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# Nonlinear Excitations in Dusty Plasma Crystals: *A new test-bed for nonlinear theories*

*(Preamble to a poster presentation)*

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and

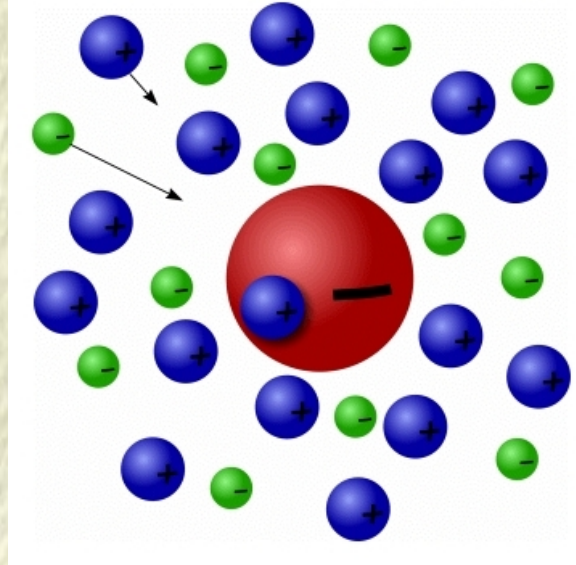
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[www.tp4.rub.de/~ioannis/conf/200710-Leiden-oral.pdf](http://www.tp4.rub.de/~ioannis/conf/200710-Leiden-oral.pdf)

## Dusty Plasmas (or *Complex Plasmas*): prerequisites

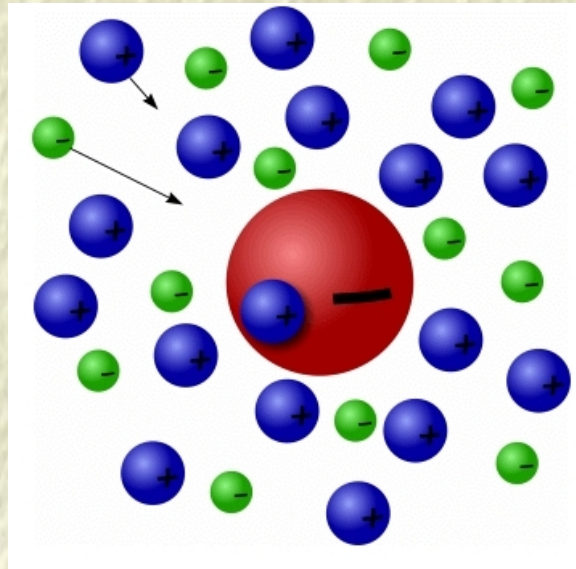


Plasma toy model:

→ *electrons*  $e^-$  (charge  $-e$ , mass  $m_e$ ),

→ *ions*  $i^+$  (charge  $+Z_i e$ , mass  $m_i$ ),

## Dusty Plasmas (or *Complex Plasmas*): prerequisites



### Dusty Plasmas (DP):

- **electrons**  $e^-$  (charge  $-e$ , mass  $m_e$ ),
- **ions**  $i^+$  (charge  $+Z_i e$ , mass  $m_i$ ),
- **charged particulates**  $\equiv$  **dust grains**  $d^\pm$  (most often  $d^-$ ):
  - charge  $Q = sZ_d e \sim \pm(10^3 - 10^4) e$ , ( $s = \pm 1$ )
  - mass  $M \sim 10^9 m_p \sim 10^{13} m_e$ ,
  - radius  $r \sim 10^{-2} \mu\text{m}$  up to  $10^2 \mu\text{m}$ .

