

*IOP Plasma Physics Group Annual Meeting, 1-4 March, 2008*

**Nonlinear Excitations in Dusty Plasma Crystals:**  
*what plasma physics can learn*  
*from nonlinear lattice theories*

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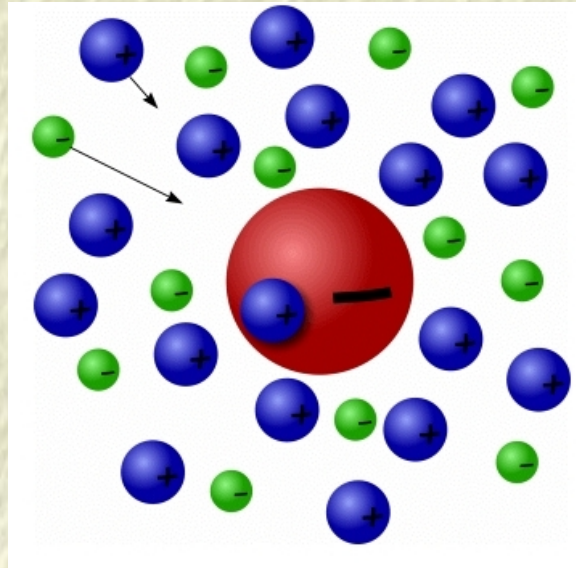


## Outline

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites.
  - (i) Focus: 1d dust crystals in lab.
  - (ii) Nonlinearity in 1d DP crystals: Origin and modelling.
2. *Transverse dust-lattice (TDL) excitations*: amplitude modulation, transverse envelope structures.
3. *Longitudinal dust-lattice (LDL) excitations*: amplitude modulation, envelope structures, solitons.
4. *1d Discrete Breather excitations (intrinsic localized modes)*
5. *Conclusions.*



# 1. Dusty Plasmas (or *Complex Plasmas*): prerequisites



Dusty Plasmas (DP):

→ *electrons*  $e^-$  (charge  $-e$ , mass  $m_e$ ),

→ *ions*  $i^+$  (charge  $+Z_i e$ , mass  $m_i$ ),

→ **charged particulates**  $\equiv$  *dust grains*  $d^\pm$  (most often  $d^-$ ):  
 charge  $Q = sZ_d e \sim \pm(10^3 - 10^4) e$ , ( $s = \pm 1$ )  
 mass  $M \sim 10^9 m_p \sim 10^{13} m_e$ ,  
 radius  $r \sim 10^{-2} \mu\text{m}$  up to  $10^2 \mu\text{m}$ .



## Where/how do dusty plasmas occur?

- *Space: cosmic debris* (silicates, graphite, amorphous carbon), *comet tails*, man-made *pollution* (Shuttle exhaust, satellites), ...
- *Earth's atmosphere*: volcanic eruptions, *extraterrestrial* origin (*meteorites*) ( $\geq 2 \cdot 10^4$  tons/yr!), *pollution*, *aerosols*, ...;
- *Technology*: Semiconductor industry, Si microchip, dust contamination, solar cell stabilization ...;
- *Laboratory*: (man-injected) melamine–formaldehyde particulates injected in *rf* or *dc* discharges.
- *Fusion devices*: plasma-surface interaction in the *divertor* region (graphite, CFCs), UFOs, ITER safety concern, ...;

Sources: P. K. Shukla & A. Mamun, book (IoP, 2002), G. E. Morfill *et al.*, 1998, etc.



## Dusty Plasma physics: unique mesoscopic features

- Studies in *slow motion* are possible due to high  $M$  i.e. *low  $Q/M$  ratio* (e.g. *dust plasma frequency*:  $\omega_{p,d} \approx 10 - 100$  Hz);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);
- Contrary to *weakly-coupled  $e - i$  plasmas* ( $\Gamma \ll 1$ ), Complex Plasmas can be *strongly coupled* and exist in “*liquid*” ( $1 < \Gamma < 170$ ) and “*crystalline*” ( $\Gamma > 170$  [IKEZI 1986]) *states*, depending on the value of:

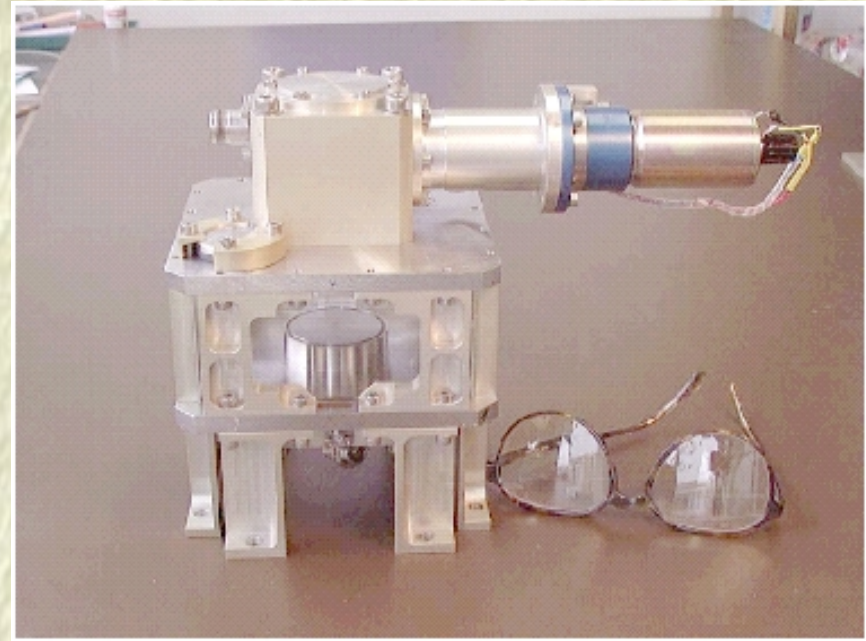
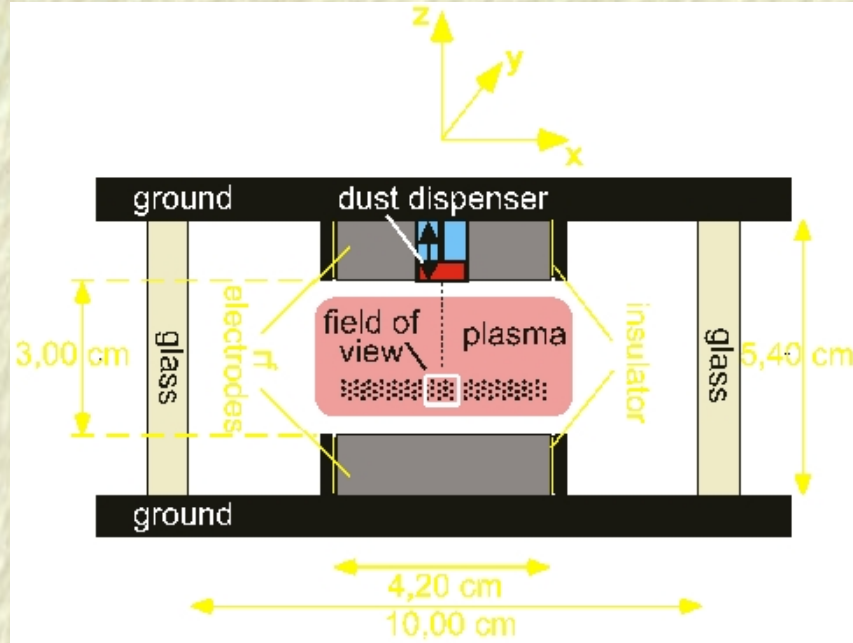
$$\Gamma_{eff} = \frac{\langle E_{potential} \rangle}{\langle E_{kinetic} \rangle} \approx \frac{\frac{Q^2}{r} e^{-r/\lambda_D}}{k_B T}$$

