Nonlinear Field-Line-Random-Walk and Generalized Compound Diffusion of Charged Particles

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1 Nonlinear field-line wandering

An improved nonlinear formulation for field-line random walk in magnetostatic turbulence has been developed in (Shalchi & Kourakis, 2007). This approach is a direct generalization of the diffusion theory proposed earlier by Matthaeus et al. (1995). For a diffusive behaviour of field-lines, the theories coincide. However, the new theory can also be applied in non-diffusive transport cases.

For the purpose of modelling field-line random walk, the turbulence model adopted has to be specified in terms of the magnetic correlation tensor $P_{ij}(\vec{k}) = \left\langle \delta B_i(\vec{k}) \delta B_j^*(\vec{k}) \right\rangle$. According to Bieber *et al.* (1994), the so-called slab/2D composite model is a realistic model for solar wind turbulence. Within the two-component model the correlation tensor has the form: $P_{xx}(\vec{k}) = P_{xx}^{slab}(\vec{k}) + P_{xx}^{2D}(\vec{k})$ with $P_{xx}^{slab}(\vec{k}) = g^{slab}(k_\parallel)\delta(k_\perp)/k_\perp$ and $P_{xx}^{2D}(\vec{k}) = g^{2D}(k_\perp)\delta(k_\parallel)k_y^2/k_\perp^3$. For the two wave spectra $g^{slab}(k_\parallel)$ and $g^{2D}(k_\perp)$, we employ the standard form (see Bieber et al. 1994)

$$g^{slab}(k_\parallel) = \frac{C(\nu)}{2\pi} l_{slab} \delta B_{slab}^2 \left(1 + k_\parallel^2 l_{slab}^2\right)^{-\nu}$$
$$g^{2D}(k_\perp) = \frac{2C(\nu)}{\pi} l_{2D} \delta B_{2D}^2 \left(1 + k_\perp^2 l_{2D}^2\right)^{-\nu} \quad (1)$$

where we have defined the constant $C(\nu) = \Gamma(\nu)/(2\sqrt{\pi}\Gamma(\nu - 1/2))$, the slab- and 2D bendover scales l_{slab} and l_{2D}, the strength of the turbulent fields δB_{slab} and δB_{2D}, and the inertial-range spectral index 2ν.

It can easily be demonstrated that, for pure slab geometry, magnetic field-line wandering behaves diffusively:

$$\left\langle (\Delta x(z))^2 \right\rangle_{|z| \to \infty} \approx 2\kappa_{FL} \mid z \mid , \quad (2)$$

for large $|z|$. A number of previous papers (e. g. Matthaeus et al. 1995) have relied on the *ad hoc* assumption that Eq. (2) is also valid in two-component turbulence. However, a rigorous formulation of field-line random walk leads to

$$\left\langle (\Delta x(z))^2 \right\rangle_{|z| \to \infty} = \left(9\sqrt{\frac{\pi}{2}} C(\nu)\right)^{2/3} \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} l_{2D}^2 \left(\frac{\mid z \mid}{l_{2D}}\right)^{4/3} \quad (3)$$

(Shalchi & Kourakis 2007). The only assumptions which have been applied to derive this result are Corrsin's independence hypothesis (Corrsin 1959) and the assumption of a Gaussian distribution of field-lines.

2 Generalized compound-diffusion of charged particles

By assuming that the particles (or, more precisely, their guiding-centers) follow the magnetic field-lines (guiding center approximation) we can formulate a relation between the perpendicular MSD of the charged particle ($\left\langle (\Delta x(t))^2 \right\rangle_P$) and the MSD of the field-lines ($\left\langle (\Delta x(z))^2 \right\rangle_{FL}$)

$$\left\langle (\Delta x(t))^2 \right\rangle_P = \int_{-\infty}^{+\infty} dz \, \left\langle (\Delta x(z))^2 \right\rangle_{FL} f_P(z,t) \, . \quad (4)$$

Here $f_P(z,t)$ denotes the particle distribution in the direction parallel to the background field.

A standard assumption in cosmic ray transport theory is the assumption of a Gaussian particle distribution

$$f_P(z,t) = \left(2\pi \left\langle (\Delta z(t))^2 \right\rangle_P\right)^{-1/2} e^{-\frac{z^2}{2\langle(\Delta z(t))^2\rangle_P}} \, . \quad (5)$$

Using Eq. (3) for the field-line MSD in combination with Eq. (5) for the particle distribution, we can evaluate Eq. (4) as

$$\left\langle (\Delta x)^2 \right\rangle_P = \alpha(\nu) \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} \left[l_{2D} \left\langle (\Delta z(t))^2 \right\rangle_P\right]^{2/3} . \quad (6)$$

with

$$\alpha(\nu) = \frac{\Gamma(7/6)}{\sqrt{\pi}} \left(18\sqrt{\frac{\pi}{2}} C(\nu)\right)^{2/3} . \quad (7)$$

