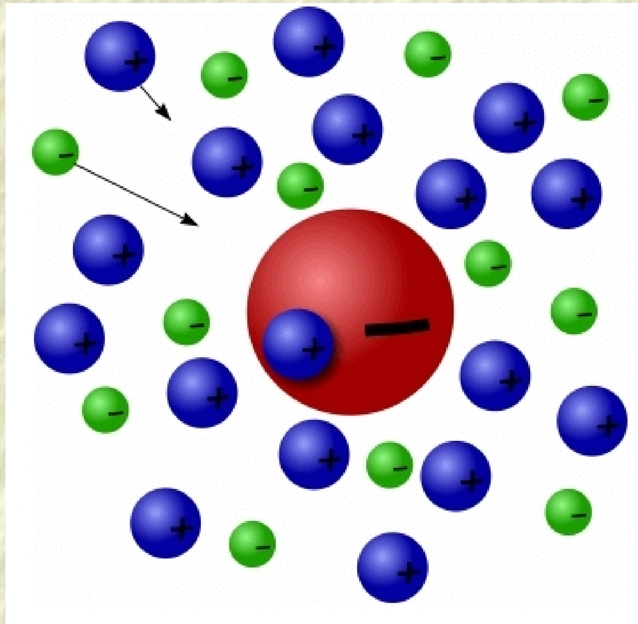
2. *Transverse* dust-lattice (*TDL*) excitations:
   amplitude modulation, transverse envelope structures.

3. *Longitudinal* dust-lattice (*LDL*) excitations:
   amplitude modulation, envelope structures, solitons.

4. 1d *Discrete Breather excitations* (*intrinsic localized modes*)

5. Conclusions.

Part 1. DP – Dusty Plasmas (or *Complex Plasmas*): definition, characteristics, modelling



- Ingredients:
  - *electrons* e^- (charge $-e$, mass m_e),
  - *ions* i^+ (charge $+Z_i e$, mass m_i), *and*
  - charged particulates $\equiv$ *dust grains* d (most often d^-):

$$\text{charge } Q = sZ_d e \sim \pm(10^3 - 10^4)\, e, \quad (s = \pm 1)$$
$$\text{mass } M \sim 10^9\, m_p \sim 10^{13}\, m_e,$$
$$\text{radius } r \sim 10^{-2}\, \mu m \text{ up to } 10^2\, \mu\text{m}.$$

Where/how do dusty plasmas occur?

- *Space: cosmic debris* (silicates, graphite, amorphous carbon), comet tails, man-made pollution (Shuttle exhaust, satellites), ...
- *Earth's atmosphere:* volcanic eruptions, *extraterrestrial* origin (*meteorites*) ($\geq 2 \cdot 10^4$ tons/yr!), *pollution*, *aerosols*, ...;
- *Fusion devices:* plasma-surface interaction in the divertor region (graphite, CFCs), UFOs, ITER safety concern, ...;
- *Technology:* Semiconductor industry, Si microchip, dust contamination, solar cell stabilization ...;
- *Laboratory:* (man-injected) melamine–formaldehyde particulates injected in *rf* or *dc* discharges.

Sources: P. K. Shukla & A. Mamun, book (IoP, 2002), G. E. Morfill *et al.*, 1998, etc.

Dusty Plasma physics: unique mesoscopic features

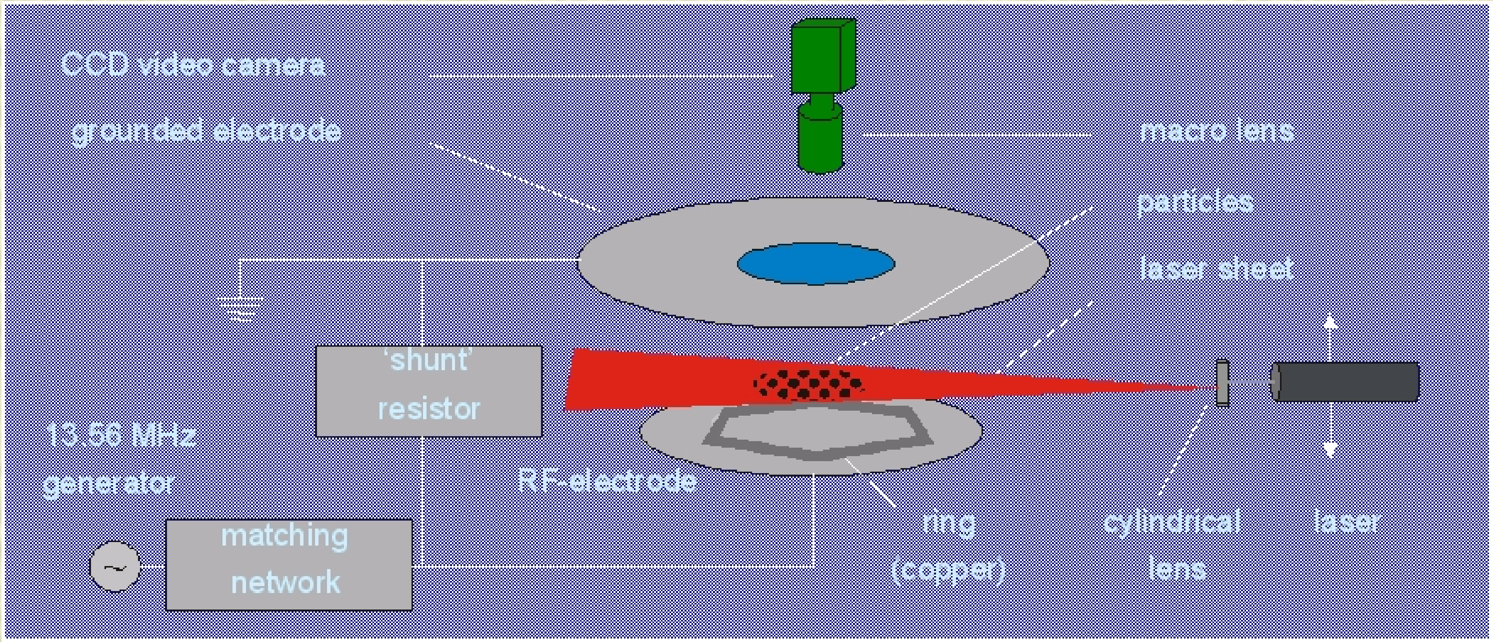
- Studies in *slow motion* are possible due to high M i.e. *low Q/M ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);
- Contrary to *weakly-coupled* $e-i$ plasmas ($\Gamma \ll 1$), Complex Plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [IKEZI 1986]) states, depending on the value of:

$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \sim \frac{Q^2}{r\,T} e^{-r/\lambda_D}$$

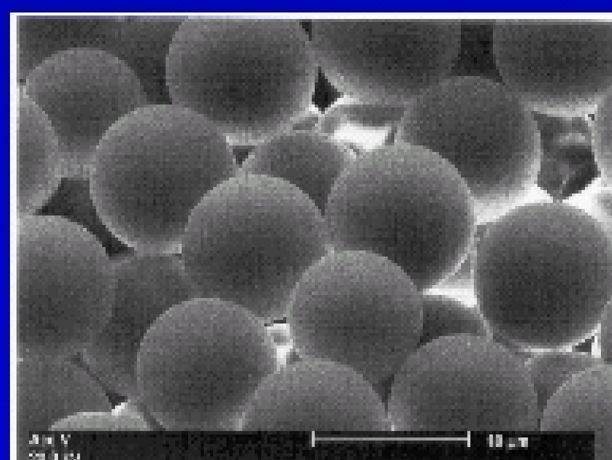
(r: inter-particle distance, T: temperature, λ_D: Debye length).