( $r$ : inter-particle distance,  $T$ : temperature,  $\lambda_D$ : Debye length,  $k_B$ : Boltzmann's constant).

→ ***Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)***



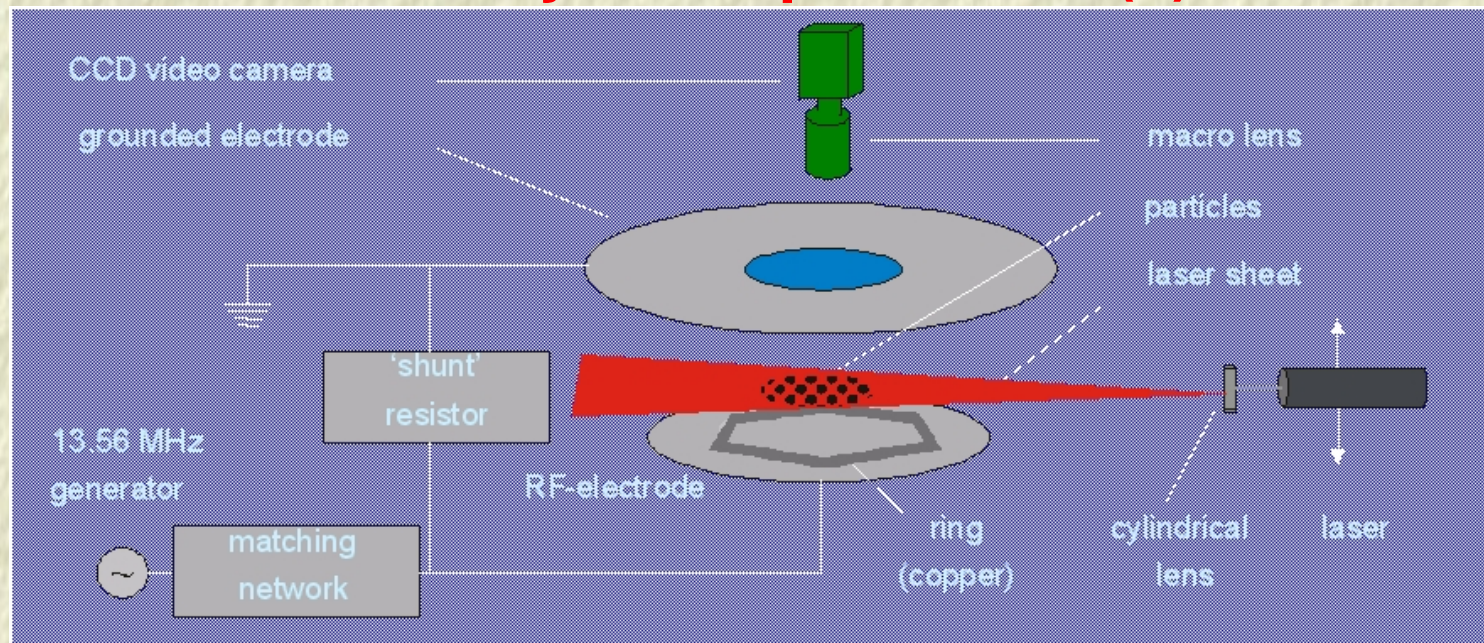
## Dust Crystal experiments (1):



- Theoretical prediction: 1986 [H. Ikezi, *Phys. Fluids* **29**, 1764 (1986)];
- Experimental realization: 1994  
[H. Thomas *et al.* *PRL* **73**, 652 (1994); Chu & Lin *J. Phys. D* **27** 296 (1994), Hayashi & Tachibana, *Jap. J. Appl. Phys.* **33** L804 (1994)];

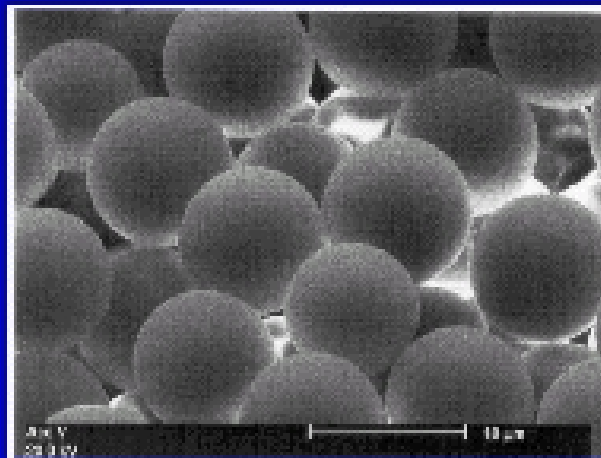


## Dust Crystal experiments (2):



### Particles:

Melamine-Formaldehyde  
diameter: few  $\mu\text{m}$



### Gas:

noble gas (argon, krypton)  
pressure: few Pa ... 100 Pa (=1 mbar)

