Nonlinear excitations in dusty plasma (Debye) crystals: a new test-bed for nonlinear theories

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1. Introduction

Nonlinear processes in *strongly coupled* dusty plasmas (DP) have attracted theoretical interest recently, motivated by recent experiments. Dust (quasi-)lattices (DL) (2D or 3D) are typically formed in the sheath region above the negative electrode in discharge experiments, horizontally suspended at a levitated equilibrium position, at $z = z_0$, where gravity and electric (and/or magnetic) forces balance. Appropriate trapping potentials have also enabled the realization of 1D lattices, dominated by electrostatic interactions.

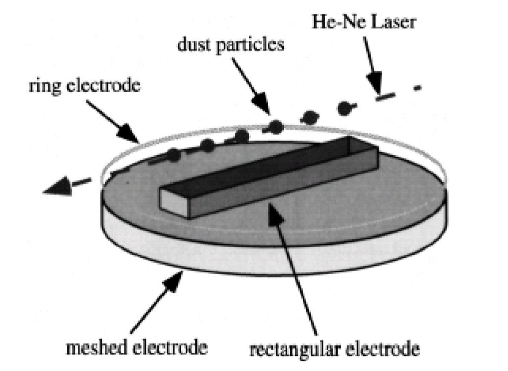


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode. (from [1a])

The linear regime of low-frequency oscillations in DP crystals, in the longitudinal (acoustic mode) and transverse (in-plane, shear acoustic mode and vertical, off-plane optical mode) direction(s), is now quite well understood. However, the *nonlinear* (NL) behaviour of DP crystals is little explored, and has lately attracted experimental [1-3] and theoretical [4-7] interest. We have recently considered the coupling among the horizontal ($\sim \hat{x}$) and vertical (off-plane, $\sim \hat{z}$) degrees of freedom in dust mono-layers; a set of NL equations for coupled longitudinal and transverse dust lattice (LDL, TDL) motion was thus derived [4].

Here, we review the nonlinear dust grain excitations which may occur in a DP crystal (assumed quasi-one-dimensional and infinite, composed from identical grains, of equilibrium charge q and mass M, located at $x_n = n\, r_0,\ n \in \mathcal{N}$). Damping is omitted here.

2. Transverse envelope structures (continuum)

Taking into account the intrinsic nonlinearity of the sheath electric (and/or magnetic) potential, the vertical (off-plane) n−th grain displacement $\delta z_n = z_n - z_0$ in a dust crystal (where $n = ..., -1, 0, 1, 2, ...$), obeys the equation

$$\frac{d^2\delta z_n}{dt^2} + \nu\,\frac{d(\delta z_n)}{dt} + \omega_{T,0}^2\,(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,. \quad (1)$$

(where coupling anharmonicity and second+ neighbor interactions are omitted)

The characteristic frequency

$$\omega_{T,0} = \left[-qU'(r_0)/(M r_0)\right]^{1/2}$$

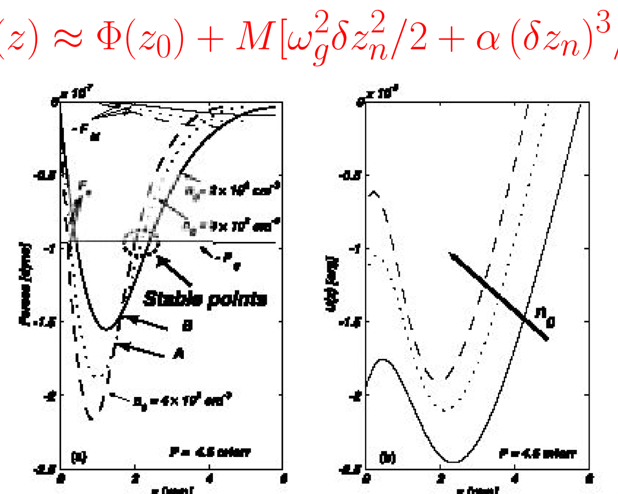
is related to the (electrostatic) interaction potential; for a Debye-Hückel potential: $U_D(r) = (q/r)\,e^{-r/\lambda_D}$, one has

$$\omega_{0,D}^2 = \omega_{DL}^2\,\exp(-\kappa)\,(1+\kappa)/\kappa^3$$

$\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$ is the characteristic dust-lattice frequency; λ_D is the Debye length; $\kappa = r_0/\lambda_D$ is the DP lattice parameter.