$\rightarrow$ *Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)*

Dust Crystal experiments:



Particles:
Melamine-Formaldehyde
diameter: few µm



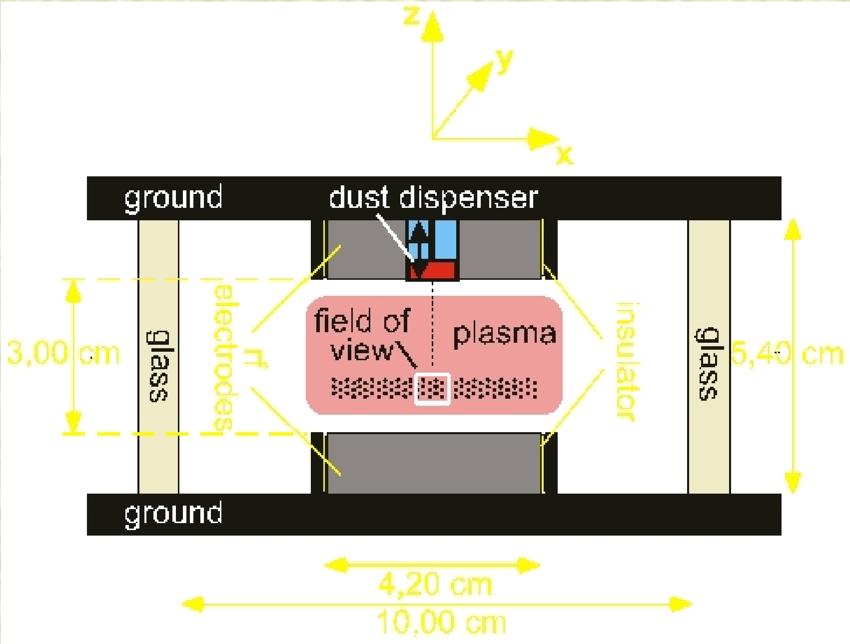
Gas:
noble gas (argon, krypton)
pressure: few Pa ... 100 Pa (=1 mbar)

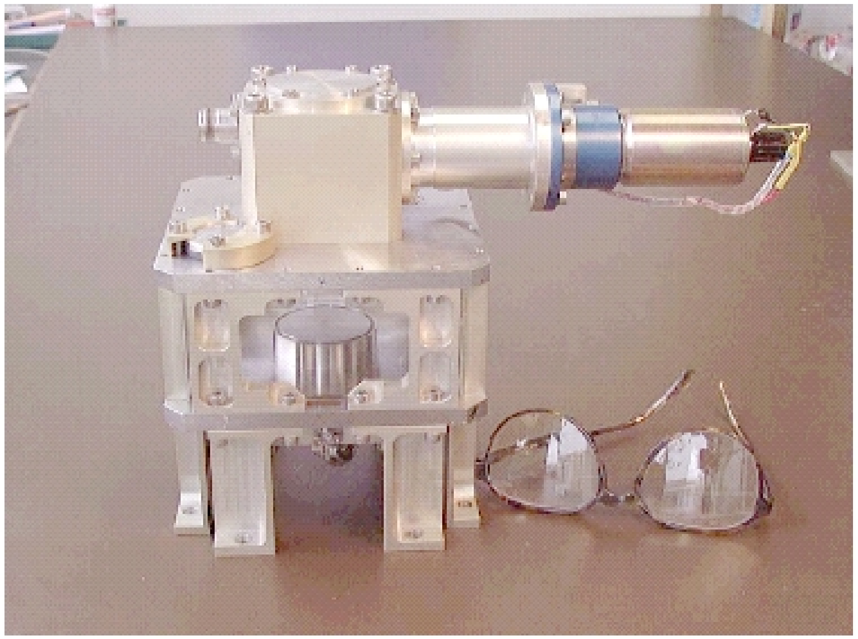
Ionisation fraction: 10^{-6} - 10^{-7}

Temperatures:
kT_e ~2-4 eV (electrons)
kT_i ~0.03 eV (ions)
kT_s ~0.025 eV (microparticles) in crystalline state

[Source: Thomas *et al.*, PRL 1994].

Dust Crystal experiments:





- Theoretical prediction: 1986 [H. Ikezi, Phys. Fluids **29**, 1764 (1986)];
- Experimental realization: 1994

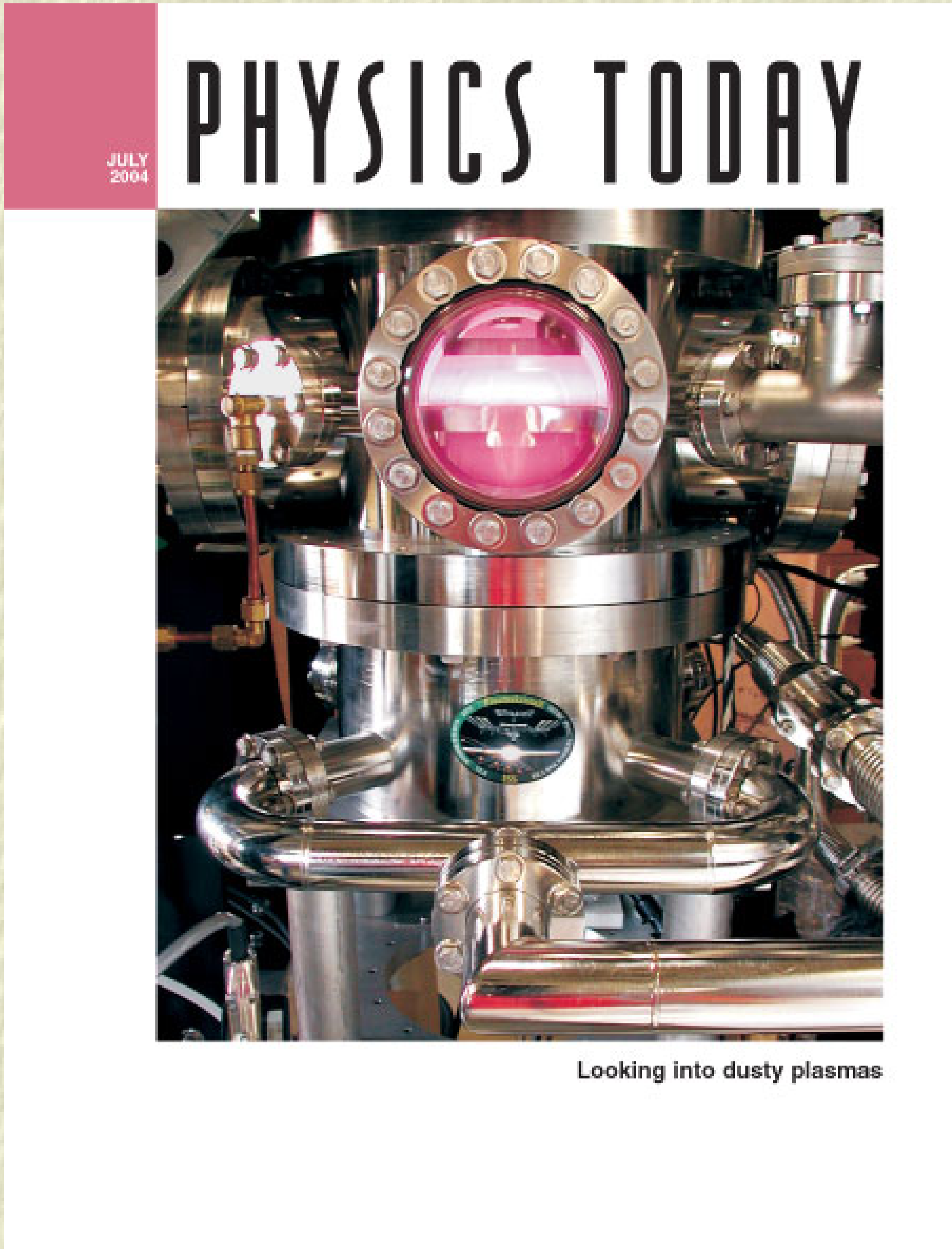
  [H. Thomas *et al.* PRL **73**, 652 (1994); Chu & Lin I J. Phys. D **27** 296 (1994), Hayashi & Tachibana, Jap. J. Appl. Phys. **33** L804 (1994)];

- Today, various experimental groups active worldwide: *G E Morfill* (MPIeP Garching, Germany), *A Piel* (Kiel, Germany), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *S Vladimirov* (Sydney, AUS), *S Takamura* (Nagoya, Japan) ...