**Ionisation** fraction:  $10^{-6}$  -  $10^{-7}$

### Temperatures:

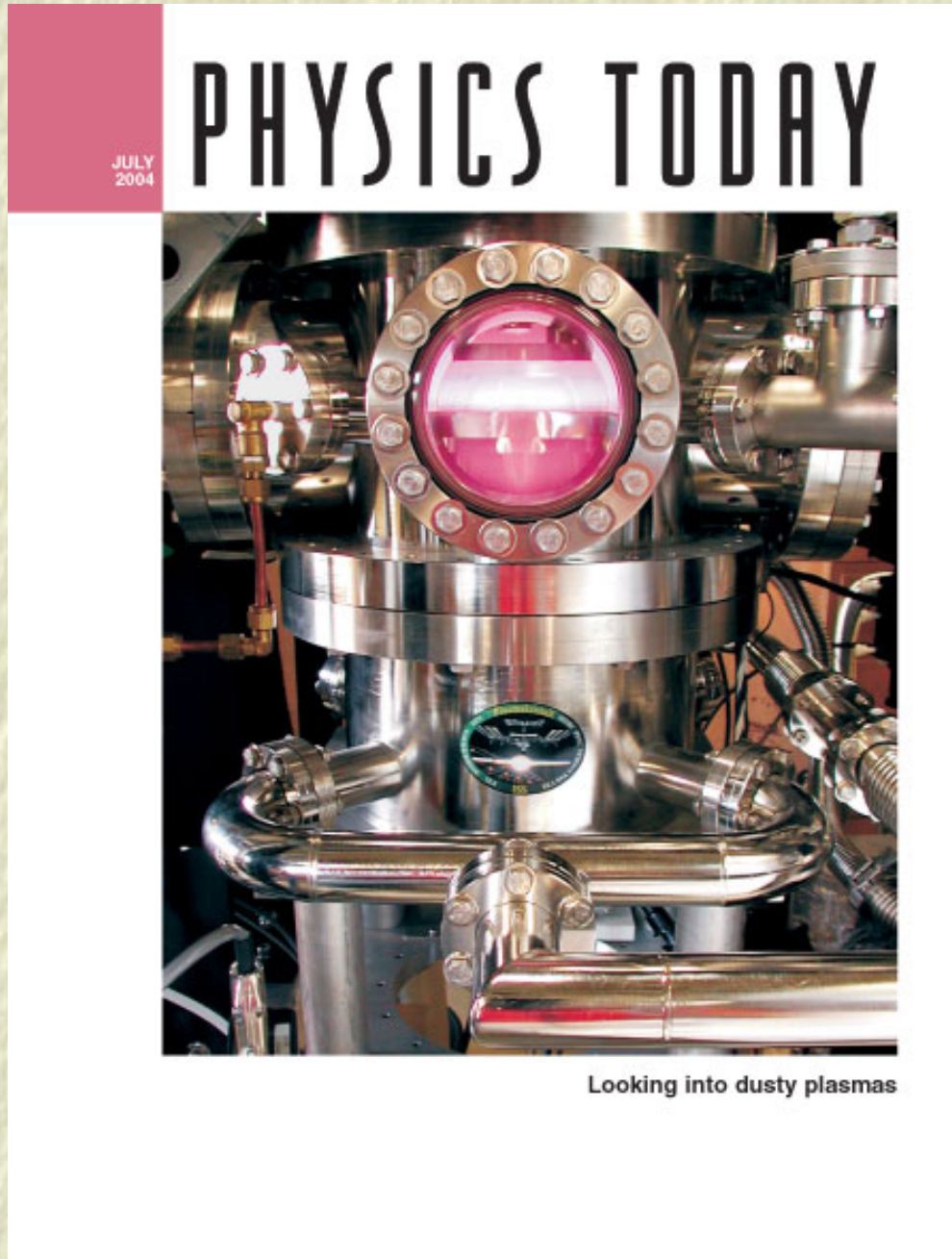
$kT_e \sim 2\text{-}4$  eV (electrons)  
 $kT_i \sim 0.03$  eV (ions)  
 $kT_p \sim 0.025$  eV (microparticles)  
in crystalline state

[Source: H. Thomas, A. Melzer *et al.*, PRL 1994].



- Today, various experimental groups active worldwide:  
*G E Morfill* (MPleP Garching, Germany), *A Piel* (Kiel, Germany), *A Melzer* (Greifswald, Germany), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *S Vladimirov* (Sydney, AUS), *S Takamura* (Nagoya, Japan) ...
- Experiments aboard the *International Space Station (ISS)*;
- Mesoscopic analog of micro-structures; research focus:
  - phase transitions, crystallization processes,
  - relaxation times, diffusion effects,
  - phase space distribution (visually observable!),
  - L & NL waves: *harmonic generation, solitons, vortices, ...*
- 3D, 2D (hexagonal, mostly), 1d lattice configurations possible  
*cf. Talk by D. Samsonov.*





Looking into dusty plasmas



## Focus on 1d DP crystals: Model Hamiltonian

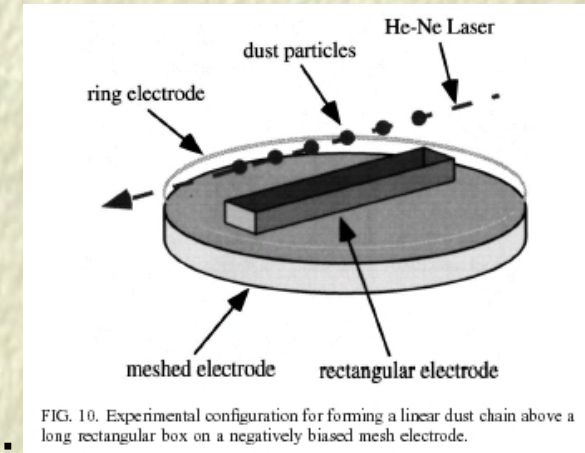
$$H = \sum_n \frac{1}{2} M \left( \frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$

Terms include:

- *Kinetic energy*;
- $\Phi_{ext}(\mathbf{r}_n)$  accounts for '*external*' force fields: may account for *confinement potentials* and/or *sheath electric* forces, i.e.  $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$ .
- Coupling:  $U_{int}(r_{nm})$  is the *interaction potential energy*;

Q.: **Nonlinearity: Origin: where from ?**

**Effect: which consequence(s) ?**





## Nonlinearity in 1d DPCs: Where does it come from? (1)

→ *Sheath environment (anharmonic vertical potential):*

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2}M\omega_g^2(\delta z_n)^2 + \frac{1}{3}M\alpha(\delta z_n)^3 + \frac{1}{4}M\beta(\delta z_n)^4 + \dots$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z(0)$ ;  $\alpha, \beta, \omega_g$  are defined experimentally