## Dusty Plasma physics: unique mesoscopic features

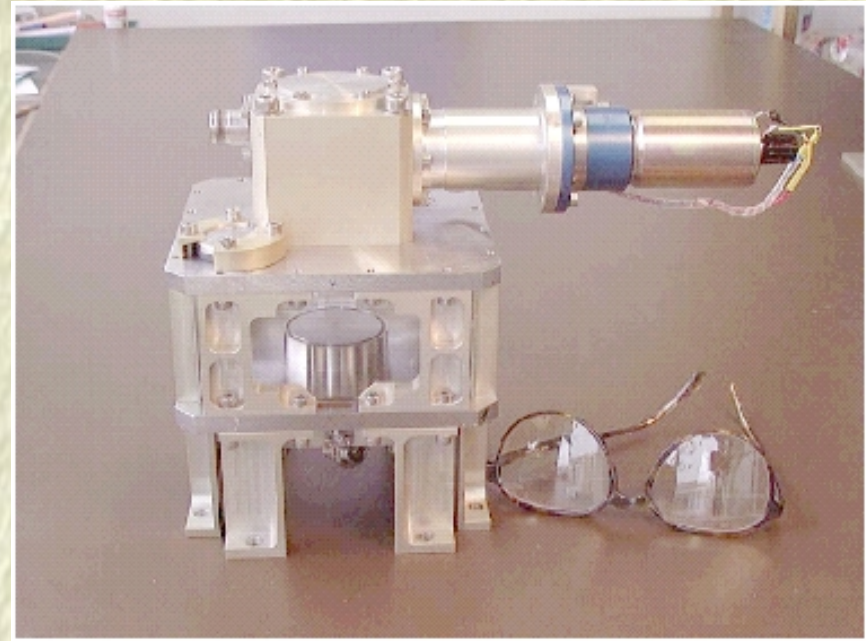
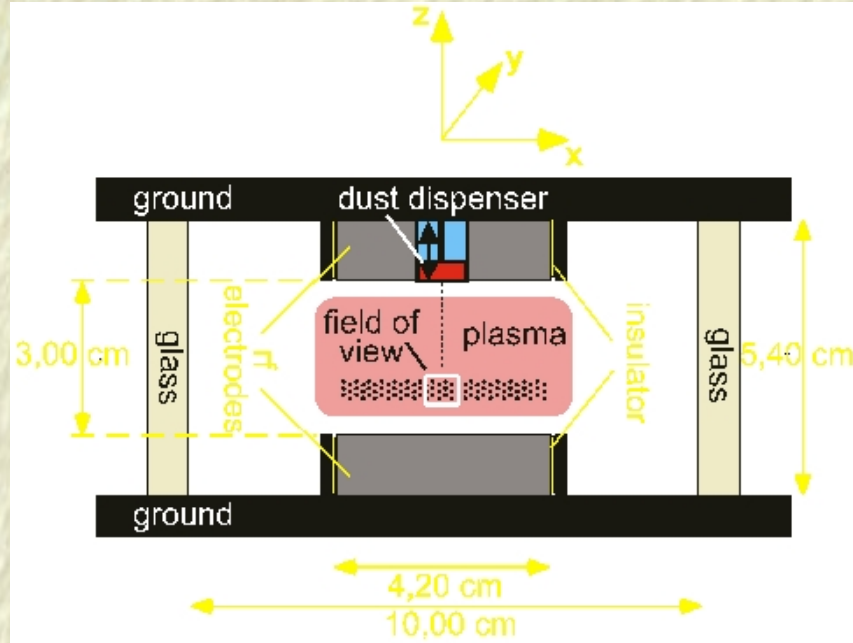
- Studies in *slow motion* are possible due to high  $M$  i.e. *low  $Q/M$  ratio* (e.g. *dust plasma frequency*:  $\omega_{p,d} \approx 10 - 100$  Hz);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);
- Contrary to *weakly-coupled  $e - i$  plasmas* ( $\Gamma \ll 1$ ), Complex Plasmas can be *strongly coupled* and exist in “*liquid*” ( $1 < \Gamma < 170$ ) and “*crystalline*” ( $\Gamma > 170$  [IKEZI 1986]) *states*, depending on the value of:

$$\Gamma_{eff} = \frac{\langle E_{potential} \rangle}{\langle E_{kinetic} \rangle} \approx \frac{\frac{Q^2}{r} e^{-r/\lambda_D}}{k_B T}$$

( $r$ : inter-particle distance,  $T$ : temperature,  $\lambda_D$ : Debye length,  $k_B$ : Boltzmann's constant).

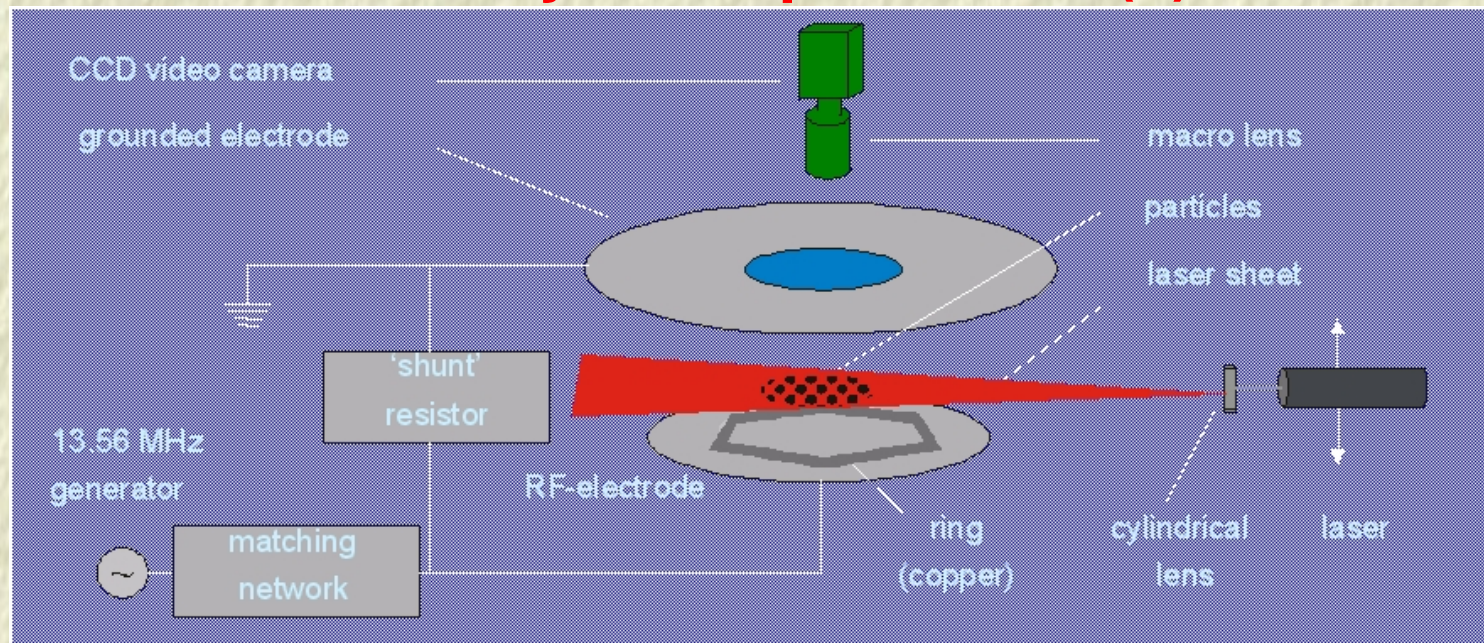
→ **Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)**