- Purpose built experiments on board the *International Space Station (ISS)*;

- Mesoscopic analog of micro-structures; research includes:
  – phase transitions, crystallization processes,
  – relaxation times, diffusion effects,
  – phase space distribution (visually observable!),
  – L & NL waves: *harmonic generation, solitons, vortices*, ...

- 3d, 2d (triangular, mostly), 1d lattice configurations possible.


JULY
2004
PHYSICS TODAY
Looking into dusty plasmas

1d DP crystal: Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt}\right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

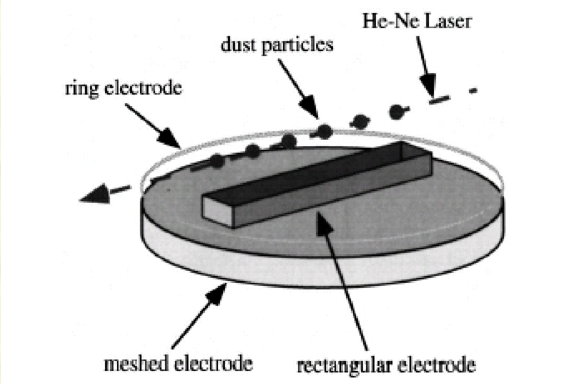


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

where:

– *Kinetic energy*;

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *'external' force fields*: may account for *confinement potentials* and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.

– Coupling: $U_{int}(r_{nm})$ is the *interaction potential energy*;

Q.: Nonlinearity: *Origin: where from ?*
Effect: which consequence(s) ?

Nonlinearity: Where does it come from?

(i) *Sheath* environment (*anharmonic* vertical potential):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2} M \omega_g^2 (\delta z_n)^2 + \frac{1}{3} M \alpha \, (\delta z_n)^3 + \frac{1}{4} M \beta \, (\delta z_n)^4 + \ldots$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z_{(0)}$; α, β, ω_g are defined experimentally

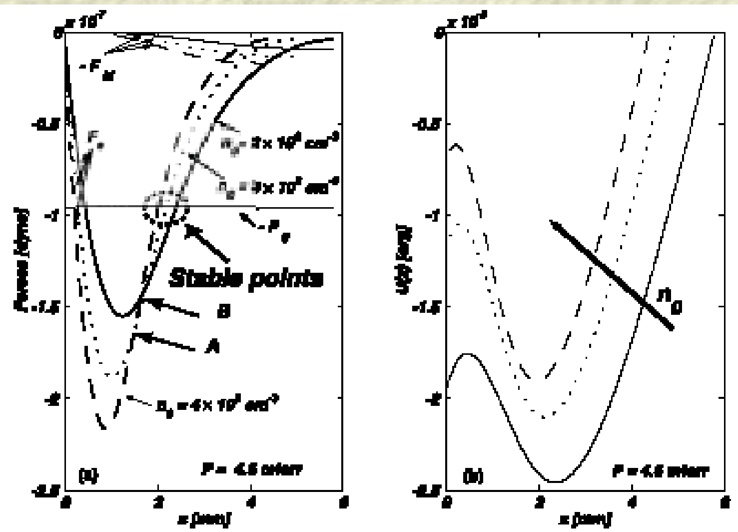


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2 \times 10^8 cm^{-3}$ (solid line), $n_0 = 3 \times 10^8 cm^{-3}$ (dashed line), $n_0 = 4 \times 10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$.

Source: Sorasio *et al.* (2002).

Nonlinearity: Where from? *(continued ...)*

(ii) *Interactions* between grains: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened: $r_0/\lambda_D \approx 1$), *anharmonic*; typically: $U_{Debye}(r) = \frac{q^2}{r} \exp{(-r/\lambda_D)}$.

Expanding $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$ near equilibrium:

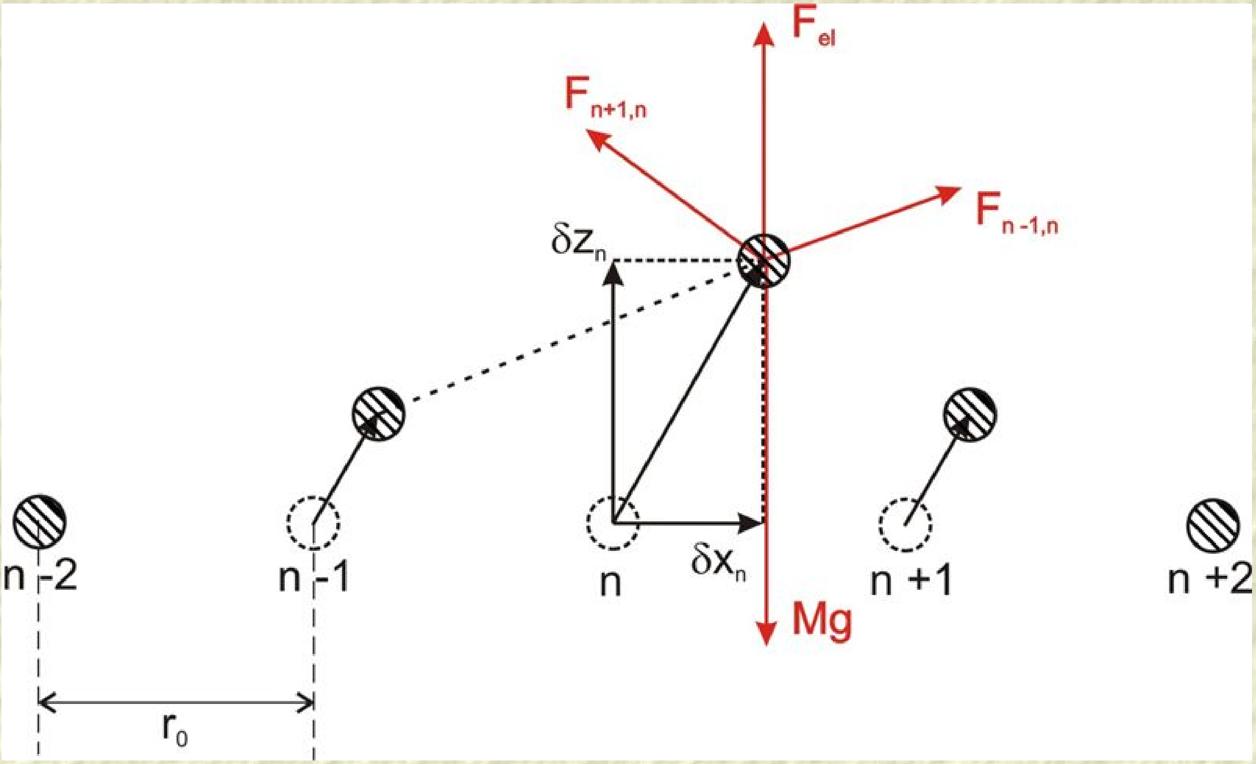
$\Delta x_{nm} = x_n - x_{n-m} \approx m r_0, \qquad \Delta z_{nm} = z_n - z_{n-m} \approx 0$,

one obtains:

$$
\begin{aligned}
U_{nm}(r) \;\approx\; & \frac{1}{2} M \omega_{L,0}^2 (\Delta x_{nm})^2 + \frac{1}{2} M \omega_{T,0}^2 (\Delta z_{nm})^2 \\
& + \frac{1}{3} u_{30} (\Delta x_{nm})^3 + \frac{1}{4} u_{40} (\Delta x_{nm})^4 + \ldots + \frac{1}{4} u_{04} (\Delta z_{nm})^4 + \\
& + \frac{1}{2} u_{12} (\Delta x_{nm})(\Delta z_{nm})^2 + \frac{1}{4} u_{22} (\Delta x_{nm})^2 (\Delta z_{nm})^2 + \ldots
\end{aligned}
$$