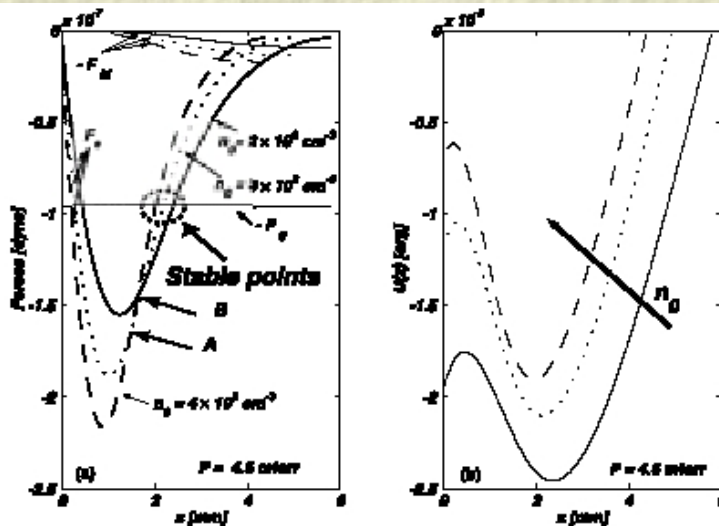


Figure 3: (a) Forces and (b) trapping potential profiles  $U(z)$  as function of distance from the electrode for:  $n_0 = 2 \times 10^8 \text{ cm}^{-3}$  (solid line),  $n_0 = 3 \times 10^8 \text{ cm}^{-3}$  (dashed line),  $n_0 = 4 \times 10^8 \text{ cm}^{-3}$  (dotted line). The parameters are:  $P = 4.6 \text{ mtorr}$ ,  $T_e = 1 \text{ eV}$ ,  $T_i = T_n = 0.05 \text{ eV}$ ,  $R = 2.5 \text{ } \mu\text{m}$ ,  $\rho_d = 1.5 \text{ g cm}^{-3}$ ,  $\phi_w = 6 \text{ V}$ .

Source: Sorasio *et al.* (2002).



## Nonlinearity: Where from? (2)

→ *Interactions between grains*: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened:  $r_0/\lambda_D \approx 1$ ), *anharmonic*; typically:  $U_{Debye}(r) = \frac{q^2}{r} \exp(-r/\lambda_D)$ .

Expanding  $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$  near equilibrium:

$$\Delta x_{nm} = x_n - x_{n-m} \approx mr_0, \quad \Delta z_{nm} = z_n - z_{n-m} \approx 0,$$

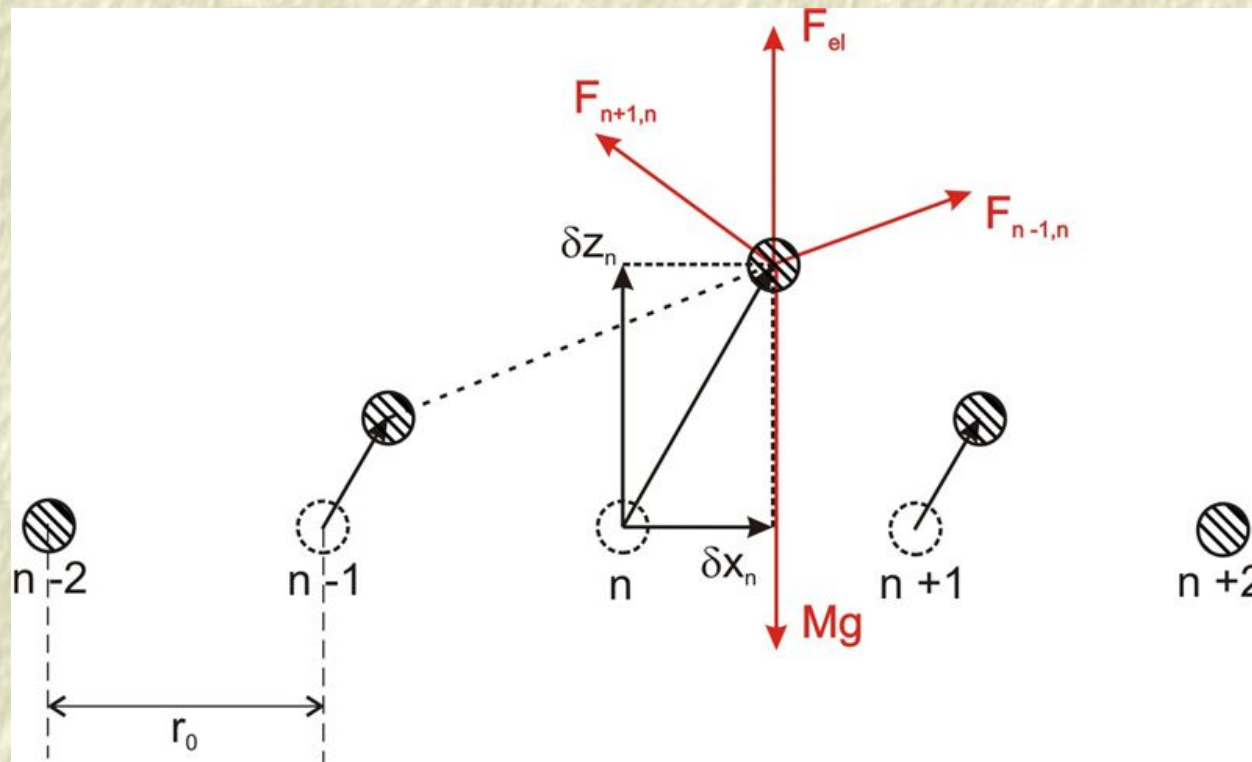
one obtains:

$$\begin{aligned} U_{nm}(r) \approx & \frac{1}{2}M\omega_{L,0}^2(\Delta x_{nm})^2 + \frac{1}{2}M\omega_{T,0}^2(\Delta z_{nm})^2 \\ & + \frac{1}{3}u_{30}(\Delta x_{nm})^3 + \frac{1}{4}u_{40}(\Delta x_{nm})^4 + \dots + \frac{1}{4}u_{04}(\Delta z_{nm})^4 + \\ & + \frac{1}{2}u_{12}(\Delta x_{nm})(\Delta z_{nm})^2 + \frac{1}{4}u_{22}(\Delta x_{nm})^2(\Delta z_{nm})^2 + \dots \end{aligned}$$



## Nonlinearity: Where from? (3)

→ *Coupling* among degrees of freedom induces nonlinearity: anisotropic motion, *not* confined along principal axes ( $\sim \hat{x}, \hat{z}$ ).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]



## Discrete coupled equations of motion

$$\begin{aligned}
 \frac{d^2(\delta x_n)}{dt^2} &= \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n) \\
 &- a_{20} \left[ (\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right] + a_{30} \left[ (\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right] \\
 &\quad + a_{02} \left[ (\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2 \right] \\
 &- a_{12} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2 \right], \\
 \frac{d^2(\delta z_n)}{dt^2} &= \omega_{0,T}^2 (2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2 \delta z_n \\
 &- K_1 (\delta z_n)^2 - K_2 (\delta z_n)^3 + \frac{a_{02}}{r_0} \left[ (\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3 \right] \\
 &+ 2 a_{02} \left[ (\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1}) \right] \\
 &- a_{12} \left[ (\delta x_{n+1} - \delta x_n)^2 (\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2 (\delta z_n - \delta z_{n-1}) \right].
 \end{aligned}$$