## Dust Crystal experiments (1):



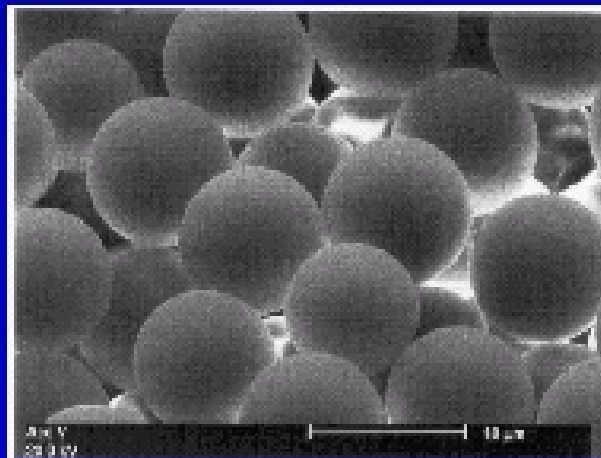
- Theoretical prediction: 1986 [H. Ikezi, *Phys. Fluids* **29**, 1764 (1986)];
- Experimental realization: 1994  
 [H. Thomas, A. Melzer *et al.* *PRL* **73**, 652 (1994); Chu & Lin *J. Phys. D* **27** 296 (1994), Hayashi & Tachibana, *Jap. J. Appl. Phys.* **33** L804 (1994)];

## Dust Crystal experiments (2):



### Particles:

Melamine-Formaldehyde  
diameter: few  $\mu\text{m}$



### Gas:

noble gas (argon, krypton)  
pressure: few Pa ... 100 Pa (=1 mbar)

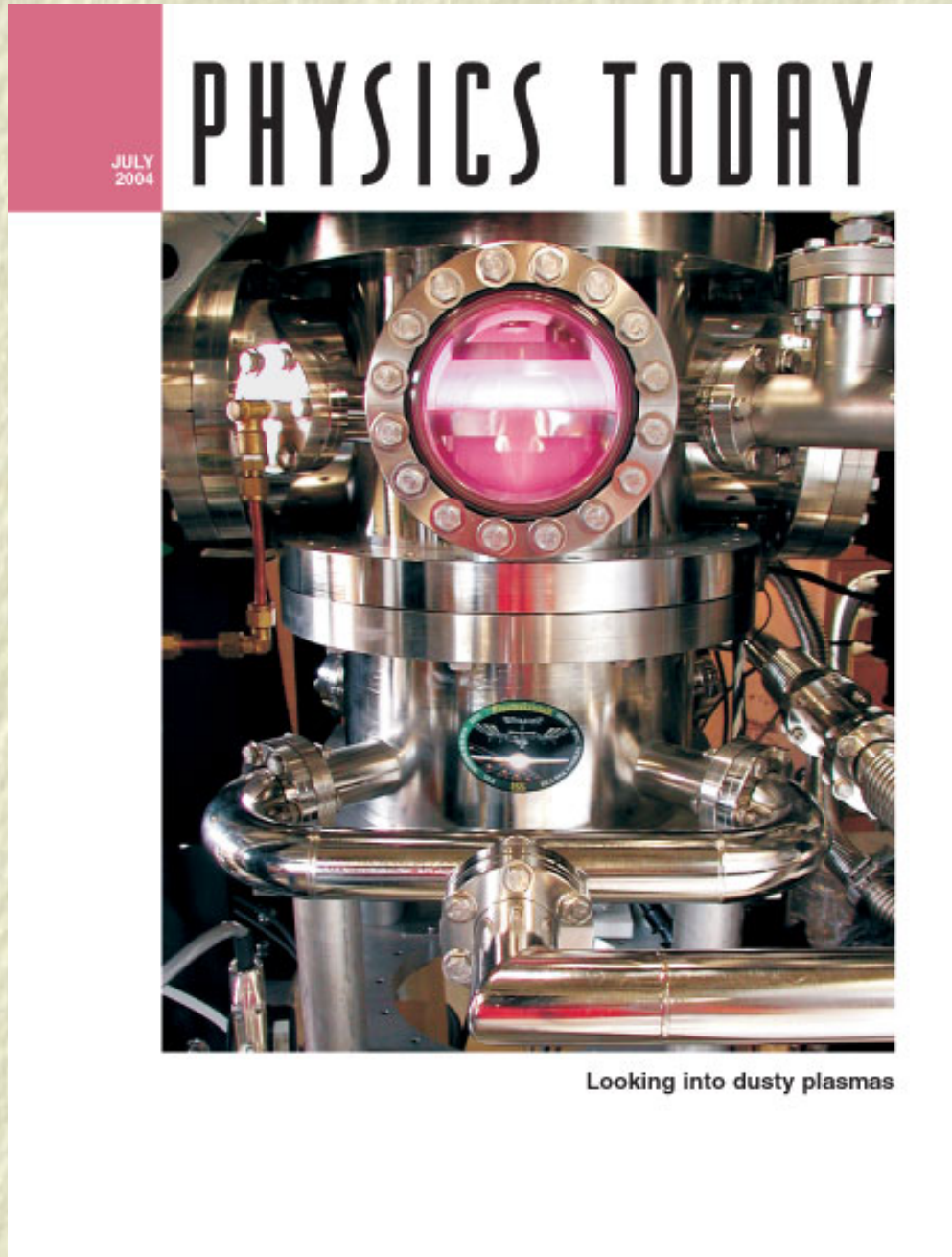
**Ionisation** fraction:  $10^{-6}$  -  $10^{-7}$

### Temperatures:

$kT_e \sim 2\text{-}4$  eV (electrons)  
 $kT_i \sim 0.03$  eV (ions)  
 $kT_p \sim 0.025$  eV (microparticles)  
in crystalline state

[Source: H. Thomas, A. Melzer *et al.*, PRL 1994].

