Hamiltonian Lattice Dynamical Systems, October 15-19, 2007

Nonlinear Excitations in Dusty Plasma Crystals:

A new test-bed for nonlinear theories

(Preamble to a poster presentation)

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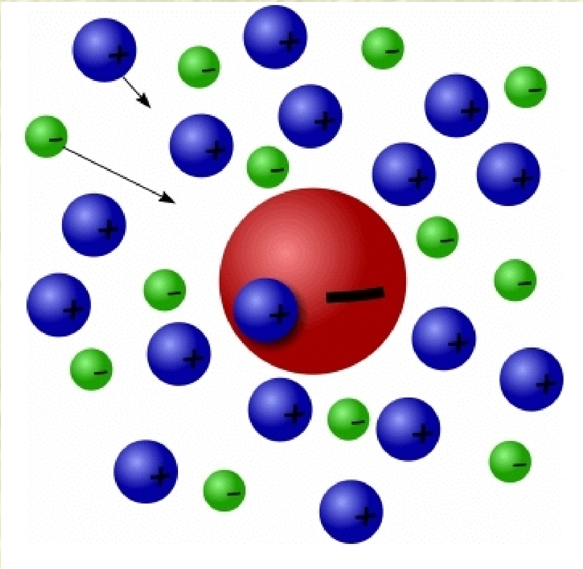
Dusty Plasmas (or *Complex Plasmas*): prerequisites

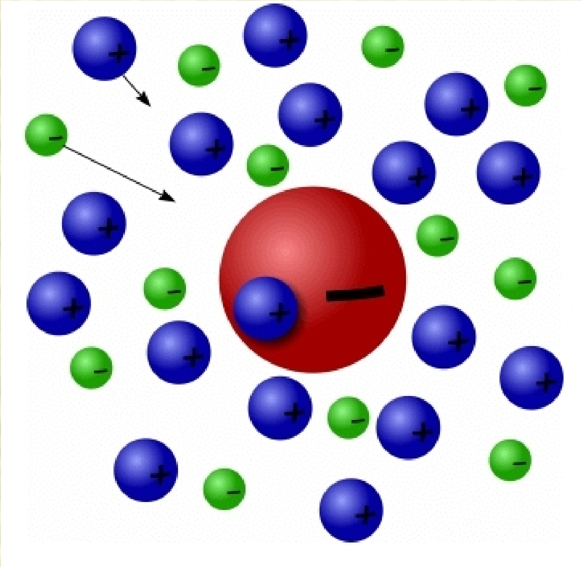


Plasma toy model:

→ *electrons* e^- (charge $-e$, mass m_e),

→ *ions* i^+ (charge $+Z_i e$, mass m_i),

Dusty Plasmas (or *Complex Plasmas*): prerequisites



Dusty Plasmas (DP):

→ *electrons* e^- (charge $-e$, mass m_e),

→ *ions* i^+ (charge $+Z_i e$, mass m_i),

→ charged particulates ≡ *dust grains* $d^\pm$ (most often d^-):

charge $Q = sZ_d e \sim \pm(10^3 - 10^4)\,e, \quad (s = \pm 1)$
mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,
radius $r \ \sim\ 10^{-2}\,\mu m$ up to $10^2\,\mu$m.