## Continuum coupled equations of motion

$$\ddot{u} - c_L^2 u_{xx} - \frac{c_L^2}{12} r_0^2 u_{xxxx} =$$

$$- 2 a_{20} r_0^3 u_x u_{xx} + 3 a_{30} r_0^4 (u_x)^2 u_{xx}$$

$$- a_{12} r_0^4 [(w_x)^2 u_{xx} + 2 w_x w_{xx} u_x] + 2 a_{02} r_0^3 w_x w_{xx},$$

$$\ddot{w} + c_T^2 w_{xx} + \frac{c_T^2}{12} r_0^2 w_{xxxx} + \omega_g^2 w =$$

$$- K_1 w^2 - K_2 w^3 + 3 a_{02} r_0^3 (w_x)^2 w_{xx}$$

$$+ 2 a_{02} r_0^3 (u_x w_{xx} + w_x u_{xx}) - a_{12} r_0^4 [(u_x)^2 w_{xx} + 2 u_x u_{xx} w_x],$$



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## 2. Transverse DP-lattice excitations

The vertical  $n$ -th grain displacement  $\delta z_n = z_n - z_{(0)}$  obeys

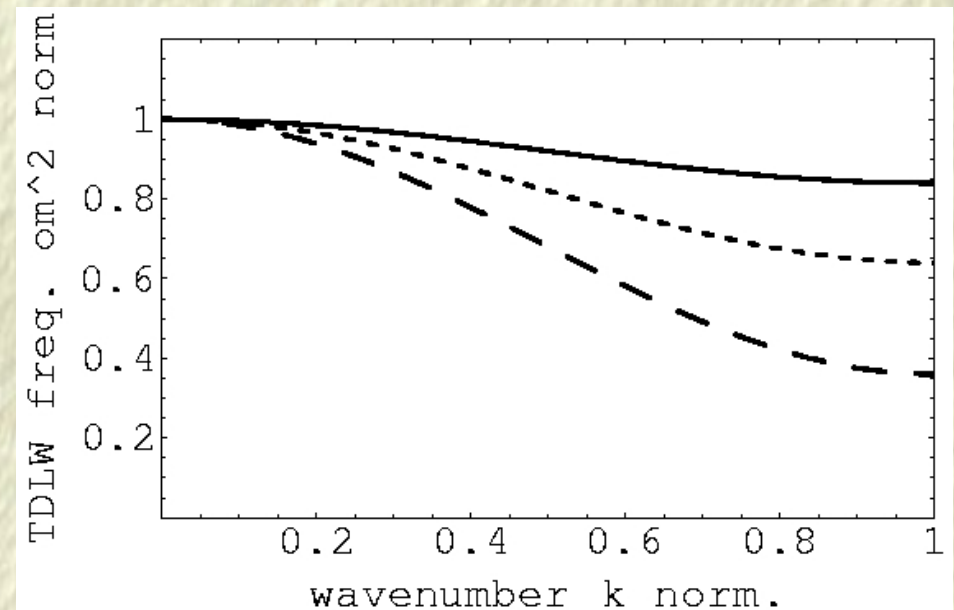
$$\frac{d^2(\delta z_n)}{dt^2} + \omega_{T,0}^2 (\delta z_{n+1} + \delta z_{n-1} - 2\delta z_n) + \omega_g^2 \delta z_n = 0$$

\*  $\omega_{T,0} = [-qU'(r_0)/(Mr_0)]^{1/2} = \omega_{DL}^2 \exp(-\kappa) (1 + \kappa)/\kappa^3$  (†)

\*  $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$ ;  $\lambda_D$  is the *Debye length*;

\* *Optical* dispersion relation  
(backward wave,  $v_g < 0$ ):

$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(kr_0/2)$$





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$$* \omega_{T,0} = \left[ -qU'(r_0)/(Mr_0) \right]^{1/2} = \omega_{DL}^2 \exp(-\kappa) (1 + \kappa)/\kappa^3 \quad (\dagger)$$

$$* \omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}; \quad \lambda_D \text{ is the Debye length};$$

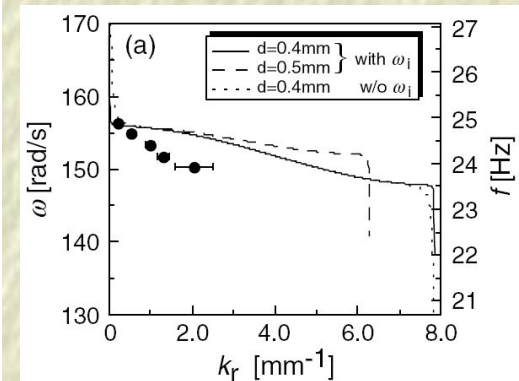
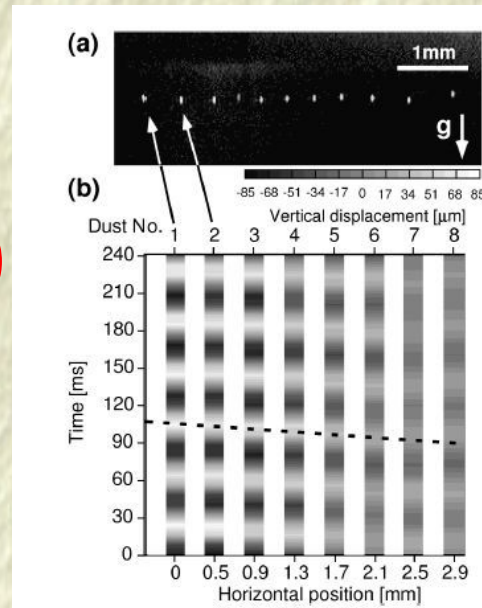
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$$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(kr_0/2)$$

$\dagger$  Cf. experiment:

T. Misawa *et al.*, *PRL* **86**, 1219 (2001)

(Nagoya, Japan).