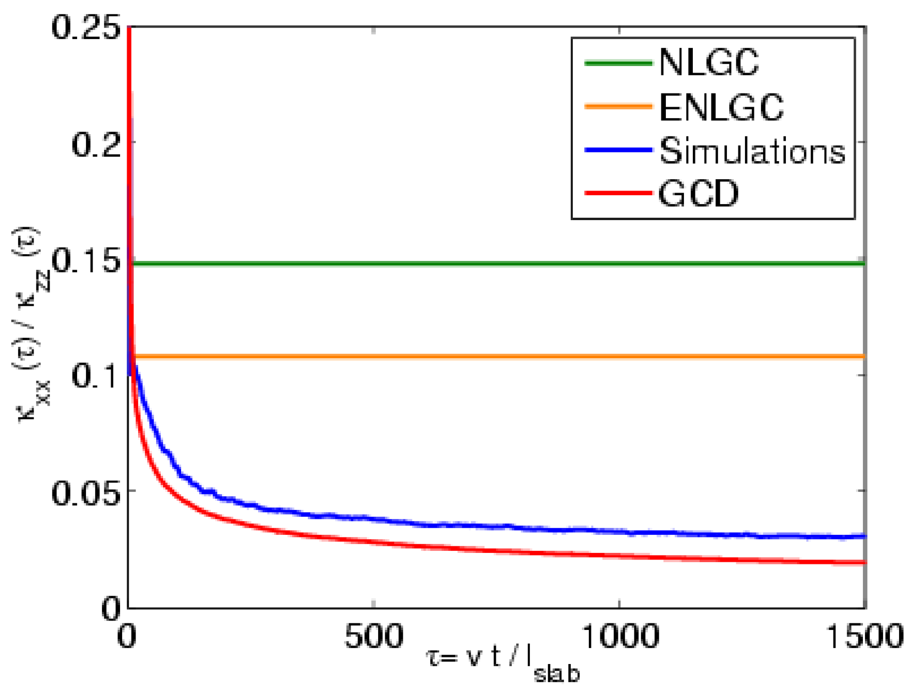
A (time-dependent) diffusion coefficient, as obtained from test-particle simulations, can be defined as $\kappa_{xx}(t) = \left\langle (\Delta x)^2 \right\rangle / (2t)$. In general, one may adopt the assumption $< (\Delta z(t))^2 >_P \sim t^{b_\parallel + 1}$, implying a parallel diffusion coefficient $\kappa_{zz} \sim t^{b_\parallel}$. Assuming $\kappa_{xx} \sim t^{b_\perp}$, it is straightforward to find from Eq. (6) the relation

$$b_\perp = \frac{2b_\parallel - 1}{3} . \quad (8)$$

Therefore, knowledge of $b_\parallel$ (e.g., from simulation data) leads to an evaluation of $b_\perp$, within this model. For instance if parallel transport behaves diffusively ($b_\parallel = 0$) we find $b_\perp = -1/3$ (*subdiffusion*). A diffusive behaviour of perpendicular transport ($b_\perp = 0$) can only be obtained for $b_\parallel = 1/2$ (superdiffusion).