Dusty Plasma physics: unique mesoscopic features

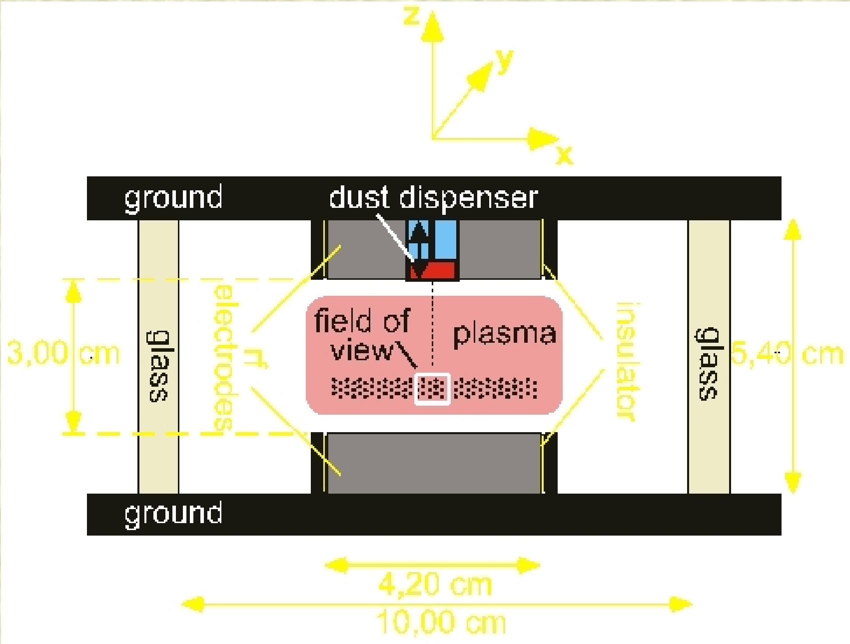
- Studies in *slow motion* are possible due to high M i.e. *low* Q/M *ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);
- Contrary to *weakly-coupled* $e - i$ plasmas ($\Gamma \ll 1$), Complex Plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [IKEZI 1986]) states, depending on the value of:

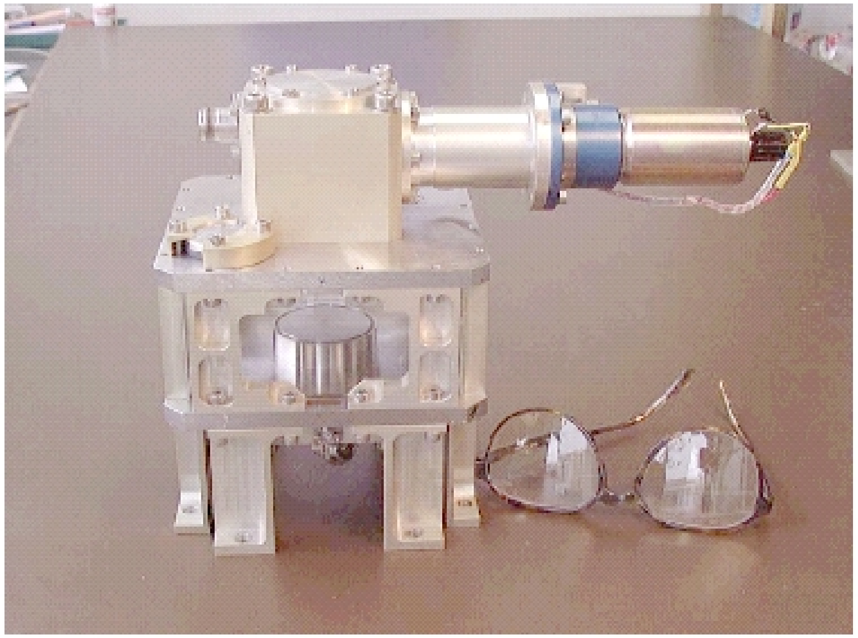
$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \approx \frac{\frac{Q^2}{r}\, e^{-r/\lambda_D}}{k_B T}$$

(r: inter-particle distance, T: temperature, λ_D: Debye length, k_B: Boltzmann's constant).

$\rightarrow$ *Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)*

Dust Crystal experiments (1):