Nonlinearity: Where from? *(continued ...)*

(iii) Mode *coupling* also induces non linearity:
anisotropic motion, *not* confined along one of the main axes ($\sim \hat{x}$, $\hat{z}$).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

Discrete coupled equations of motion

$$\frac{d^2(\delta x_n)}{dt^2} + \nu \frac{d(\delta x_n)}{dt} = \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$- a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right]$$
$$+ a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right] + a_{02} \left[(\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2 \right]$$
$$- a_{12} \left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2 \right],$$

$$\frac{d^2(\delta z_n)}{dt^2} + \nu \frac{d(\delta z_n)}{dt} = \omega_{0,T}^2 (2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2 \, \delta z_n$$
$$- K_1 (\delta z_n)^2 - K_2 (\delta z_n)^3 + \frac{a_{02}}{r_0} \left[(\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3 \right]$$
$$+ 2\, a_{02} \left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1}) \right]$$
$$- a_{12} \left[(\delta x_{n+1} - \delta x_n)^2 (\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2 (\delta z_n - \delta z_{n-1}) \right].$$

Continuum coupled equations of motion

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} =$$
$$-\,2\,a_{20}\,r_0^3\,u_x\,u_{xx} + 3\,a_{30}\,r_0^4\,(u_x)^2\,u_{xx}$$
$$-\,a_{12}\,r_0^4\,[(w_x)^2\,u_{xx} + 2w_x w_{xx} u_x] + 2\,a_{02}\,r_0^3\,w_x\,w_{xx}\,,$$

$$\ddot{w} + \nu\,\dot{w} + c_T^2\,w_{xx} + \frac{c_T^2}{12}\,r_0^2\,w_{xxxx} + \omega_g^2\,w =$$
$$-\,K_1\,w^2 - K_2\,w^3 + 3\,a_{02}\,r_0^3\,(w_x)^2\,w_{xx}$$
$$+\,2\,a_{02}\,r_0^3\,(u_x\,w_{xx} + w_x\,u_{xx}) - a_{12}\,r_0^4\,[(u_x)^2\,w_{xx} + 2u_x u_{xx} w_x]\,,$$

Part 2. *Transverse* excitations

The vertical $n-$th grain displacement $\delta z_n = z_n - z_{(0)}$ obeys

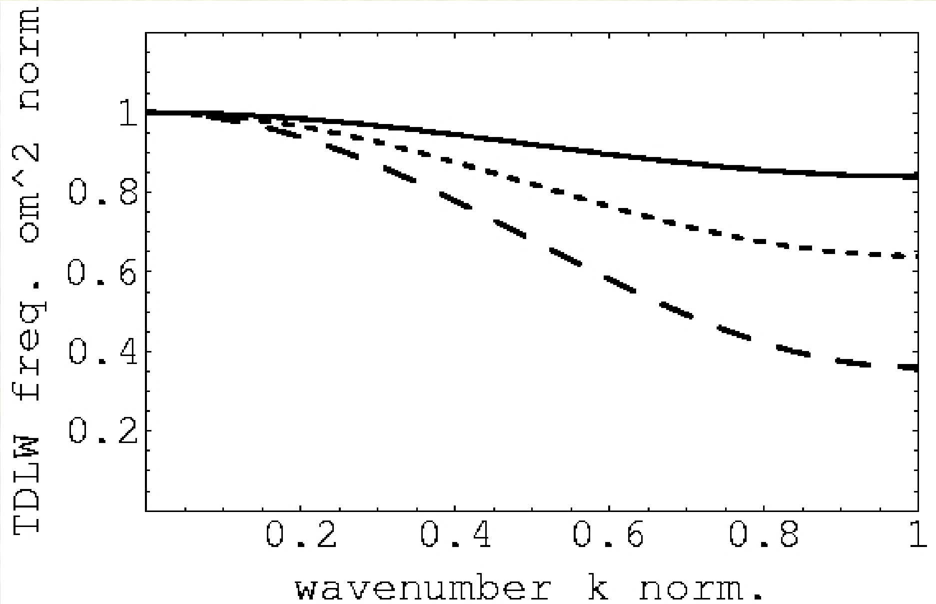
$$\frac{d^2(\delta z_n)}{dt^2} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n \right) + \omega_g^2\, \delta z_n = 0$$

* $\omega_{T,0} = \left[-qU'(r_0)/(M r_0) \right]^{1/2} = \omega_{DL}^2 \exp(-\kappa)\,(1+\kappa)/\kappa^3 \quad (\dagger)$

* $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$; λ_D is the *Debye length*;

* *Optical* dispersion relation (*backward* wave, $v_g < 0$):

$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(k r_0/2)$$



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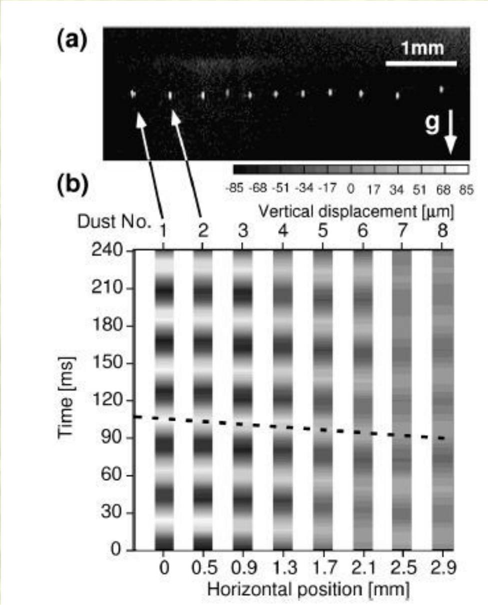
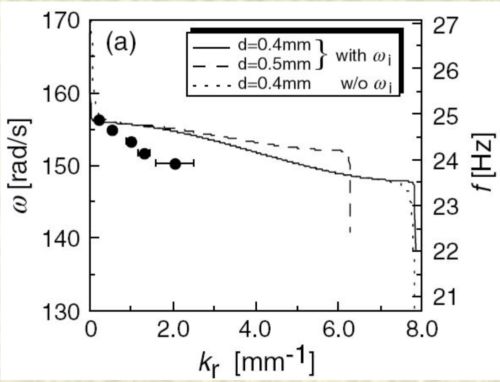
* *Optical* dispersion relation (*backward* wave, $v_g < 0$) †:

$$\omega^2 \, = \omega_g^2 - 4\omega_{T,0}^2 \, \sin^2(kr_0/2)$$

† Cf. experiment:
T. Misawa *et al.*, *PRL* **86**, 1219 (2001)
(Nagoya, Japan).