- Today, various experimental groups active worldwide:  
*G E Morfill* (MPleP Garching, Germany), *A Piel* (Kiel, Germany), *A Melzer* (Greifswald, Germany), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *S Vladimirov* (Sydney, AUS), *S Takamura* (Nagoya, Japan) ...
- Experiments aboard the *International Space Station (ISS)*;
- Mesoscopic analog of micro-structures; research focus:
  - phase transitions, crystallization processes,
  - relaxation times, diffusion effects,
  - phase space distribution (visually observable!),
  - L & NL waves: *harmonic generation, solitons, vortices, ...*
- 3D, 2D (hexagonal, mostly), 1D lattice configurations possible ( $\rightarrow$  *video*).



Looking into dusty plasmas



## 1D DP crystal: Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left( \frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

Terms include:

– *Kinetic energy*;

–  $\Phi_{ext}(\mathbf{r}_n)$  accounts for '*external*' force fields:

may account for *confinement potentials*

and/or *sheath electric* forces, i.e.  $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$ .

– Coupling:  $U_{int}(r_{nm})$  is the *interaction potential energy*;

Q.: **Nonlinearity: Origin: where from ?**

**Effect: which consequence(s) ?**

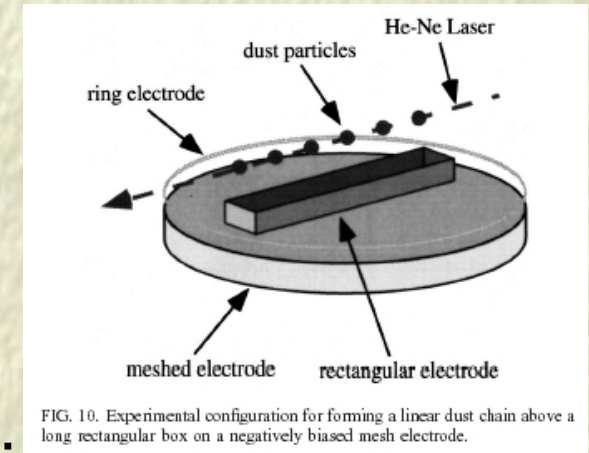


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

## Nonlinearity in 1D DPCs: Where does it come from? (1)

→ *Sheath environment (anharmonic vertical potential):*

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2}M\omega_g^2(\delta z_n)^2 + \frac{1}{3}M\alpha(\delta z_n)^3 + \frac{1}{4}M\beta(\delta z_n)^4 + \dots$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z(0)$ ;  $\alpha, \beta, \omega_g$  are defined experimentally

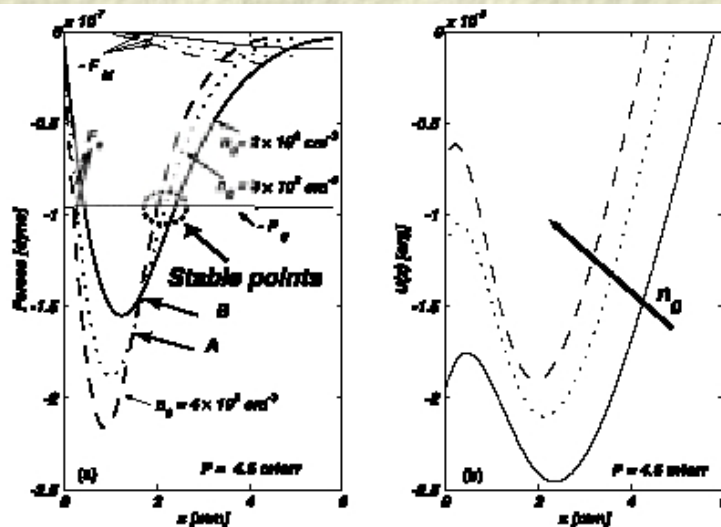


Figure 3: (a) Forces and (b) trapping potential profiles  $U(z)$  as function of distance from the electrode for:  $n_0 = 2 \times 10^8 \text{ cm}^{-3}$  (solid line),  $n_0 = 3 \times 10^8 \text{ cm}^{-3}$  (dashed line),  $n_0 = 4 \times 10^8 \text{ cm}^{-3}$  (dotted line). The parameters are:  $P = 4.6 \text{ mtorr}$ ,  $T_e = 1 \text{ eV}$ ,  $T_i = T_n = 0.05 \text{ eV}$ ,  $R = 2.5 \text{ } \mu\text{m}$ ,  $\rho_d = 1.5 \text{ g cm}^{-3}$ ,  $\phi_w = 6 \text{ V}$ .

Source: Sorasio *et al.* (2002).

## Nonlinearity: Where from? (2)

→ *Interactions between grains*: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened:  $r_0/\lambda_D \approx 1$ ), *anharmonic*; typically:  $U_{Debye}(r) = \frac{q^2}{r} \exp(-r/\lambda_D)$ .