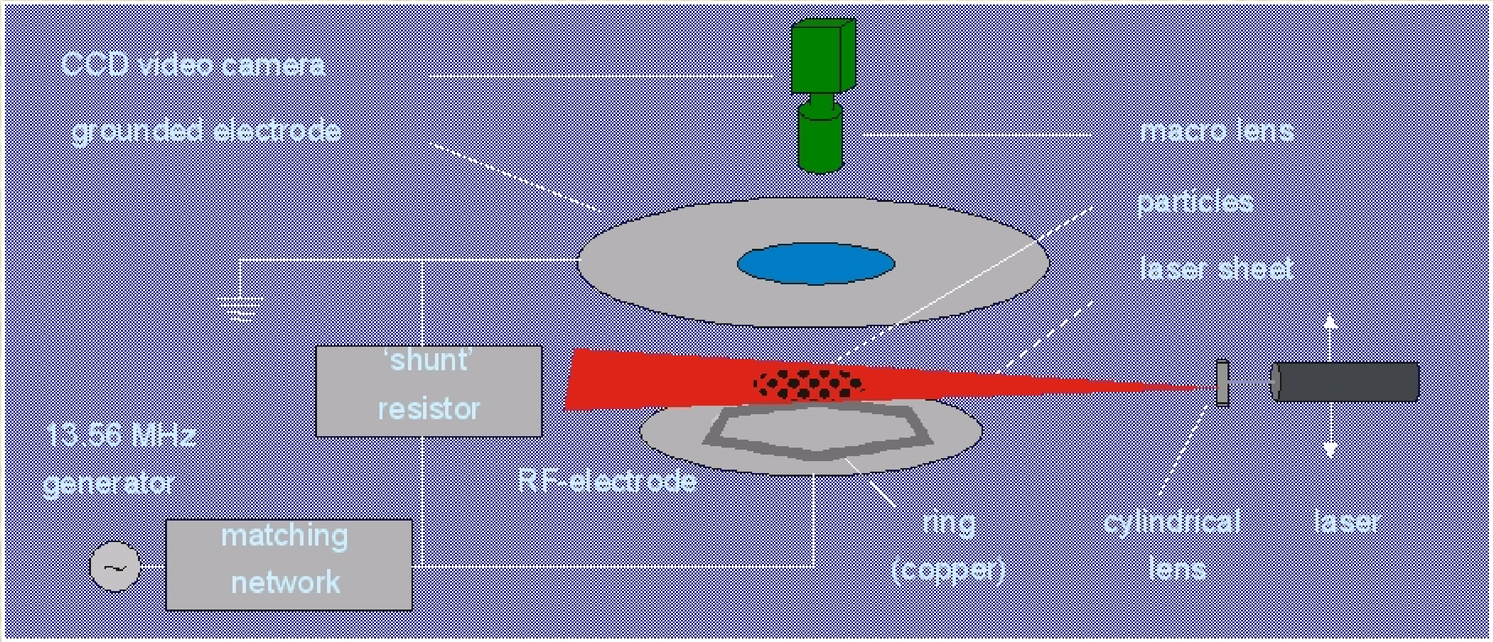


- Theoretical prediction: 1986 [H. Ikezi, Phys. Fluids **29**, 1764 (1986)];

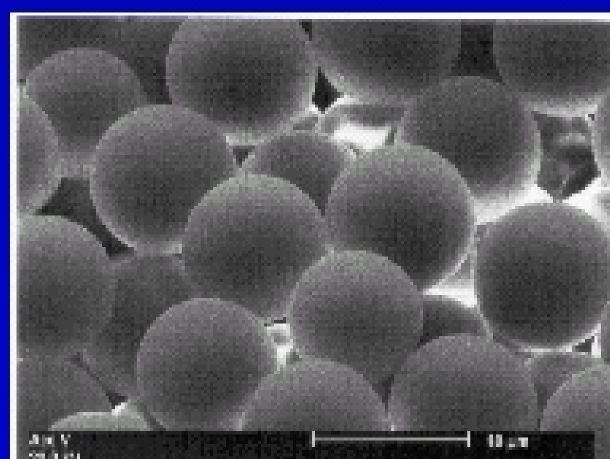
- Experimental realization: 1994

[H. Thomas, A. Melzer *et al.* PRL **73**, 652 (1994); Chu & Lin I J. Phys. D **27** 296 (1994), Hayashi & Tachibana, Jap. J. Appl. Phys. **33** L804 (1994)];

Dust Crystal experiments (2):



Particles:
Melamine-Formaldehyde
diameter: few μm



Gas:
noble gas (argon, krypton)
pressure: few Pa ... 100 Pa (=1 mbar)

Ionisation fraction: 10^{-6} - 10^{-7}

Temperatures:
kT_e ~2-4 eV (electrons)
kT_i ~0.03 eV (ions)
kT ~0.025 eV (microparticles)
in crystalline state