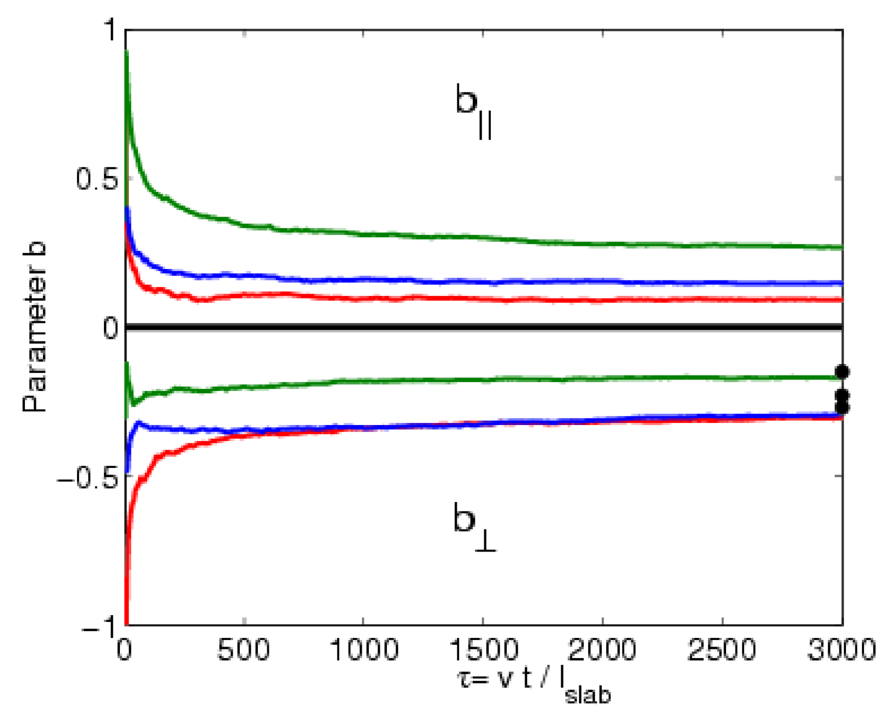
3 Test particle simulations

We have investigated test-particle dynamics for the following set of parameters: $l_{2D} = 0.1\, l_{slab}$, $\nu = 5/6$, and 20%/80% slab/2D composite geometry. In Fig. 1, we have depicted the ratio of perpendicular and parallel diffusion coefficients κ_{xx}/κ_{zz} as a function of the dimensionless time $\tau = vt/l_{slab}$ for dimensionless cosmic ray rigidity value $R = R_L/l_{slab} = 0.001$.



A small value of R has been chosen, to ensure that the guiding-center approximation is valid. The results of the Generalized Compound Diffusion (GCD) model are compared to those obtained from the NLGC- and ENLGC-theories (see Matthaeus et al. 2003, Shalchi 2006), and also to test-particle simulations.

By assuming the simple form $\tilde{\kappa}_{xx}(t) = a\tau^b$ we can deduce the time dependence from numerical data, by using $b = (\ln \tilde{\kappa}_{xx}(\tau) - \ln a)/\ln \tau \approx (\ln \tilde{\kappa}_{xx}(\tau))/\ln \tau$ in the large time limit ($\tilde{\kappa}_{xx}$ denotes the dimensionless diffusion coefficient obtained by the simulations).



The exponents for the parallel $b_\parallel$ and perpendicular $b_\perp$ diffusion coefficients are depicted in Fig. 2 for different values of the parameter R ($R = 0.001$ (red), $R = 0.01$ (blue), $R = 0.1$ (green)) in comparison to the theoretical result (dots).

4 Comparison with solar wind observations

It is not a trivial task to compare our new (non-diffusive) result with solar wind observations. First, we replace the parallel mean square deviation in Eq. (6) by a diffusive behavior ($< (\Delta z(t))^2 >_P \approx 2\kappa_\parallel t$) to get

$$\left\langle (\Delta x)^2 \right\rangle_P = \alpha(\nu) \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} \left(2 l_{2D} \kappa_\parallel t\right)^{2/3} . \quad (9)$$

One thus obtains for the (time-dependent) perpendicular diffusion coefficient

$$\kappa_\perp(t) = 2^{-1/3} \alpha(\nu) \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} \left(l_{2D}\kappa_\parallel\right)^{2/3} t^{-1/3} . \quad (10)$$

To proceed, we average over the scattering time $t_c = \lambda_\parallel / v$ to get

$$\overline{\kappa}_\perp = \frac{3}{2^{4/3}} \alpha(\nu) \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} \left(l_{2D}\kappa_\parallel\right)^{2/3} \left(\frac{v}{\lambda_\parallel}\right)^{1/3} . \quad (11)$$

By using $\lambda_i = 3\kappa_i / v$, we find an analytical expression for the perpendicular mean free path

$$\overline{\lambda}_\perp = \left(\frac{3}{2}\right)^{4/3} \alpha(\nu) \left(\frac{\delta B_{2D}}{B_0}\right)^{4/3} l_{2D}^{2/3} \lambda_\parallel^{1/3} . \quad (12)$$

For $\nu = 5/6$ and $\delta B_{2D}^2 / B_0^2 = 0.8$, as proposed by Bieber et al. (1994) we obtain

$$\overline{\lambda}_\perp = 0.75 \, l_{2D}^{2/3} \, \lambda_\parallel^{1/3} . \quad (13)$$

Palmer (1982) suggested that the parallel mean free path in the solar wind is $0.08AU \leq \lambda_{\parallel,Palmer} \leq 0.3AU$ and the perpendicular mean free path is $\lambda_{\perp,Palmer} \approx 0.007AU$. By taking the average value for the parallel mean free path $\lambda_{\parallel,Palmer} \approx 0.2$ and by applying Eq. (13) we find $\lambda_{\perp,GCD} \approx 0.009AU$ (for $l_{2D} = 0.1 l_{slab} \approx 0.003AU$) which is close to the measured value. Obviously, there is a very good agreement between solar wind observations and our theoretical approach.

The content of this poster will be published in:

Shalchi, A. & Kourakis, I., submitted to Physics of Plasmas (2007); e-print at: http://arxiv.org/pdf/astro-ph/0703366.

Shalchi, A. & Kourakis, I., Astronomy & Astrophysics, in press (2007).

The exact references cited in the text above can be found therein. *For further contact and/or reprint requests: ate@tp4.rub.de*.