Expanding  $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$  near equilibrium:

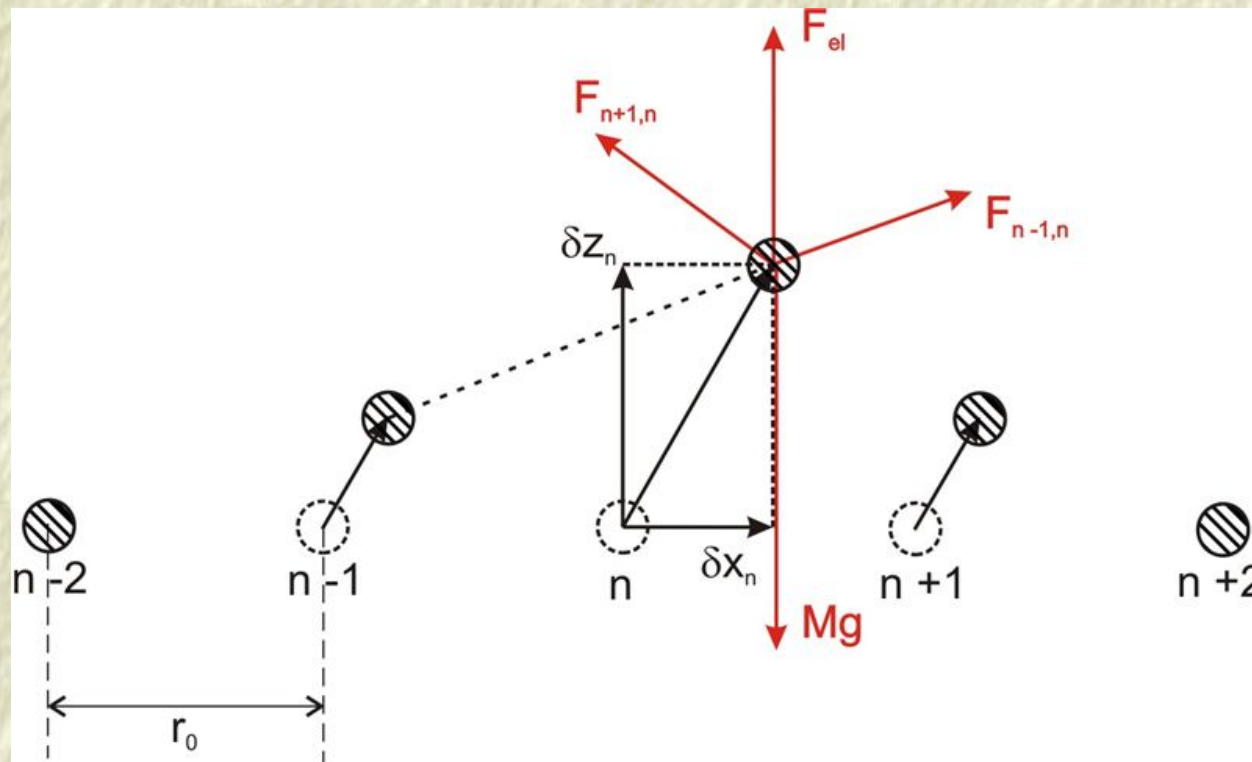
$$\Delta x_{nm} = x_n - x_{n-m} \approx mr_0, \quad \Delta z_{nm} = z_n - z_{n-m} \approx 0,$$

one obtains:

$$\begin{aligned} U_{nm}(r) \approx & \frac{1}{2}M\omega_{L,0}^2(\Delta x_{nm})^2 + \frac{1}{2}M\omega_{T,0}^2(\Delta z_{nm})^2 \\ & + \frac{1}{3}u_{30}(\Delta x_{nm})^3 + \frac{1}{4}u_{40}(\Delta x_{nm})^4 + \dots + \frac{1}{4}u_{04}(\Delta z_{nm})^4 + \\ & + \frac{1}{2}u_{12}(\Delta x_{nm})(\Delta z_{nm})^2 + \frac{1}{4}u_{22}(\Delta x_{nm})^2(\Delta z_{nm})^2 + \dots \end{aligned}$$

## Nonlinearity: Where from? (3)

→ *Coupling* among degrees of freedom induces nonlinearity: anisotropic motion, *not* confined along principal axes ( $\sim \hat{x}, \hat{z}$ ).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

## Discrete coupled equations of motion

$$\begin{aligned}
 \frac{d^2(\delta x_n)}{dt^2} &= \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n) \\
 &- a_{20} \left[ (\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right] + a_{30} \left[ (\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right] \\
 &\quad + a_{02} \left[ (\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2 \right] \\
 &- a_{12} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2 \right], \\
 \\
 \frac{d^2(\delta z_n)}{dt^2} &= \omega_{0,T}^2 (2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2 \delta z_n \\
 &- K_1 (\delta z_n)^2 - K_2 (\delta z_n)^3 + \frac{a_{02}}{r_0} \left[ (\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3 \right] \\
 &+ 2 a_{02} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1}) \right] \\
 &- a_{12} \left[ (\delta x_{n+1} - \delta x_n)^2 (\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2 (\delta z_n - \delta z_{n-1}) \right].
 \end{aligned}$$

## Discrete coupled equations of motion

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$

$$-a_{20} \left[ (\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right] + a_{30} \left[ (\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right]$$

~~$$+ a_{02} \left[ (\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2 \right]$$~~