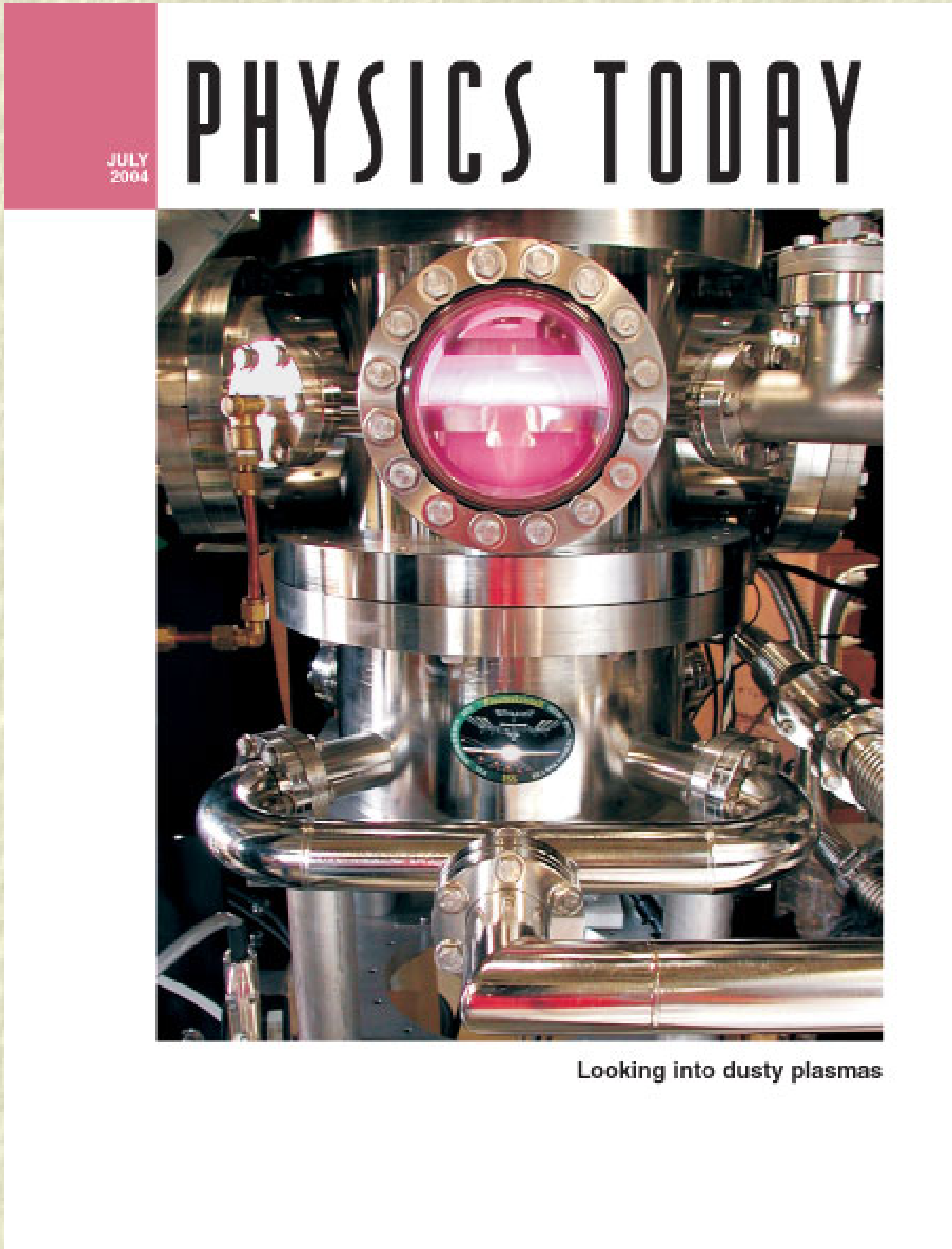
[Source: H. Thomas, A. Melzer *et al.*, PRL 1994].

- Today, various experimental groups active worldwide: *G E Morfill* (MPIeP Garching, Germany), *A Piel* (Kiel, Germany), *A Melzer* (Greifswald, Germany), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *S Vladimirov* (Sydney, AUS), *S Takamura* (Nagoya, Japan) ...

- Experiments aboard the *International Space Station (ISS)*;

- Mesoscopic analog of micro-structures; research focus:
  – phase transitions, crystallization processes,
  – relaxation times, diffusion effects,
  – phase space distribution (visually observable!),
  – L & NL waves: *harmonic generation, solitons, vortices*, ...