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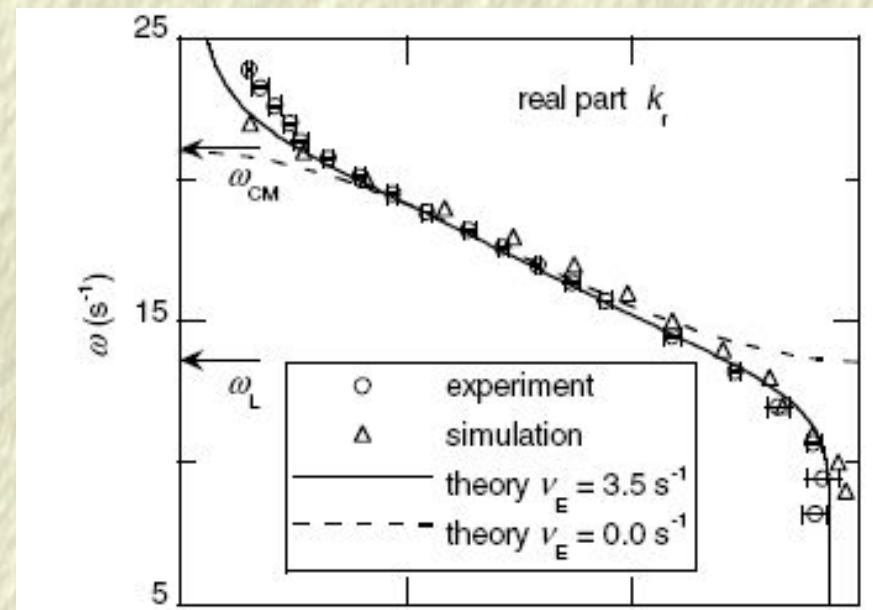
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B. Liu *et al.*, *PRL* **91**, 255003 (2003)

(Iowa, USA).

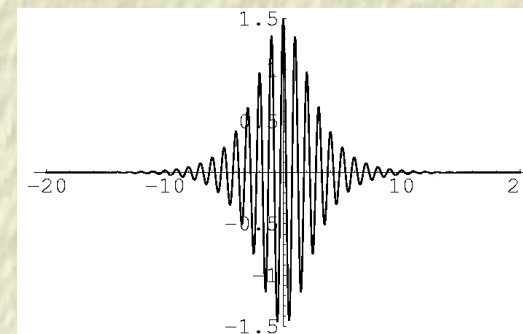
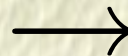
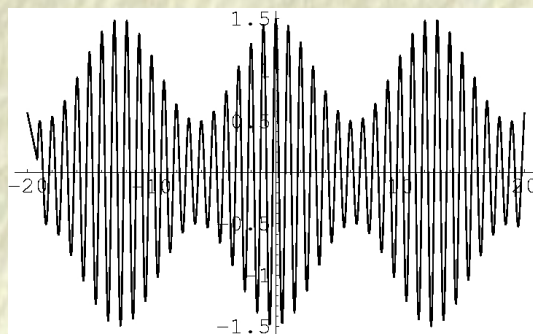
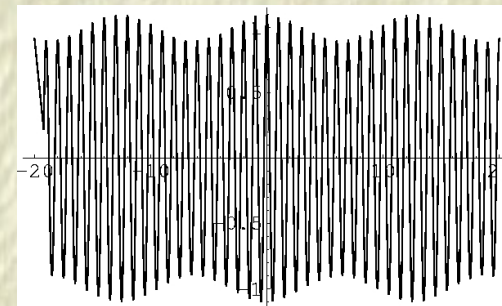
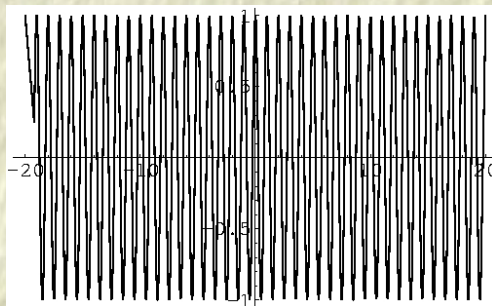




## What if *nonlinearity* is taken into account?

$$\frac{d^2 \delta z_n}{dt^2} + \omega_{T,0}^2 (\delta z_{n+1} + \delta z_{n-1} - 2 \delta z_n) + \omega_g^2 \delta z_n + \alpha (\delta z_n)^2 + \beta (\delta z_n)^3 = 0.$$

\* *Intermezzo: NL wave amplitude modulation → localisation!*





## Large amplitude oscillations - envelope structures

A reductive perturbation (multiple scale) technique, viz.

$t \rightarrow \{t_0, t_1 = \epsilon t, t_2 = \epsilon^2 t, \dots\}$ ,  $x \rightarrow \{x_0, x_1 = \epsilon x, x_2 = \epsilon^2 x, \dots\}$   
yields ( $\epsilon \ll 1$ ; damping omitted):

$$\delta z_n \approx \epsilon (A e^{i\phi_n} + \text{c.c.}) + \epsilon^2 \alpha \left[ -\frac{2|A|^2}{\omega_g^2} + \left( \frac{A^2}{3\omega_g^2} e^{2i\phi_n} + \text{c.c.} \right) \right] + \dots$$

( $\phi_n = nkr_0 - \omega t$ ); the harmonic amplitude  $A(X, T)$ :

- depends on the *slow* variables  $\{X, T\} = \{\epsilon(x - v_g t), \epsilon^2 t\}$  ;
- obeys the ***nonlinear Schrödinger equation*** (NLSE):

$$i \frac{\partial A}{\partial T} + P \frac{\partial^2 A}{\partial X^2} + Q |A|^2 A = 0, \quad (1)$$

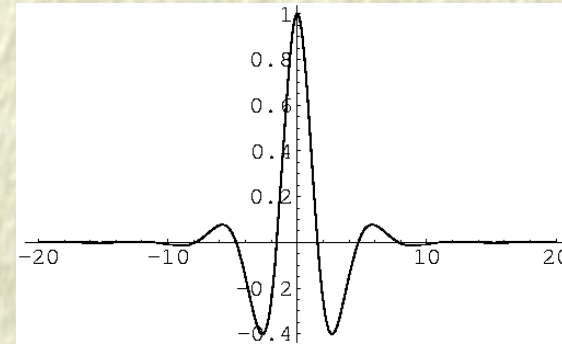
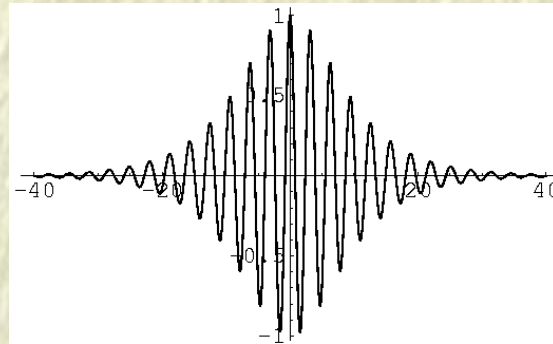
- ***Dispersion coefficient***:  $P = \omega''(k)/2 \rightarrow$  see dispersion relation;
- ***Nonlinearity coefficient***:  $Q = [10\alpha^2/(3\omega_g^2) - 3\beta]/2\omega$ .

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); **11**, 3665 (2004).]