~~$$- a_{12} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2 \right],$$~~

$$\frac{d^2(\delta z_n)}{dt^2} = \omega_{0,T}^2 (2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2 \delta z_n$$

$$- K_1 (\delta z_n)^2 - K_2 (\delta z_n)^3 + \frac{a_{02}}{r_0} \left[ (\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3 \right]$$

~~$$+ 2 a_{02} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1}) \right]$$~~

~~$$- a_{12} \left[ (\delta x_{n+1} - \delta x_n)^2 (\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2 (\delta z_n - \delta z_{n-1}) \right].$$~~

## Continuum coupled equations of motion

$$\ddot{u} - c_L^2 u_{xx} - \frac{c_L^2}{12} r_0^2 u_{xxxx} =$$

$$- 2 a_{20} r_0^3 u_x u_{xx} + 3 a_{30} r_0^4 (u_x)^2 u_{xx}$$

$$- a_{12} r_0^4 [(w_x)^2 u_{xx} + 2 w_x w_{xx} u_x] + 2 a_{02} r_0^3 w_x w_{xx},$$

$$\ddot{w} + c_T^2 w_{xx} + \frac{c_T^2}{12} r_0^2 w_{xxxx} + \omega_g^2 w =$$

$$- K_1 w^2 - K_2 w^3 + 3 a_{02} r_0^3 (w_x)^2 w_{xx}$$

$$+ 2 a_{02} r_0^3 (u_x w_{xx} + w_x u_{xx}) - a_{12} r_0^4 [(u_x)^2 w_{xx} + 2 u_x u_{xx} w_x],$$

## Overview of existing results (→ poster)


1. **1D: *Transverse dust-lattice (TDL) motion*** ( $\sim$  NL KG, inv. disp.):
  - Envelope (NLS) solitons [IK & P K Shukla, *Phys. Plasmas* **11**, 1384 (2004)]
  - DBs (ILMs) [V. Koukoulouyannis & IK, *PRE* **76**, 016402 (2007)]
2. **1D: *Longitudinal dust-lattice (LDL) motion*** ( $\sim$  FPU):
  - *Asymmetric* envelope structures (coupled 0th/1st harmonics) [IK & P K Shukla, *Phys. Plasmas* **11**, 3665 (2004)]
  - KdV vs. eKdV / Bq solitons [IK & PKS, *Eur. Phys. J. D* **29**, 247 (2004)]  
Rem.: experimentally observed (compressive case only)
3. **2D: *In-plane (“LDL”) motion in hexagonal DP crystals***:
  - Envelope structures [Farokhi, IK & PKS, *Phys. Plasmas* **13**, 122304 (2006)]
4. **2D: *Out-of-plane (TDL) motion in hexagonal DP crystals***:
  - DBs → *Presentation by Vassilis Koukoulouyannis.*



## Future considerations & perspectives

1. *LDL-DBs* ? ( $\sim$  FPU);
2. *Damping* (dissipative system), ion drag, wake potentials, ...
3. *Mixed T-L Mode*: coupled FPU-NLKG Eqs. (ongoing work);
4. *2D hexagonal dust lattices*: vortices ? (seen experimentally);
5. *Experimental feedback*:
  - establish & pursue contacts,
  - seek confirmation of results, ...


→ *Still a lot to be done ...*



## Nonlinear excitations in dusty plasma (Debye) crystals: a new test-bed for nonlinear theories

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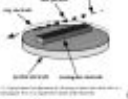
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### 1. Introduction

Nonlinear processes in strongly coupled dusty plasma (DP) have attracted theoretical interest recently, motivated by recent experiments. Our group (Kourakis *et al.*, 2010) has recently focused on the dust-ring geometry where negative electrostatic dust rings are experimentally supported at a limited equilibrium position, at  $z = z_0$ , where gravity and electric (and/or magnetic) forces balance. Appropriate trapping potentials have also enabled the realization of 1D lattices, constructed by electrostatic interactions.



The basic regime of low-frequency oscillations in DP crystals, in the longitudinal (acoustic mode) and transverse (p-plane, shear acoustic mode and vertical, out-of-plane optical mode) directions, is now quite well understood. However, the nonlinear (NL) behaviour of DP crystals is little explored, and has lately attracted experimental [1] and theoretical [2-4] interest. We have recently considered the coupling among the horizontal ( $x$ - $y$ ) and vertical ( $z$ -plane,  $\sim z$ ) degree of freedom in dust mono-layers, a set of NL equations for coupled longitudinal and transverse dust lattice (LL), TLL) motion was thus derived [4].

Here, we review the nonlinear dust grain excitations which may occur in a DP crystal (vertical quadrupole-dimensional and lateral, composed from identical grains, of equilibrium charge  $q_0$  and mass  $M$ , located at  $\mathbf{r}_i = n\mathbf{a}_0$ ,  $n \in \mathbb{N}$ ). Depending on context here:

### 2. Transverse envelope structures (continuum)

Taking into account the intrinsic nonlinearity of the sheath electric (and/or magnetic) potential, the vertical (oblique)  $n$ -th grain displacement  $u_n = u_n(x, y, z)$  in a dust crystal (where  $n = -1, 0, 1, 2, \dots$ ), obeys the equation

$$\frac{\partial^2 u_n}{\partial t^2} + \gamma \frac{\partial u_n}{\partial t} + \omega_{Dn}^2 (4u_{n+1} + 4u_{n-1} - 2u_n) + \omega_{Dn}^2 u_n^3 = \alpha (u_n^2)^2 + \beta (u_n^2)^3 = 0, \quad (1)$$