- 3D, 2D (hexagonal, mostly), 1D lattice configurations possible *(→ video)*.


JULY
2004
PHYSICS TODAY
Looking into dusty plasmas

1D DP crystal: Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

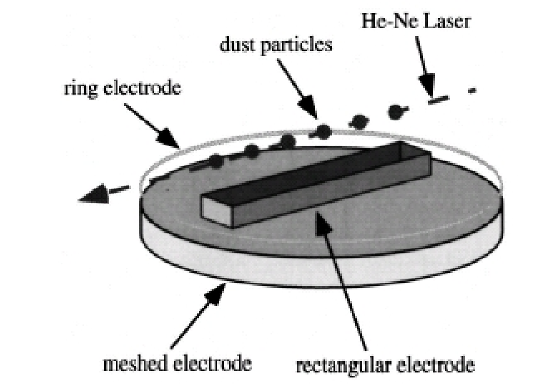


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

Terms include:

– *Kinetic energy*;

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *'external' force fields*:
may account for *confinement potentials*
and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.

– Coupling: $U_{int}(r_{nm})$ is the *interaction potential energy*;

Q.: Nonlinearity: *Origin: where from ?*
Effect: which consequence(s) ?

Nonlinearity in 1D DPCs: Where does it come from? (1)

→ *Sheath* environment (*anharmonic* vertical potential):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2} M \omega_g^2 (\delta z_n)^2 + \frac{1}{3} M \alpha \, (\delta z_n)^3 + \frac{1}{4} M \beta \, (\delta z_n)^4 + \ldots$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z_{(0)}$; α, β, ω_g are defined experimentally

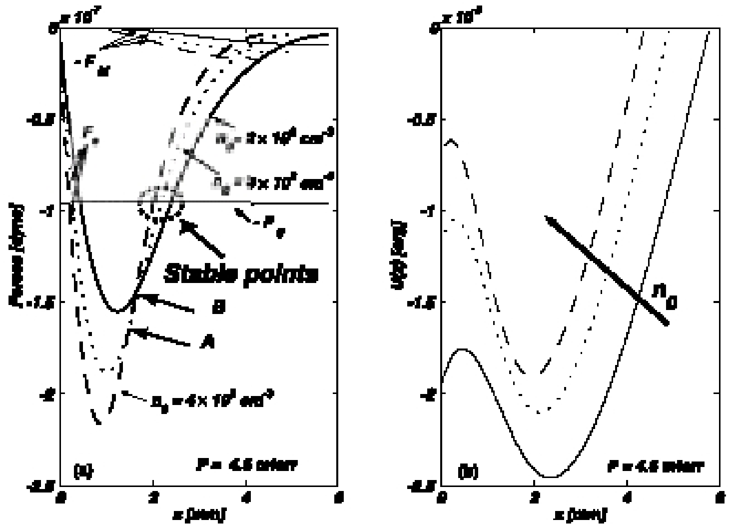


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2 \times 10^8 cm^{-3}$ (solid line), $n_0 = 3 \times 10^8 cm^{-3}$ (dashed line), $n_0 = 4 \times 10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1\ eV$, $T_i = T_n = 0.05\ eV$, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\, cm^{-3}$, $\phi_w = 6\ V$.

Source: Sorasio *et al.* (2002).

Nonlinearity: Where from? (2)

→ *Interactions* between grains: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened: $r_0/\lambda_D \approx 1$), *anharmonic*; typically: $U_{Debye}(r) = \frac{q^2}{r} \exp\left(-r/\lambda_D\right)$.

Expanding $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$ near equilibrium:

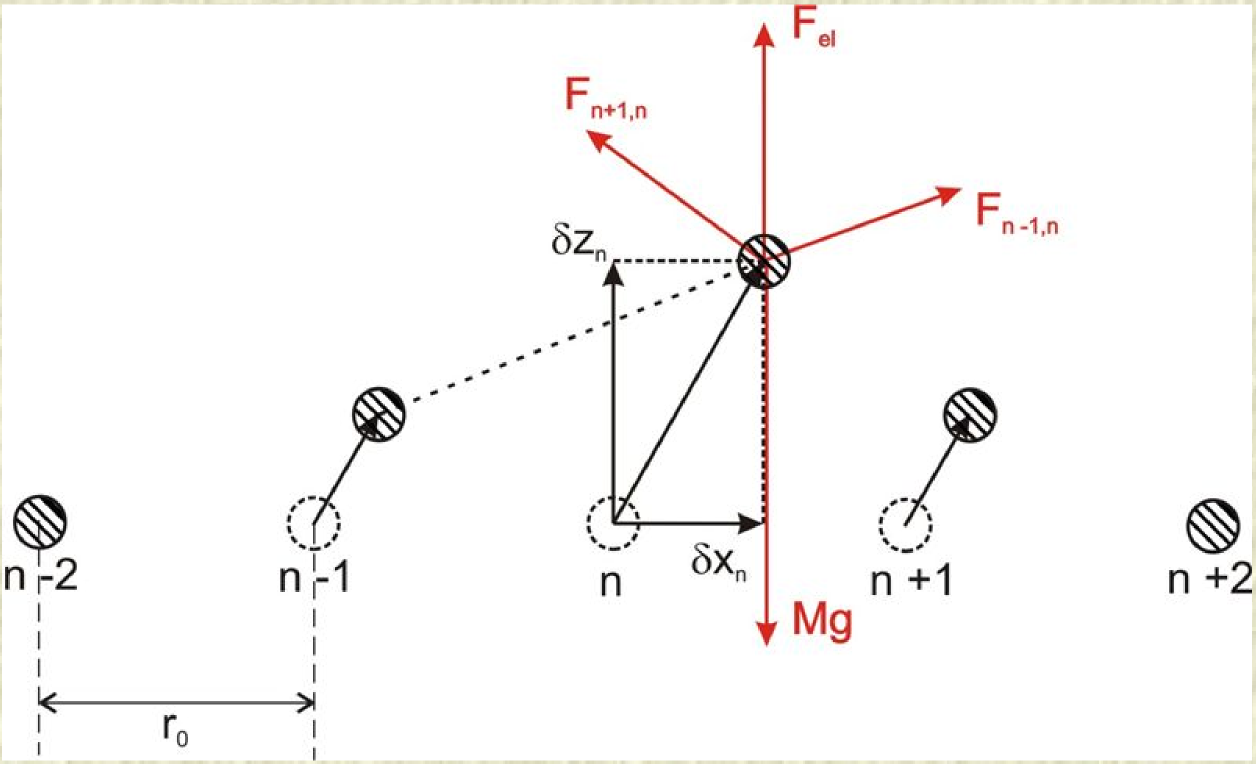
$\Delta x_{nm} = x_n - x_{n-m} \approx m r_0, \qquad \Delta z_{nm} = z_n - z_{n-m} \approx 0$,

one obtains:

$$
\begin{aligned}
U_{nm}(r) \approx\ & \frac{1}{2} M \omega_{L,0}^2 (\Delta x_{nm})^2 + \frac{1}{2} M \omega_{T,0}^2 (\Delta z_{nm})^2 \\
& + \frac{1}{3} u_{30} (\Delta x_{nm})^3 + \frac{1}{4} u_{40} (\Delta x_{nm})^4 + \ldots + \frac{1}{4} u_{04} (\Delta z_{nm})^4 + \\
& + \frac{1}{2} u_{12} (\Delta x_{nm}) (\Delta z_{nm})^2 + \frac{1}{4} u_{22} (\Delta x_{nm})^2 (\Delta z_{nm})^2 + \ldots
\end{aligned}
$$

Nonlinearity: Where from? (3)

→ *Coupling* among degrees of freedom induces nonlinearity: anisotropic motion, *not* confined along principal axes ($\sim \hat{x}$, $\hat{z}$).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

Discrete coupled equations of motion

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 \,(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$

$$-a_{20}\left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right] + a_{30}\left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]$$

$$+ a_{02}\left[(\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2\right]$$

$$-a_{12}\left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2\right],$$

$$\frac{d^2(\delta z_n)}{dt^2} = \omega_{0,T}^2 \,(2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2\,\delta z_n$$

$$- K_1\,(\delta z_n)^2 - K_2\,(\delta z_n)^3 + \frac{a_{02}}{r_0}\left[(\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3\right]$$

$$+ 2\,a_{02}\left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})\right]$$

$$- a_{12}\left[(\delta x_{n+1} - \delta x_n)^2(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2(\delta z_n - \delta z_{n-1})\right].$$