The on-site electric potential near equilibrium ($z = z_0$) reads

$$\Phi(z) \approx \Phi(z_0) + M[\omega_g^2 \delta z_n^2/2 + \alpha\,(\delta z_n)^3/3 + \beta\,(\delta z_n)^4/4] + ...$$



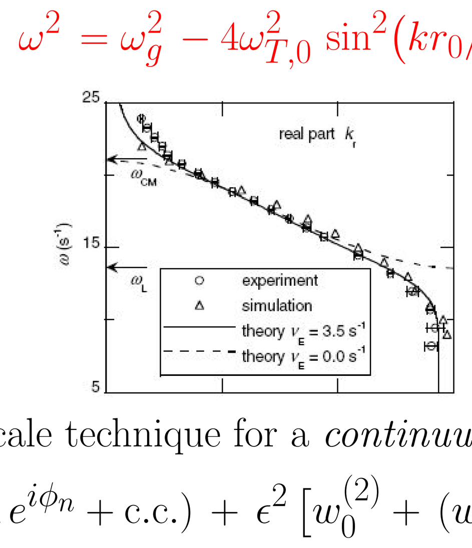
(G. Sorasio *et al.*, 2002)

(in account of the electric and/or magnetic field inhomogeneity and charge variations), which is related to the overall vertical force

$$F(z) = F_{el/m}(z) - Mg \equiv -\partial\Phi(z)/\partial z\,.$$

Linear excitations, viz. $\delta z_n \sim \cos\phi_n$ (here $\phi_n = nkr_0 - \omega t$; k and ω are the wavenumber and frequency; damping is neglected) obey the *optic*-like *discrete backward wave* [1] dispersion relation

$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2\,\sin^2\!\left(kr_0/2\right) \equiv \omega_T^2\,. \quad (2)$$



(from [1c])

A multiple scale technique for a *continuum* wavepacket gives [5]:

$$\delta z_n \approx \epsilon\,(A\,e^{i\phi_n} + \text{c.c.}) + \epsilon^2\left[w_0^{(2)} + (w_2^{(2)}\,e^{2i\phi_n} + \text{c.c.})\right] + ...$$

where $w_0^{(2)} \sim |A|^2$, $w_2^{(2)} \sim A^2$.

The amplitude A obeys the *nonlinear Schrödinger equation*:

$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2\,A = 0\,, \quad (3)$$

where $\{X, T\}$ are the *slow* variables $\{\epsilon(x - v_g t), \epsilon^2 t\}$.

The *dispersion coefficient* $P_T = \omega_T''(k)/2$ takes negative (positive) values for low (high) k.

The *nonlinearity coefficient* $Q = \left[10\alpha^2/(3\omega_g^2) - 3\,\beta\right]/2\omega_T$ is positive for *all* known experimental values of α, β [3].

For small wavenumbers k (where $PQ < 0$), TDLWs will be modulationally stable, and may propagate in the form of dark/grey envelope excitations (*hole* solitons or *voids* [5].

For larger k, *modulational instability* may lead to the formation of bright (*pulse*) envelope solitons.

Exact expressions for these excitations can be found in [5].