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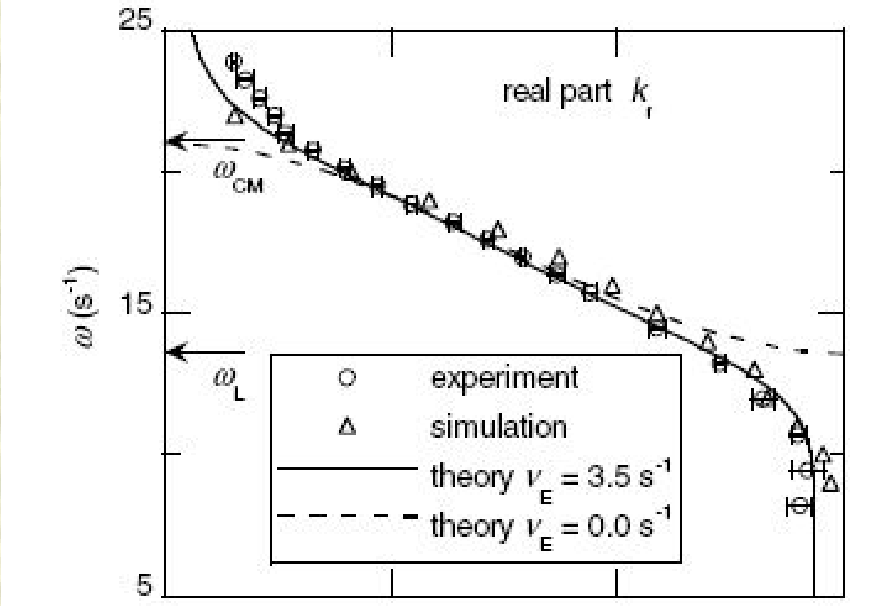
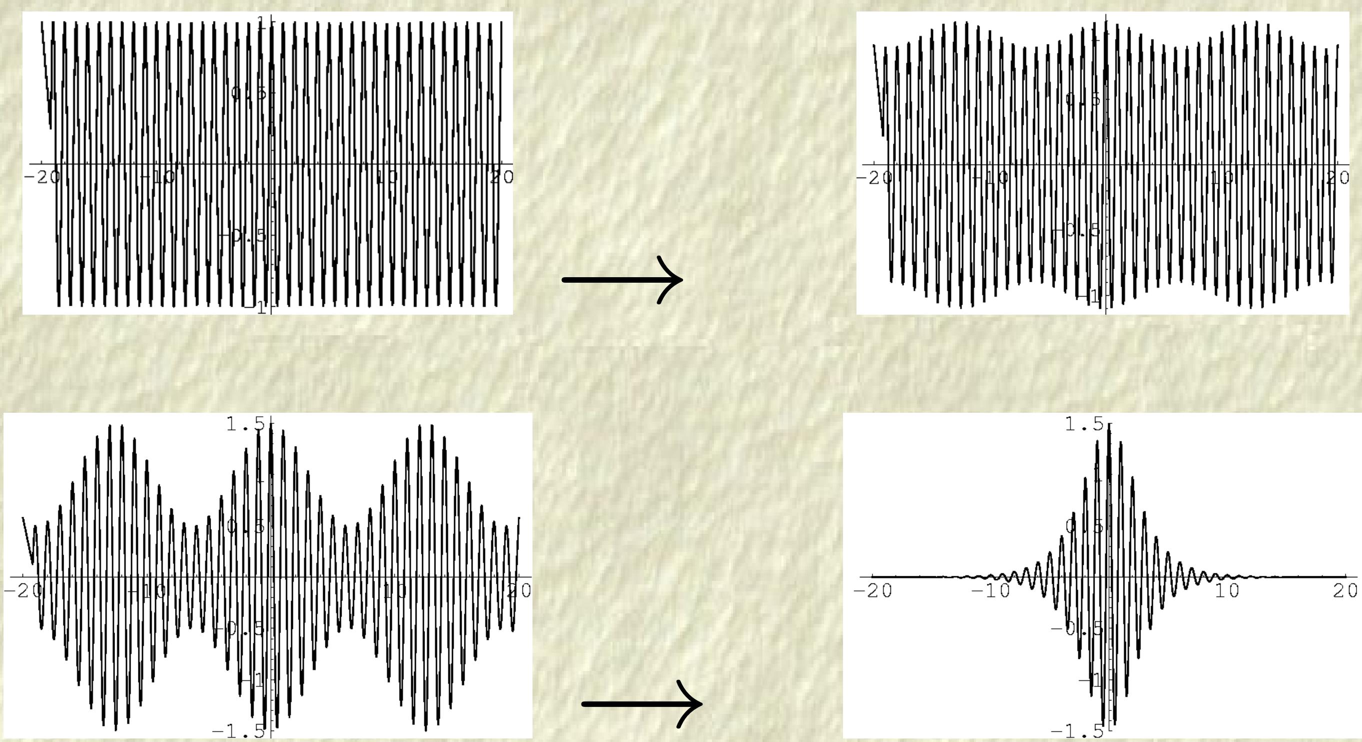


† Cf. experiment:
B. Liu *et al.*, *PRL* **91**, 255003 (2003)
(Iowa, USA).

What if *nonlinearity* is taken into account?

$$\frac{d^2\delta z_n}{dt^2} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n \right) + \omega_g^2\, \delta z_n$$
$$+\alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,.$$

* *Intermezzo:* The mechanism of *wave amplitude modulation*:



Large amplitude oscillations - envelope structures

A reductive perturbation (multiple scale) technique, viz.

$t \to \{t_0, t_1 = \epsilon t, t_2 = \epsilon^2 t, \ldots\}, \ x \to \{x_0, x_1 = \epsilon x, x_2 = \epsilon^2 x, \ldots\}$

yields ($\epsilon \ll 1$; damping omitted):

$$\delta z_n \approx \epsilon\,(A\,e^{i\phi_n} + \text{c.c.}) + \epsilon^2\,\alpha\left[-\frac{2|A|^2}{\omega_g^2} + \left(\frac{A^2}{3\omega_g^2}\,e^{2i\phi_n} + \text{c.c.}\right)\right] + \ldots$$

($\phi_n = nkr_0 - \omega t$); the harmonic amplitude $A(X,T)$:

– depends on the *slow* variables $\{X,T\} = \{\epsilon(x - v_g t), \epsilon^2 t\}$;

– obeys the *nonlinear Schrödinger equation* (NLSE):

$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2\,A = 0\,, \qquad (1)$$

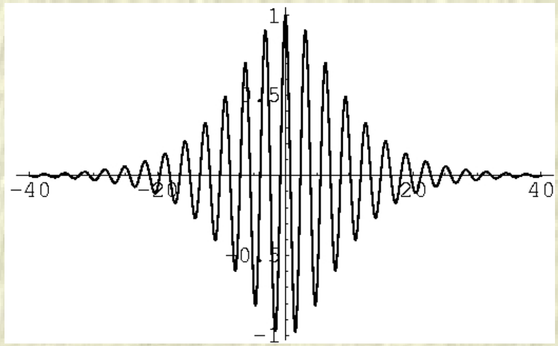
– *Dispersion coefficient*: $P = \omega''(k)/2$ $\to$ see dispersion relation;

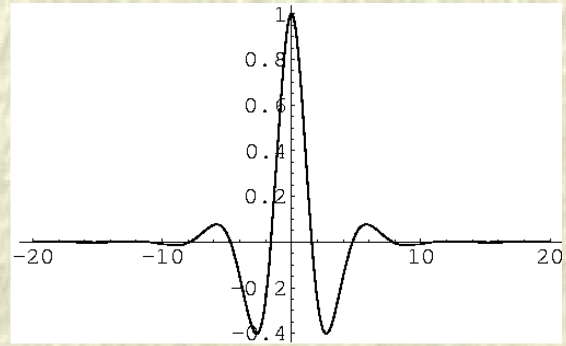
– *Nonlinearity coefficient*: $Q = \left[10\alpha^2/(3\omega_g^2) - 3\,\beta\right]/2\omega$.