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$$- K_1\,(\delta z_n)^2 - K_2\,(\delta z_n)^3 + \frac{a_{02}}{r_0}\left[(\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3\right]$$

$$+ 2\,a_{02}\left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})\right]$$

$$- a_{12}\left[(\delta x_{n+1} - \delta x_n)^2(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2(\delta z_n - \delta z_{n-1})\right].$$

Continuum coupled equations of motion

$$\ddot{u} - c_L^2\, u_{xx} - \frac{c_L^2}{12}\, r_0^2\, u_{xxxx} =$$
$$- 2\, a_{20}\, r_0^3\, u_x\, u_{xx} + 3\, a_{30}\, r_0^4\, (u_x)^2\, u_{xx}$$
$$- a_{12}\, r_0^4\, [(w_x)^2\, u_{xx} + 2 w_x w_{xx} u_x] + 2\, a_{02}\, r_0^3\, w_x\, w_{xx}\,,$$

$$\ddot{w} + c_T^2\, w_{xx} + \frac{c_T^2}{12}\, r_0^2\, w_{xxxx} + \omega_g^2\, w =$$
$$- K_1\, w^2 - K_2\, w^3 + 3\, a_{02}\, r_0^3\, (w_x)^2\, w_{xx}$$
$$+ 2\, a_{02}\, r_0^3\, (u_x\, w_{xx} + w_x\, u_{xx}) - a_{12}\, r_0^4\, [(u_x)^2\, w_{xx} + 2 u_x u_{xx} w_x]\,,$$

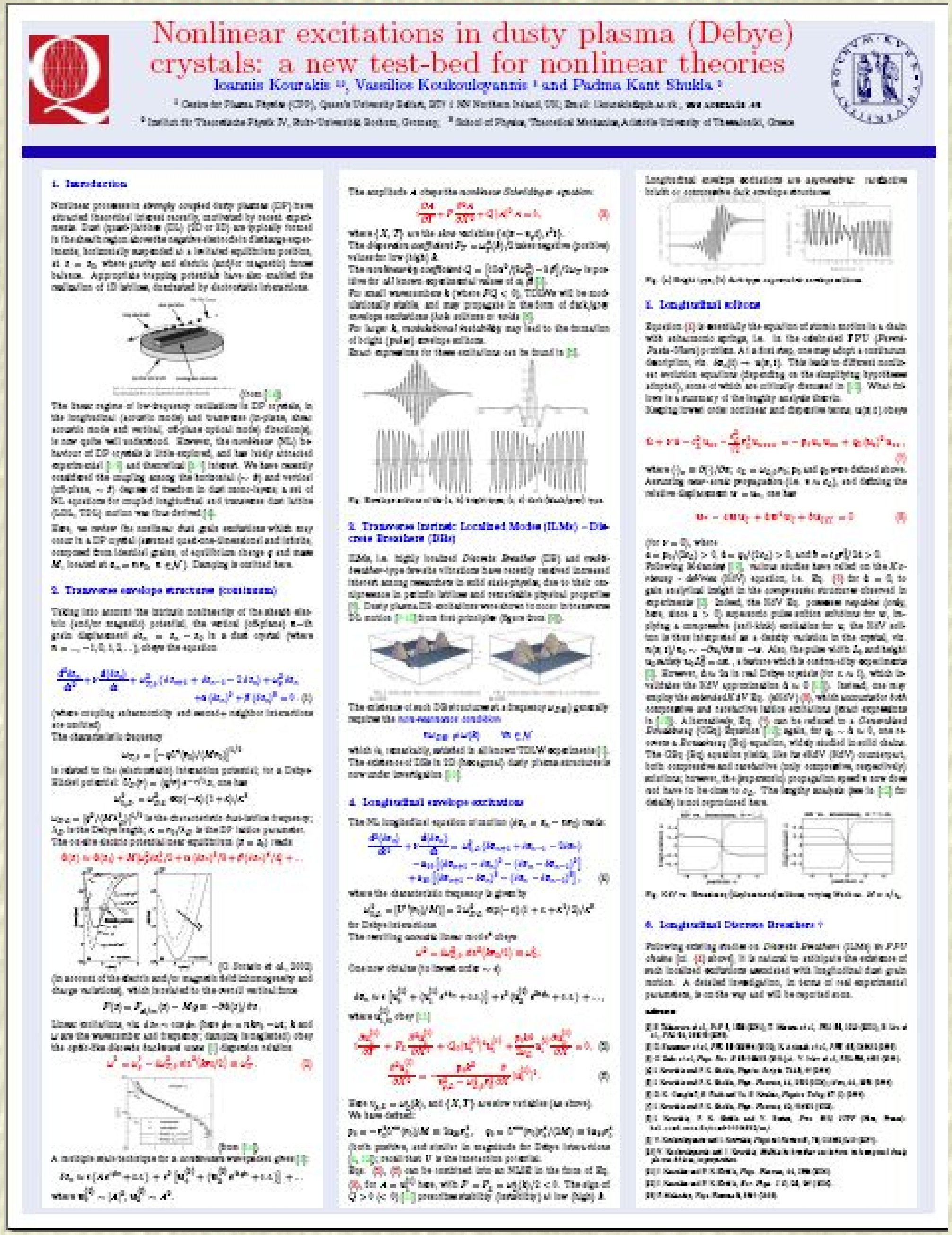
Overview of existing results ($\rightarrow$ poster)