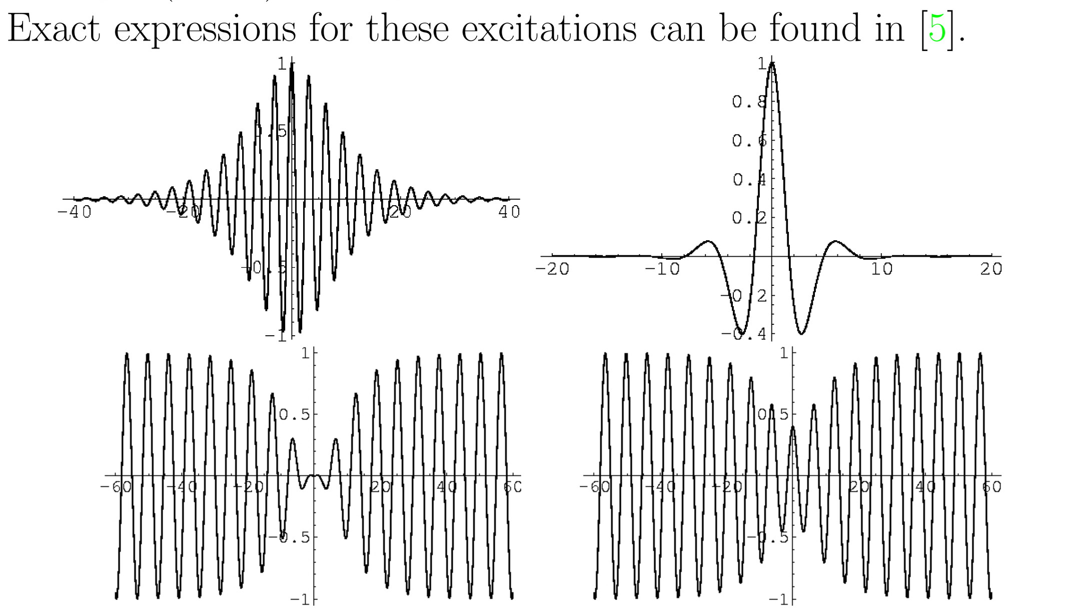


Fig. Envelope solitons of the (a, b) bright type; (c, d) dark (black/grey) type.

3. Transverse Intrinsic Localized Modes (ILMs) – Discrete Breathers (DBs)

ILMs, i.e. highly localized *Discrete Breather* (DB) and *multi-breather*-type few-site vibrations have recently received increased interest among researchers in solid state physics, due to their omnipresence in periodic lattices and remarkable physical properties [6]. Dusty plasma DB excitations were shown to occur in transverse DL motion [7-10] from first principles (figure from [9]).

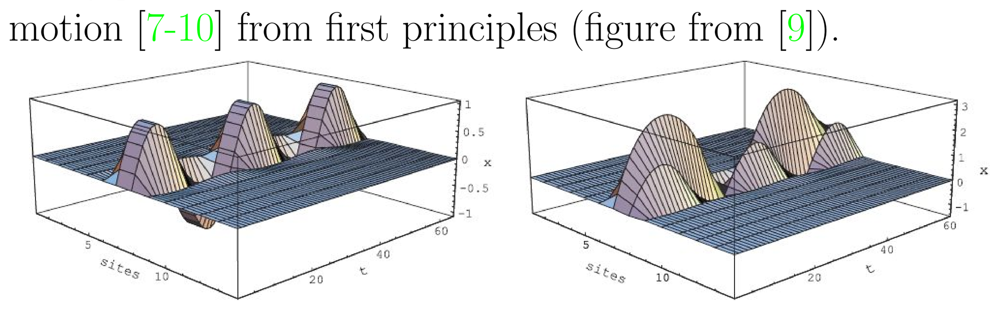


FIG. 5. (Color online) Time evolution of a 1:1 two-breather for ϵ=−0.016.

FIG. 6. (Color online) Time evolution of a 2:3 two-breather for ϵ=−0.003.

The existence of such DB structures at a frequency ω_{DB}) generally requires the *non-resonance condition*

$$n\omega_{DB} \neq \omega(k) \qquad \forall n \in \mathcal{N}$$

which *is*, remarkably, satisfied in all known TDLW experiments [1].

The existence of DBs in 2D (hexagonal) dusty plasma structures is now under investigation [10].