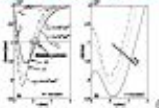
(where coupling phenomenology and second-neighbour interactions are omitted). The characteristic frequency

$$\omega_{Dn} = \left[ \frac{q_0^2 (n_0^2 / M)}{4\pi n_0^2} \right]^{1/2}$$

is related to the (electrostatic) interaction potential, for a Debye-Hückel potential:  $\phi_D(r) = (q_0/r) e^{-r/\lambda_D}$ , one has

$$\omega_{Dn}^2 = \omega_{D0}^2 \exp(-|n-1|/\lambda_D)$$

$\omega_{D0} = \left[ \frac{q_0^2 (M n_0^2)}{4\pi} \right]^{1/2}$  is the characteristic dust lattice frequency;  $\lambda_D$  is the Debye length;  $\alpha = n_0 q_0 / M$  is the DP lattice parameter. The on-site electrostatic potential near equilibrium ( $z = z_0$ ) reads


$$\phi(z) = \phi_0(z) + M \omega_{Dn}^2 z^2 + \alpha (z/\lambda_D)^2 + \beta (z/\lambda_D)^4 + \dots$$


(I. Kourakis *et al.*, 2010)

In contrast of the electric and/or magnetic field inhomogeneity and charge neutrality, which prevailed in the overall vertical force

$$F(z) = F_{el}(z) - Mg = -3B(z)/z^2.$$

Linear excitations, viz.  $u_n = \text{const} \cdot \exp(i(kx + \omega t))$ , with  $k$  and  $\omega$  are the wavenumber and frequency, damping (neglected) obey the on-site electric field near equilibrium (1) dispersion relation

$$\omega^2 = \omega_{Dn}^2 + \alpha (k/\lambda_D)^2 + \beta (k/\lambda_D)^4. \quad (2)$$


(from [4])

A multiple scale technique for a continuous wavepacket gives [4]

$$u_n = \epsilon (A e^{i(kx + \omega t + \dots)} + c.c.) + \epsilon^2 (u^{(2)} e^{i(kx + \omega t + \dots)} + c.c.) + \dots$$

where  $\epsilon = |k| \lambda_D \ll 1$ .

The amplitude  $A$  obeys nonlinear Schrödinger equation:

$$i \frac{\partial A}{\partial T} + P \frac{\partial^2 A}{\partial X^2} + Q |A|^2 A = 0, \quad (3)$$

where  $(X, T)$  are the slow variables ( $x = \epsilon X, t = \epsilon T$ ). The dispersion coefficient  $P = \omega_{Dn}^2 (2\omega_{Dn}^2)^{-1}$  is positive (negative) whenever low (high)  $\lambda_D$ . The nonlinearity coefficient  $Q = (3\alpha^2 / 4\omega_{Dn}^2) - 3\beta / 4\omega_{Dn}^2$  (independent of all known experimental values of  $\alpha, \beta$ ) [4]. For small wavenumbers  $k$  (where  $P|Q| < 1$ ), TLLs will be predominantly stable, and may propagate in the form of dust-grain envelope excitations (dark solitons or vortices) [5]. For large  $k$ , modulational instability may lead to the formation of bright (white) envelope solitons. Such expressions for these excitations can be found in [4].


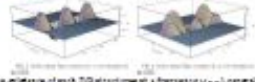


Fig. 1: Envelope solitons in (a), bright (b), (c) and dark (d) type, respectively.

### 3. Transverse Lateral Localized Mode (LLM) - Dustic Breathers (DBs)

LLMs, i.e. highly localized Dustic Breathers (DBs) and modulation-type dustic vibrations have recently received increased interest among researchers in solid state physics, due to their correspondence to specific lattice and structural properties [6]. Dusty plasma DB excitations were shown to occur in transverse DB motion [7-10] from first principles (Bogoliubov-Fokker-Planck).



The existence of such DB structures at a frequency  $\omega_{DB}$  generally requires the non-resonance condition

$$\omega_{DB} \neq \omega_{Dn} \quad \forall n \in \mathbb{N}$$

which is remarkably violated in all known TLLW excitations [4]. The existence of LLMs in 1D (helical) dusty plasma structures is now under investigation [11].

### 4. Longitudinal envelope excitations

The NL longitudinal equation of motion ( $\Delta z = z_0 - \epsilon z$ ) reads:

$$\frac{\partial^2 (u_n)}{\partial t^2} + \gamma \frac{\partial u_n}{\partial t} + \omega_{Dn}^2 (2u_{n+1} + 2u_{n-1} - 2u_n) = -2\alpha_1 (u_{n+1} - u_n)^2 - (2\alpha_2 - 2\alpha_1) (u_n - u_{n-1})^2 + 2\alpha_2 (u_{n-1} - u_n)^2 - (\alpha_2 - 2\alpha_1) u_n^3, \quad (4)$$

where the characteristic frequency is given by

$$\omega_{Dn}^2 = \left[ \frac{q_0^2 (n_0^2 / M)}{4\pi} \right] (2u_{n+1} + 2u_{n-1} - 2u_n) + \alpha + \beta (k/\lambda_D)^2$$

for Debye interactions. The resulting acoustic linear mode obeys

$$\omega^2 = \omega_{Dn}^2 + \alpha (k/\lambda_D)^2 + \beta (k/\lambda_D)^4.$$

One can obtain (at lowest order  $\epsilon$ )

$$u_n = \epsilon \left[ \frac{1}{2} (A e^{i(kx + \omega t + \dots)} + c.c.) + \epsilon^2 (u^{(2)} e^{i(kx + \omega t + \dots)} + c.c.) + \dots \right],$$

where  $\epsilon = |k| \lambda_D \ll 1$  (see [4]).