1. 1D: *Transverse* dust-lattice (*TDL*) motion ($\sim$ NL KG, inv. disp.):
   - $\rightarrow$ Envelope (NLS) solitons [IK & P K Shukla, Phys. Plasmas **11**, 1384 (2004)]
   - $\rightarrow$ DBs (ILMs) [V. Koukouloyannis & IK, PRE **76**, 016402 (2007)]
2. 1D: *Longitudinal* dust-lattice (*LDL*) motion) ($\sim$ FPU):
   - $\rightarrow$ *Asymmetric* envelope structures (coupled 0th/1st harmonics) [IK & P K Shukla, Phys. Plasmas **11**, 3665 (2004)]
   - $\rightarrow$ KdV vs. eKdV / Bq solitons [IK & PKS, Eur. Phys. J. D **29**, 247 (2004)]
     Rem.: experimentally observed (compressive case only)
3. 2D: *In-plane* ("LDL") motion in *hexagonal* DP crystals:
   - $\rightarrow$ Envelope structures [Farokhi, IK & PKS, Phys. Plasmas **13**, 122304 (2006)]
4. 2D: *Out-of-plane* (TDL) motion in *hexagonal* DP crystals:
   - $\rightarrow$ DBs $\rightarrow$ *Presentation by Vassilis Koukouloyannis*.

Future considerations & perspectives

1. *LDL-DBs ?* ($\sim$ FPU);
2. *Damping* (dissipative system), ion drag, wake potentials, ...
3. Mixed *T-L Mode*: coupled FPU-NLKG Eqs. (ongoing work);
4. *2D hexagonal dust lattices*: vortices ? (seen experimentally);
5. *Experimental feedback*:
   - establish & pursue contacts,
   - seek confirmation of results, ...

$\rightarrow$ *Still a lot to be done ...*


Nonlinear excitations in dusty plasma (Debye) crystals: a new test-bed for nonlinear theories
Ioannis Kourakis, Vassilios Koukouloyannis and Padma Kant Shukla
1. Introduction

→ *poster:*

Thank You !

Ioannis Kourakis

Acknowledgments: P K Shukla (RUB, Bochum, Germany), V Koukouloyannis (AUTh, Greece), V. Basios (U.L.B., Brussels), T Bountis (Patras, Greece), B Farokhi (Arak, Iran).

Material from:

I Kourakis & P K Shukla, *Phys. Plasmas*, **11**, 2322 (2004);

idem, *Phys. Plasmas*, **11**, 3665 (2004).

idem, *Phys. Plasmas*, **11**, 1384 (2004).

idem, *European Phys. J. D*, **29**, 247 (2004).

V Koukouloyannis & IK, PRE **76**, 016402 (2007).

Available at: `www.tp4.rub.de/~ioannis`

ioannis@tp4.rub.de → *Poster(s).*