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); **11**, 3665 (2004).]

Modulational stability analysis & envelope structures

- $PQ > 0$: *Modulational instability* of the carrier, *bright solitons*:





→ *TDLW*s: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

Rem.: $Q > 0$ for *all* known experimental values of α, β.

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)]

Source: G. Sorasio *et al.* (2002).

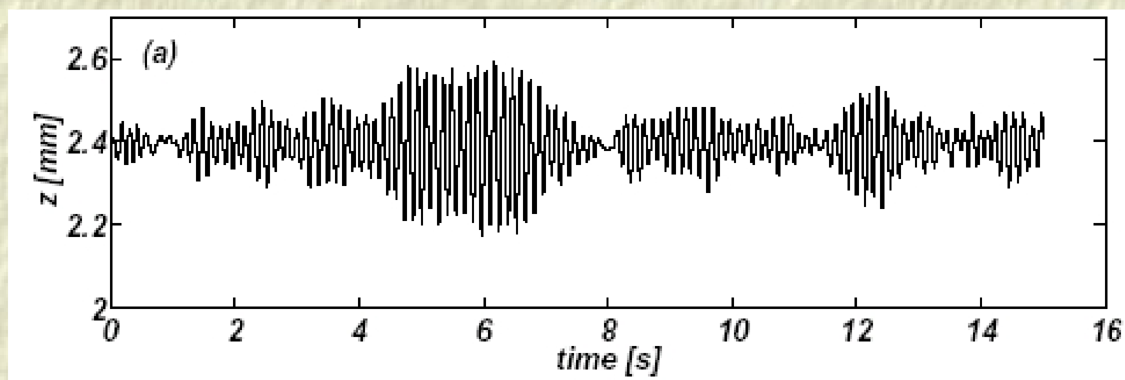
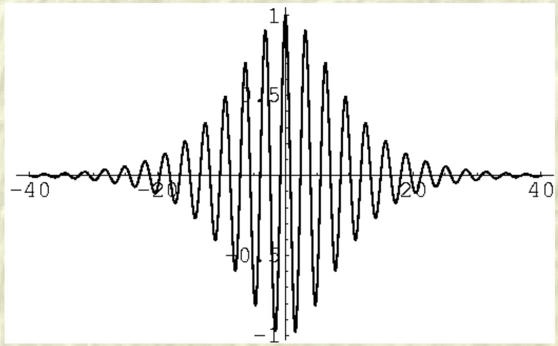
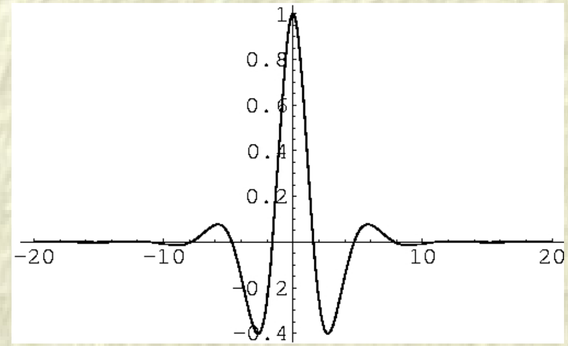


Figure 9: Dust grain oscillations induced by a 1% fluctuation in plasma density. The simulation parameters are: $P = 0.9\ mtorr$, $n_0 = 0.8 \times 10^8\ cm^{-3}$, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6\ V$, $\varsigma_t = 0.06$, $\varsigma_p = 1\% n_0$

Modulational stability analysis & envelope structures

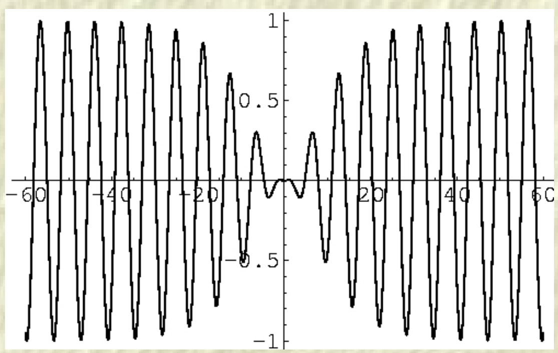
- $PQ > 0$: *Modulational instability* of the carrier, *bright solitons*:

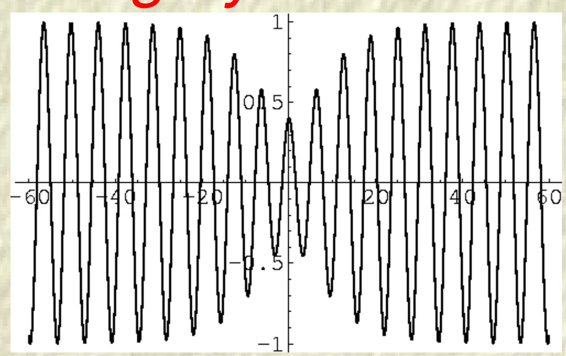




$\rightarrow$ *TDLW*s: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

- $PQ < 0$: Carrier wave is *stable*, *dark/grey solitons*:





$\rightarrow$ *TDLW*s: possible for *long* wavelengths i.e. $k < k_{cr}$.

Rem.: $Q > 0$ for *all* known experimental values of α, β

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)].

Part 3a. (*nonlinear*) *longitudinal* excitations.

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 \, (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$-a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right]$$
$$+ \, a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]$$

– $\delta x_n = x_n - nr_0$: longitudinal dust grain displacements

– Cf. *Fermi-Pasta-Ulam (FPU) problem*:
anharmonic spring chain model.

Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$$\delta x_n \approx \epsilon \left[u_0^{(1)} + \, (u_1^{(1)} \, e^{i\phi_n} + \text{c.c.})\right] + \epsilon^2 \, (u_2^{(2)} \, e^{2i\phi_n} + \text{c.c.}) \, + \ldots \, ,$$

[Harmonic generation; Cf. experiments: K. Avinash PoP 2004].

where the amplitudes now obey the coupled equations:

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L \frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0 \, |u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L} u_1^{(1)} \frac{\partial u_0^{(1)}}{\partial X} = 0 \, ,$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - c_L^2} \frac{\partial}{\partial X} |u_1^{(1)}|^2 \equiv R(k) \frac{\partial}{\partial X} |u_1^{(1)}|^2$$

– $Q_0 = -\frac{k^2}{2\omega} \left(q_0 \, k^2 + \frac{2p_0^2}{c_L^2 r_0^2} \right) ; \; P_L = \omega_L''(k)/2 \, ;$

– $v_{g,L} = \omega_L{}'(k)$; $\{X, T\}$ are *slow* variables;

– $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$, $q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$.

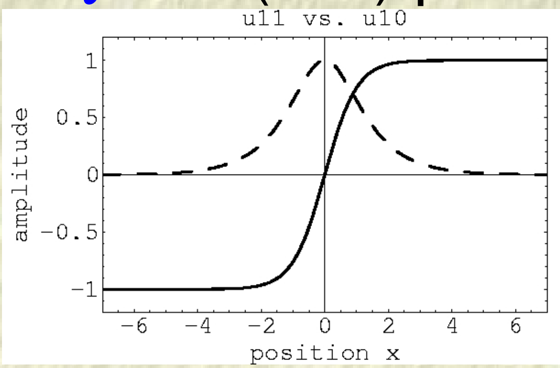
– $R(k) > 0$, since $\forall \, k$ $\quad v_{g,L} < \omega_{L,0} \, r_0 \equiv c_L$ *(sound velocity)*.