4. Longitudinal envelope excitations

The NL longitudinal equation of motion ($\delta x_n = x_n - nr_0$) reads:

$$\begin{aligned}\frac{d^2(\delta x_n)}{dt^2} + \nu\,\frac{d(\delta x_n)}{dt} &= \omega_{0,L}^2\,(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n) \\ &\quad - a_{20}\left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right] \\ &\quad + a_{30}\left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]\,, \quad (4)\end{aligned}$$

where the characteristic frequency is given by

$$\omega_{0,L}^2 = [U''(r_0)/M)] = 2\,\omega_{DL}^2\,\exp(-\kappa)\,(1 + \kappa + \kappa^2/2)/\kappa^3$$

for Debye interactions.

The resulting *acoustic* linear mode[4] obeys

$$\omega^2 = 4\omega_{L,0}^2\,\sin^2\!\left(kr_0/2\right) \equiv \omega_L^2\,.$$

One now obtains (to lowest order $\sim \epsilon$)

$$\delta x_n \approx \epsilon\left[u_0^{(1)} + (u_1^{(1)}\,e^{i\phi_n} + \text{c.c.})\right] + \epsilon^2\,(u_2^{(2)}\,e^{2i\phi_n} + \text{c.c.}) + ...\,,$$

where $u_{1/0}^{(1)}$ obey [11]

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L\,\frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0\,|u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L}\,u_1^{(1)}\frac{\partial u_0^{(1)}}{\partial X} = 0\,, \quad (5)$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - \omega_{L,0}^2 r_0^2}\,\frac{\partial}{\partial X}\,|u_1^{(1)}|^2\,. \quad (6)$$

Here $v_{g,L} = \omega_L'(k)$, and $\{X, T\}$ are slow variables (as above).

We have defined:

$p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$, $\quad q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$

(both positive, and similar in magnitude for Debye interactions [4, 12]); recall that U is the interaction potential.

Eqs. (5), (6) can be combined into an NLSE in the form of Eq. (3), for $A = u_1^{(1)}$ here, with $P = P_L = \omega_L''(k)/2 < 0$. The sign of $Q > 0$ (< 0) [11] prescribes stability (instability) at low (high) k.

Longitudinal envelope excitations are *asymmetric*: rarefactive bright or compressive dark envelope structures.

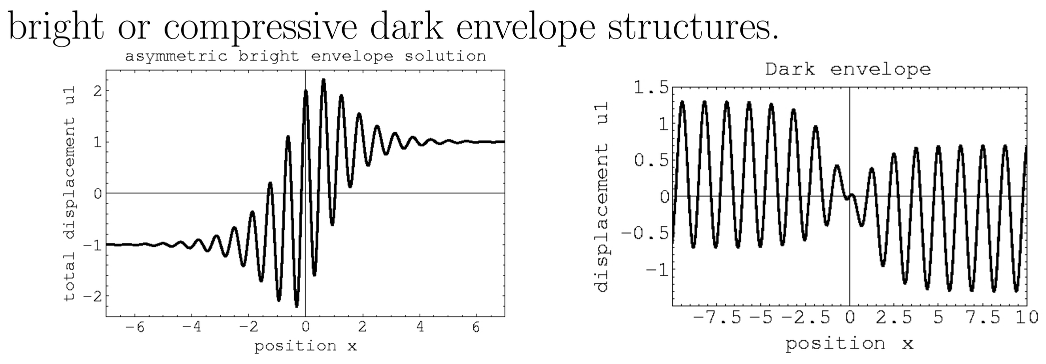


Fig. (a) Bright type; (b) dark type *asymmetric* envelope solitons.

5. Longitudinal solitons

Equation (4) is essentially the equation of atomic motion in a chain with anharmonic springs, i.e. in the celebrated **FPU** (*Fermi-Pasta-Ulam*) problem. At a first step, one may adopt a continuum description, viz. $\delta x_n(t) \to u(x, t)$. This leads to different nonlinear evolution equations (depending on the simplifying hypotheses adopted), some of which are critically discussed in [12]. What follows is a summary of the lengthy analysis therein.

Keeping lowest order nonlinear and dispersive terms, $u(x, t)$ obeys

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx} + q_0\,(u_x)^2\,u_{xx}\,, \quad (7)$$

where $(\cdot)_x \equiv \partial(\cdot)/\partial x$; $c_L = \omega_{L,0}\,r_0$; p_0 and q_0 were defined above.