$$\frac{\partial^2 A}{\partial T^2} + P_1 \frac{\partial^2 A}{\partial X^2} - Q_1 |A|^2 A + \frac{P_2 \alpha_1}{2\omega_{Dn}^2} \frac{\partial^2 A}{\partial X^2} = 0, \quad (5)$$

$$\frac{\partial^2 A}{\partial T^2} - \frac{P_2 \alpha_2}{2\omega_{Dn}^2} \frac{\partial^2 A}{\partial X^2} = 0. \quad (6)$$

Here  $\omega_{Dn} = \omega_{Dn}(k)$ , and  $(X, T)$  are slow variables (see above). We have defined:

$$P_1 = -\frac{3\alpha_1}{2\omega_{Dn}^2} (n_0^2 / M) + \frac{3\alpha_2}{2\omega_{Dn}^2} (n_0^2 / M) + \frac{3\alpha_1}{2\omega_{Dn}^2} (n_0^2 / M)$$

(both positive, and similar in magnitude for Debye interactions [4, 10]), recall that  $\Gamma$  is the interaction potential. Note  $(P_1, P_2)$  can be combined into an NLSE in the form of Eq. (3), for  $A = u_n e^{i(kx + \omega t)}$ , with  $P = P_1 + \alpha_1 (k/\lambda_D)^2 < 0$ . The sign of  $Q > 0 < 0$  [4] prescribes stability (instability) at low (high)  $k$ .

Longitudinal envelope excitations are asymmetric: radiative breath or compressive dark envelope structures.




Fig. 2: (a) Bright (b) dark type envelope excitations.

### 5. Longitudinal solitons

Equation (1) is essentially the equation of motion written in a chain with enhanced spring, i.e. in the extended FPU (Fermi-Pasta-Ulam) problem. As a first step, one may adopt a continuous description, viz.  $u_n(t) \rightarrow u(x, t)$ . The field is defined in terms of evolution equations (depending on the simplifying hypothesis adopted), some of which are critically discussed in [4]. What follows is a summary of the lengthy analysis therein.

Using lower order nonlinear and dispersion terms, one obtains

$$\partial_t^2 u + P_1 \frac{\partial^2 u}{\partial x^2} - \frac{Q_1}{2} |u|^2 u = -P_2 \alpha_1 u_{xxx} + Q_2 \alpha_2 u_{xxx}, \quad (7)$$

where  $(P_1, Q_1, Q_2, \alpha_1, \alpha_2) = (\omega_{Dn}^2, 2\omega_{Dn}^2, P_2, P_1, P_2)$  and  $Q_2$  were defined above. Assuming near-linear propagation (i.e.  $u = u_0$ ), and defining the relative displacement  $v = u - u_0$ , one has

$$v_t - \alpha v v_x + \beta v^2 v_x + \beta v_{xxx} = 0. \quad (8)$$

(for  $v = 0$ ), where

$$\alpha = P_2 \alpha_1 / \omega_{Dn}^2 > 0, \quad \beta = Q_2 \alpha_2 / \omega_{Dn}^2 > 0, \quad \text{and } \beta = P_2 \alpha_2 / \omega_{Dn}^2 > 0.$$

Following Miura [12], mutual studies have relied on the Korteweg-de Vries (KdV) equation, i.e. Eq. (8) for  $\beta = 0$ , to gain analytical insight in the compressive structures observed in experiments [3]. Indeed, the KdV Eq. possesses regular (only) breather, since  $\alpha > 0$ , asymptotic pulse soliton solutions for  $v$ , implying a compressive (anti-KdV) solution for  $u$ , the KdV solution is thus interpreted as a density variation in the crystal, viz.  $\rho(x, t) \sim -\partial_x v / \omega_{Dn} \sim -\partial_x v / \omega_{Dn}$ . Also, the pulse width  $\Delta x$  and height (relative to  $u_0$ )  $\Delta u$ , a feature which is controlled by experiment [3]. However,  $\beta \neq 0$  in real DP crystals (for  $k \neq 0$ ), which invalidates the KdV approximation  $\beta = 0$  [13]. Indeed, one may employ the extended KdV Eq. (MKdV), which accounts for both compressive and radiative lattice excitations (exact expressions in [4]). Alternatively, Eq. (8) can be reduced to a generalized Burgers-like (GBL) equation [14], again, for  $\alpha_1 = \beta = 0$ , one recovers a Burgers-like (BL) equation, widely studied in solid state. The GBL (BL) equation yields, like the KdV (KdV) counterpart, both compressive and radiative (only compressive, respectively) solutions, however, the propagation propagation speed  $v$  now does not have to be close to  $c_D$ . The lengthy analysis (see in [4] for details) is not reported here.




Fig. 3: GBL (Burgers-like) and BL (Burgers-like) equations, respectively, for  $\alpha_1 = \beta = 0$ .

### 6. Longitudinal Dustic Breathers

Following existing studies on Dustic Breathers (DBs) in FPU chains (see [4] above), it is natural to anticipate the existence of such localized excitations associated with longitudinal dust grain motion. A detailed investigation, in terms of real experimental parameters, is on the way and will be reported soon.

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# Thank You !

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→ *Poster(s)*.