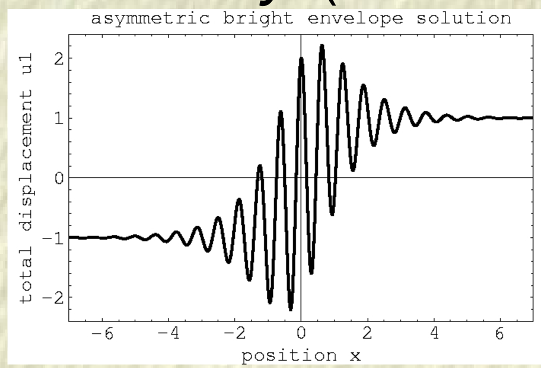
Asymmetric *longitudinal* envelope structures.

– The system of Eqs. for $u_1^{(1)}$, $u_0^{(1)}$ may be solved exactly:
$\rightarrow$ *asymmetric* envelope solutions.

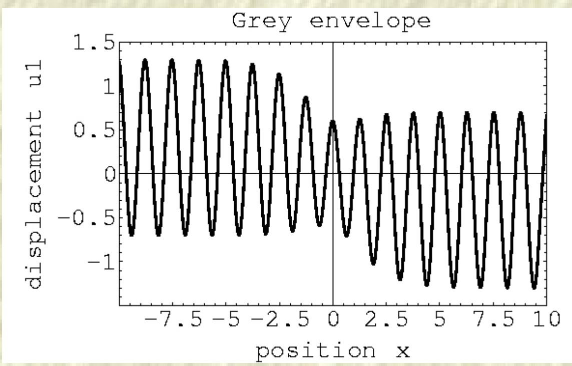
– $P = P_L = \omega_L''(k)/2 < 0$;

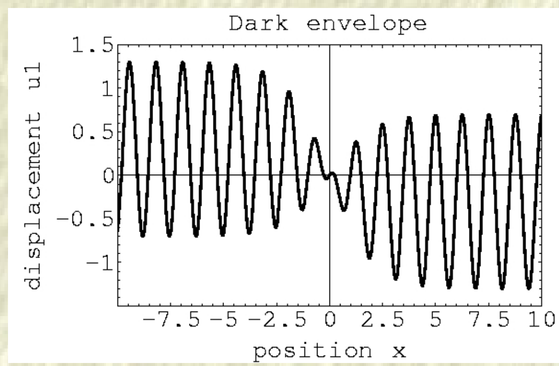
– $Q > 0$ (< 0) prescribes *stability* (instability) at *low* (high) k.





(at high k)





(at low k)

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).].

Part 3b. *Longitudinal* soliton formalism.

A link to soliton theories:

– *Continuum approximation*, viz. $\delta x_n(t) \to u(x,t)$.

– *"Standard"* description: keeping *lowest order* nonlinearity,

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} \;=\; -\,p_0\,u_x\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$ and p_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), slow profile evolution in time τ and defining the *relative displacement* $w = u_\zeta$, one obtains (for $\nu = 0$) the Korteweg-deVries Equation:

$$w_\tau \;-\; a\,w\,w_\zeta + b\,w_{\zeta\zeta\zeta} \;=\; 0$$

Defs.: $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$.

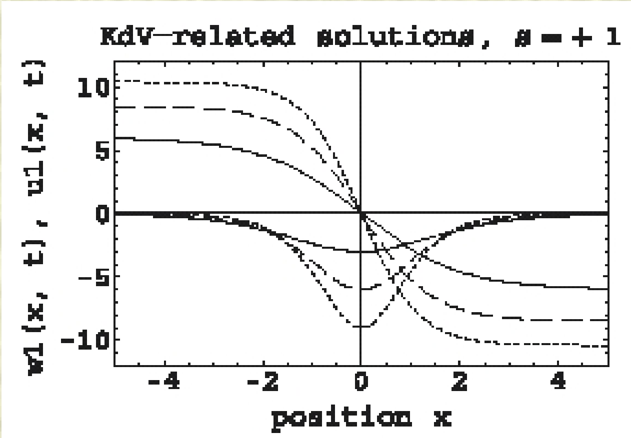
The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, since $a > 0$) solutions, in the form:

$$w_1(\zeta, \tau) = -\frac{3v}{a}\mathrm{sech}^2\Big[(v/4b)^{1/2}(\zeta - v\tau - \zeta_0)\Big]$$

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$ †.



† F. Melandsø 1996; S. Zhdanov *et al.* 2002; K. Avinash *et al.* 2003; V. Fortov *et al.* 2004.

Experimental observation of LDL solitons

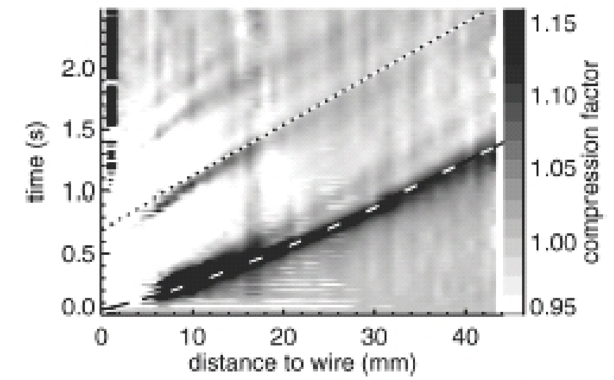


FIG. 2. Compression factor n/n_0 as a function of time and distance to the wire. Darker regions correspond to higher compression. The lower dashed curve is a fit to the trajectory of the soliton. The upper dotted line was drawn above a weak secondary pulse with Mach number $M \simeq 1$. Its slope is determined by the dust lattice wave speed $C_{DL} = 23$ mm/s in the middle of the lattice.

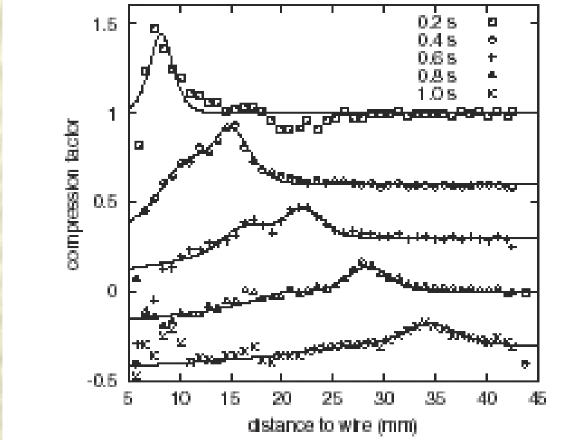


FIG. 3. Compression factor n/n_0 versus distance to the wire at different times. The solid lines show the theoretical fits to the experimental data. Two solitons are present. The fits and experimental points at later times are offset down (by 0.4, 0.7, 1.0, 1.3, respectively).

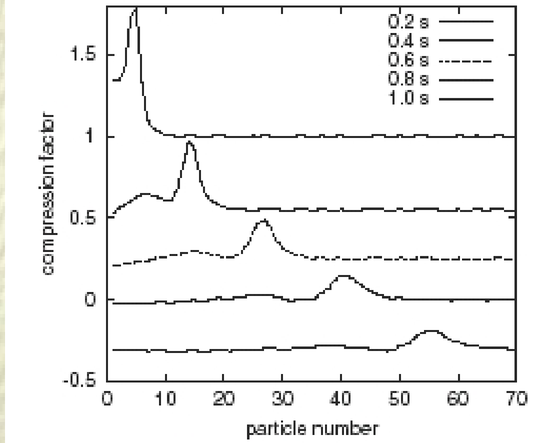


FIG. 5. Compression factor versus particle number for a simulated linear chain model. It describes the formation of two (or more) solitons by a single excitation pulse and qualitatively agrees with the experiment. The curves at later times are offset down (by 0.45, 0.75, 1.0, and 1.3, respectively).