Assuming *near-sonic propagation* (i.e. $v \approx c_L$), and defining the relative displacement $w = u_x$, one has

$$w_\tau - a\,w\,w_\zeta + \hat{a}\,w^2\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0 \quad (8)$$

(for $\nu = 0$), where

$a = p_0/(2c_L) > 0$, $\hat{a} = q_0/(2c_L) > 0$, and $b = c_L r_0^2/24 > 0$.

Following Melandsø [13], various studies have relied on the *Korteweg - deVries* (KdV) equation, i.e. Eq. (8) for $\hat{a} = 0$, to gain analytical insight in the *compressive* structures observed in experiments [2]. Indeed, the KdV Eq. possesses *negative* (*only*, here, since $a > 0$) supersonic pulse soliton solutions for w, implying a compressive (anti-kink) excitation for u; the KdV soliton is thus interpreted as a density variation in the crystal, viz. $n(x, t)/n_0 \sim -\partial u/\partial x \equiv -w$. Also, the pulse width L_0 and height u_0 satisfy $u_0 L_0^2 = cst.$, a feature which is confirmed by experiments [2]. However, $\hat{a} \approx 2a$ in real Debye crystals (for $\kappa \approx 1$), which invalidates the KdV approximation $\hat{a} \approx 0$ [12]). Instead, one may employ the *extended KdV* Eq. (eKdV) (8), which accounts for *both* compressive *and* rarefactive lattice excitations (exact expressions in [12]). Alternatively, Eq. (7) can be reduced to a *Generalized Boussinesq* (GBq) Equation [12]; again, for $q_0 \sim \hat{a} \approx 0$, one recovers a *Boussinesq* (Bq) equation, widely studied in solid chains. The GBq (Bq) equation yields, like its eKdV (KdV) counterpart, both compressive and rarefactive (only compressive, respectively) solutions; however, the (supersonic) propagation speed v now does *not* have to be close to c_L. The lengthy analysis (see in [12] for details) is not reproduced here.

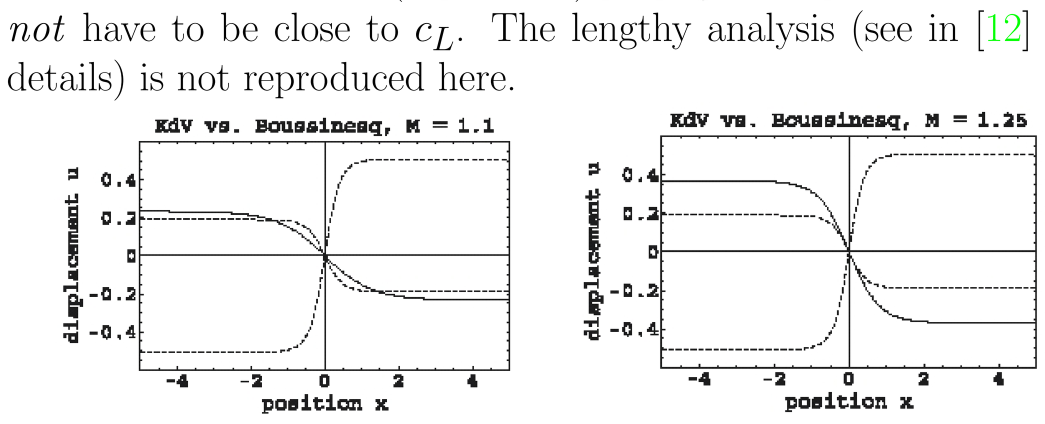


Fig. KdV vs. Boussinesq (displacement) solitons, varying Mach no. $M = v/c_L$.

6. Longitudinal Discrete Breathers ?

Following existing studies on *Discrete Breathers* (ILMs) *in FPU chains* [cf. (4) above], it is natural to anticipate the existence of such localized excitations associated with longitudinal dust grain motion. A detailed investigation, in terms of real experimental parameters, is on the way and will be reported soon.

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