[Samsonov *et al.*, PRL 2002].

- *Only compressive* solitons predicted by KdV theory;
- *Only compressive* solitons experimentally anticipated and, hence, reported;
- What about *rarefactive* longitudinal solitons?

Extended description: the Boussinesq theory

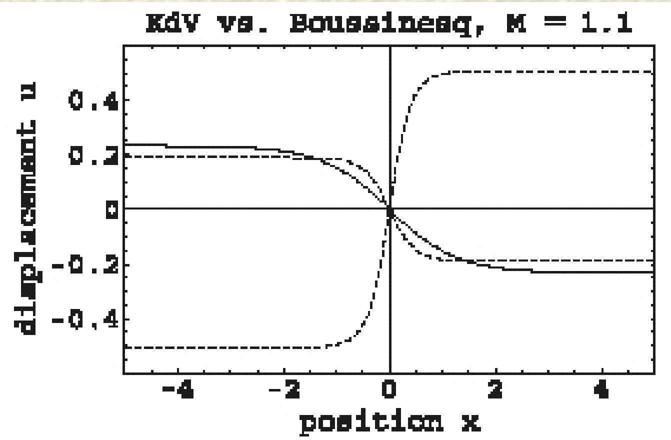
The *Generalized* *Boussinesq* (Bq) Equation (for $w = u_x$):

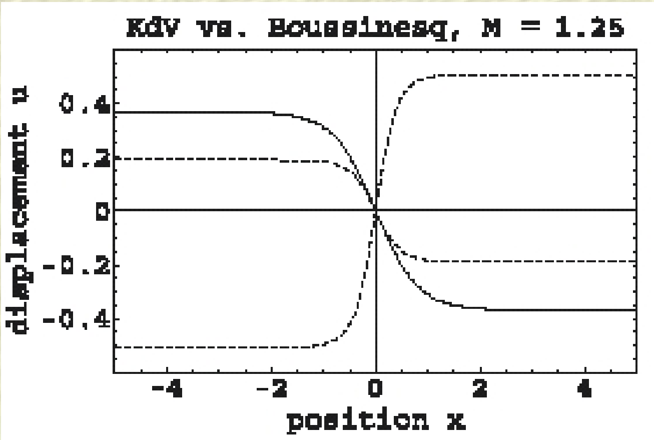
$$\ddot{w} - c_L^2 w_{xx} = \frac{c_L^2 r_0^2}{12} w_{xxxx} - \frac{p_0}{2} (w^2)_{xx} + \frac{q_0}{2} (w^3)_{xx}$$

– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...); *and, ...*

– relaxes the velocity assumption, i.e. is valid $\forall \quad v > c_L$.





[from: I Kourakis & P K Shukla, *European Phys. J. D*, **29**, 247 (2004)].

Part 4. Transverse Discrete Breathers (DBs)

- Recall the eq. of motion in the *transverse* direction:

$$\frac{d^2 u_n}{dt^2} + \omega_{T,0}^2 \,(u_{n+1} + u_{n-1} - 2\, u_n) + \omega_g^2\, u_n + \alpha\, u_n^2 + \beta\, u_n^3 = 0$$

- 1d DPCs are (intrinsically) *highly discrete* lattice config'tions: $\epsilon = \omega_0^2/\omega_g^2 \simeq 0.016$ ([Misawa et al., PRL 2001]); $\epsilon \simeq 0.181$ ([Liu et al., PRL 2003]).

- Damping may be neglected (for low plasma density and/or pressure): $\nu/\omega_g \simeq 0.00154$ ([Misawa et al., PRL 2001]).

- One may seek *discrete breather* solutions (localized modes):

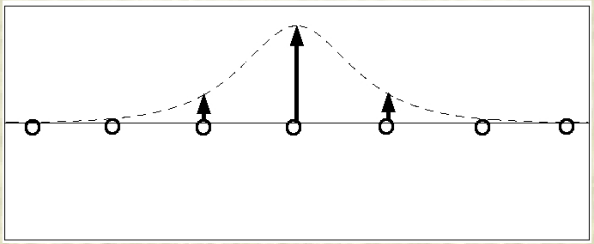
$$u_n(t) = \sum_m A_n(m) \exp(im\omega t)$$

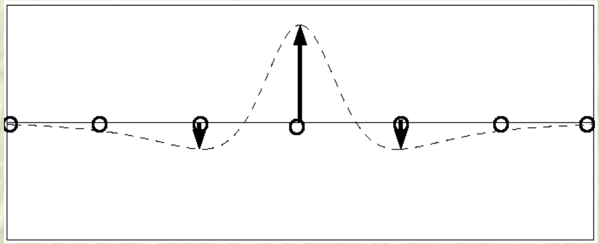
Part 4. Transverse Discrete Breathers (DBs) *(cont.)*

* The well-known *non - resonance condition* is recovered:

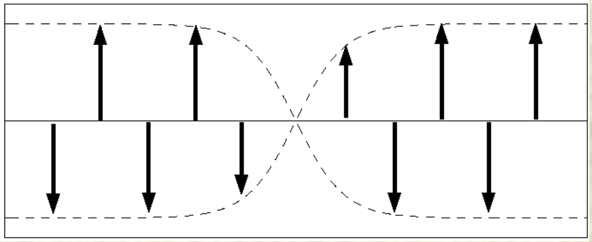
$$m\omega_B \neq \omega_T(k) \qquad \forall k,\, m = 1, 2, ...$$

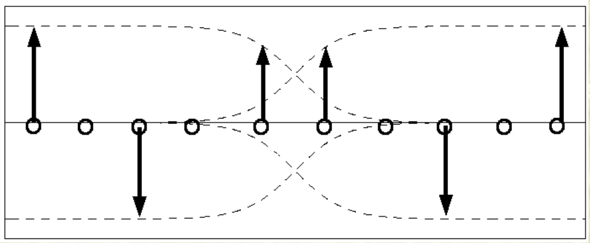
* "*Bright-type*" few-site DB solutions (localized pulses):





* +: *dark-type* DBs (holes; *dark* modes [Yu. Kivshar, *PLA* **173**, 172 (1993)]):





* Existence & stability profile established: cf. poster no. 20

[V Koukouloyannis & I Kourakis, subm. PRE; `arxiv.org/pdf/nlin.PS/0703020`].

* DB modes may also occur in the $x-$ direction ($\sim$ FPU).

Conclusions - State of Art

– *Dust crystals* provide an excellent test-field for NL theories;

– *Observations are possible at the kinetic level*: unique possibility for physical data processing & real-time analysis;

– *Technology for experiment*: *cheap* and *readily available*;

– Link between *Plasma Phys., Solid State Physics, Stat. Mech.*;

– *Theory (1d): Envelope solitons, (non-topological) solitons, Discrete Breathers*: predicted;

– *Theory (2d): Discrete Breathers, vortices, ...*: predicted;

– *Experiment:* Harmonic generation, compressive solitons, NL TDL oscillations, backward wave: observed (*Urge for more :-)*);

– Future scope: *dissipation*, *mode coupling*, ... *"(Realism!)"*.

Thank You !

Ioannis Kourakis

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Material from:

I Kourakis & P K Shukla, *Phys. Plasmas*, **11**, 2322 (2004);

idem, *Phys. Plasmas*, **11**, 3665 (2004).

idem, *Phys. Plasmas*, **11**, 1384 (2004).

idem, *European Phys. J. D*, **29**, 247 (2004).

V Koukouloyannis & IK, subm. PRE; ePrint `nlin.PS/0703020`

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