## Modulational stability analysis & envelope structures

- $PQ > 0$ : Modulational instability of the carrier, *bright solitons*:



→ *TDLWs*: possible for *short* wavelengths i.e.  $k_{cr} < k < \pi/r_0$ .

Rem.:  $Q > 0$  for *all* known experimental values of  $\alpha$ ,  $\beta$ .

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)]

Source: G. Sorasio *et al.* (2002).

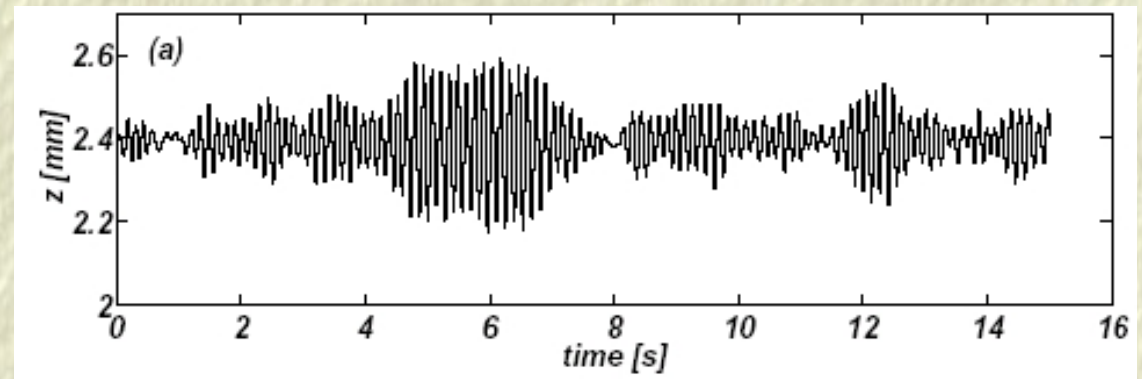
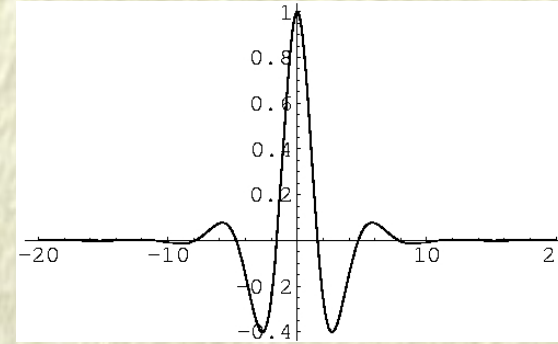
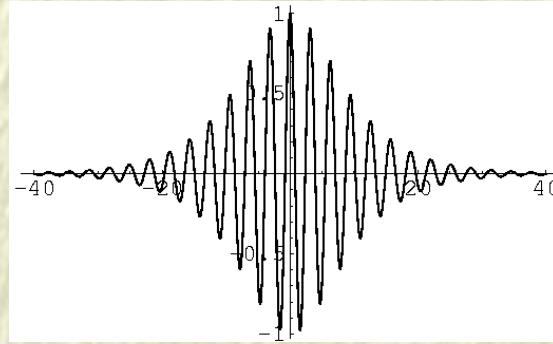


Figure 9: Dust grain oscillations induced by a 1% fluctuation in plasma density. The simulation parameters are:  $P = 0.9$  mtorr,  $n_0 = 0.8 \times 10^8$   $cm^{-3}$ ,  $T_e = 1$  eV,  $T_i = T_n = 0.05$  eV,  $R = 2.5$   $\mu m$ ,  $\rho_d = 1.5$   $g\ cm^{-3}$ ,  $\phi_w = 6$  V,  $\zeta_t = 0.06$ ,  $\zeta_p = 1\%n_0$



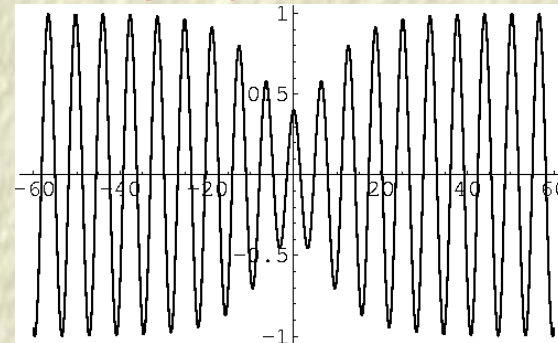
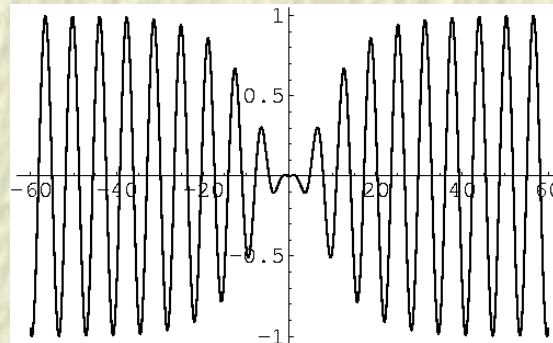
## Modulational stability analysis & envelope structures

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→ *TDLWs*: possible for *short* wavelengths i.e.  $k_{cr} < k < \pi/r_0$ .

- $PQ < 0$ : Carrier wave is *stable*, *dark/grey solitons*:



→ *TDLWs*: possible for *long* wavelengths i.e.  $k < k_{cr}$ .

Rem.:  $Q > 0$  for *all* known experimental values of  $\alpha$ ,  $\beta$

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### 3a. (*nonlinear*) *longitudinal excitations.*

The *nonlinear* equation of longitudinal motion reads:

$$\begin{aligned} \frac{d^2(\delta x_n)}{dt^2} = & \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n) \\ & - a_{20} [(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2] \\ & + a_{30} [(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3] \end{aligned}$$

- $\delta x_n = x_n - nr_0$ : longitudinal dust grain displacements
- Cf. *Fermi-Pasta-Ulam (FPU) problem*:  
anharmonic spring chain model.



## Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$$\delta x_n \approx \epsilon \left[ u_0^{(1)} + (u_1^{(1)} e^{i\phi_n} + \text{c.c.}) \right] + \epsilon^2 (u_2^{(2)} e^{2i\phi_n} + \text{c.c.}) + \dots,$$

[**Harmonic generation**; Cf. experiments: K. Avinash PoP 2004].

where the amplitudes now obey the coupled equations:

$$i \frac{\partial u_1^{(1)}}{\partial T} + P_L \frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0 |u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L} u_1^{(1)} \frac{\partial u_0^{(1)}}{\partial X} = 0,$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = - \frac{p_0 k^2}{v_{g,L}^2 - c_L^2} \frac{\partial}{\partial X} |u_1^{(1)}|^2 \equiv R(k) \frac{\partial}{\partial X} |u_1^{(1)}|^2$$

$$- Q_0 = - \frac{k^2}{2\omega} \left( q_0 k^2 + \frac{2p_0^2}{c_L^2 r_0^2} \right); \quad P_L = \omega_L''(k)/2;$$

$$- v_{g,L} = \omega_L'(k); \quad \{X, T\} \text{ are slow variables};$$

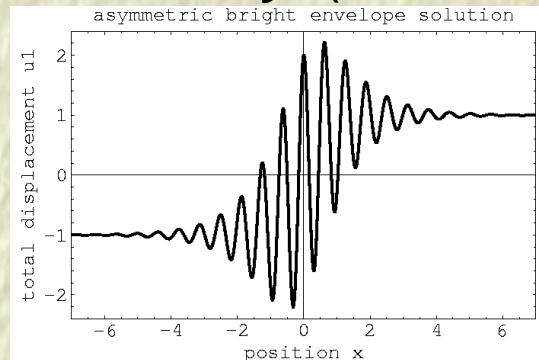
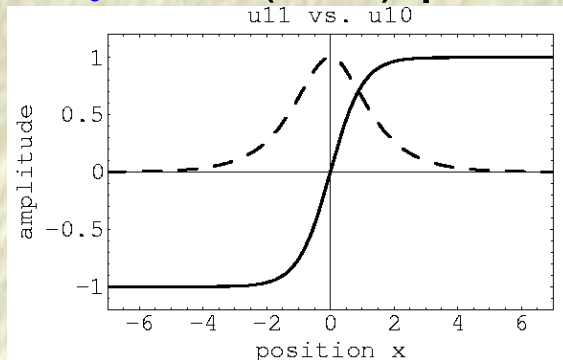
$$- p_0 = -r_0^3 U''''(r_0)/M \equiv 2a_{20} r_0^3, \quad q_0 = U''''(r_0) r_0^4 / (2M) \equiv 3a_{30} r_0^4.$$

$$- R(k) > 0, \text{ since } \forall k \quad v_{g,L} < \omega_{L,0} r_0 \equiv c_L \quad (\text{sound velocity}).$$

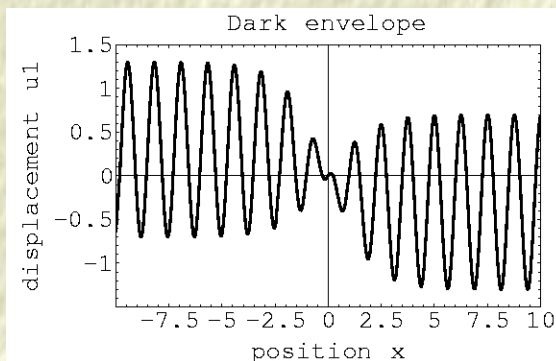
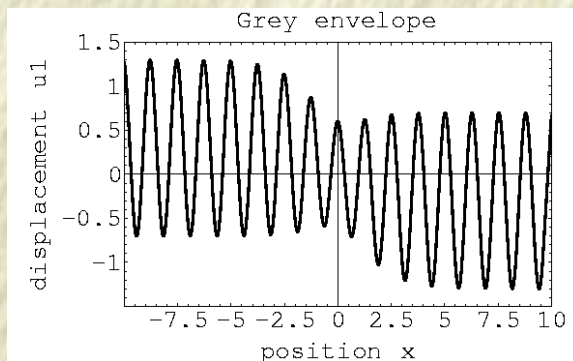


## Asymmetric longitudinal envelope structures.

- The system of Eqs. for  $u_1^{(1)}$ ,  $u_0^{(1)}$  may be solved exactly:  
     → asymmetric envelope solutions.
- $P = P_L = \omega_L''(k)/2 < 0$ ;
- $Q > 0$  ( $< 0$ ) prescribes *stability* (instability) at *low* (high)  $k$ .



(at high  $k$ )



(at low  $k$ )

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).].



### 3b. Longitudinal soliton formalism.

#### A link to soliton theories:

- Continuum approximation, viz.  $\delta x_n(t) \rightarrow u(x, t)$ .
- “Standard” description: keeping lowest order nonlinearity,

$$\ddot{u} + \nu \dot{u} - c_L^2 u_{xx} - \frac{c_L^2}{12} r_0^2 u_{xxxx} = -p_0 u_x u_{xx}$$

$c_L = \omega_{L,0} r_0$ ;  $\omega_{L,0}$  and  $p_0$  were defined above.

- For near-sonic propagation (i.e.  $v \approx c_L$ ), slow profile evolution in time  $\tau$  and defining the relative displacement  $w = u_\zeta$ , one obtains (for  $\nu = 0$ ) the Korteweg-deVries Equation:

$$w_\tau - a w w_\zeta + b w_{\zeta\zeta\zeta} = 0$$

Defs.:  $\zeta = x - vt$ ;  $a = p_0/(2c_L) > 0$ ;  $b = c_L r_0^2/24 > 0$ .



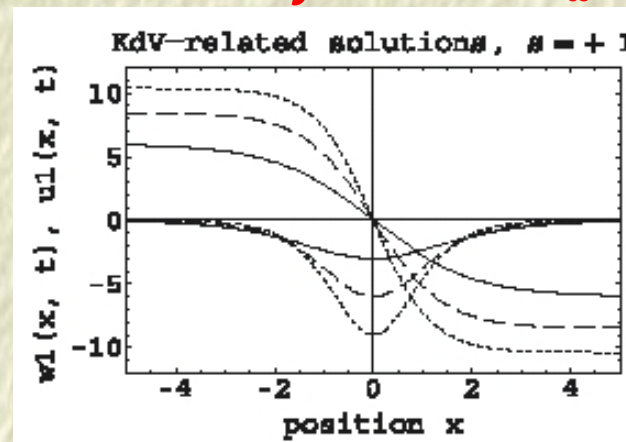
# The Korteweg-deVries (KdV) Equation

$$w_\tau - a w w_\zeta + b w_{\zeta\zeta\zeta} = 0$$

yields **compressive** (*only*, since  $a > 0$ ) solutions, in the form:

$$w_1(\zeta, \tau) = -\frac{3v}{a} \operatorname{sech}^2 \left[ (v/4b)^{1/2} (\zeta - v\tau - \zeta_0) \right]$$

– This solution is a negative pulse for  $w = u_x$ , describing a *compressive* excitation for the *displacement*  $\delta x = u$ , i.e. a localized increase of **density**  $n \sim -u_x$ <sup>†</sup>.



<sup>†</sup> F. Melandsø 1996; S. Zhdanov *et al.* 2002; K. Avinash *et al.* 2003; V. Fortov *et al.* 2004.



## Experimental observation of LDL solitons

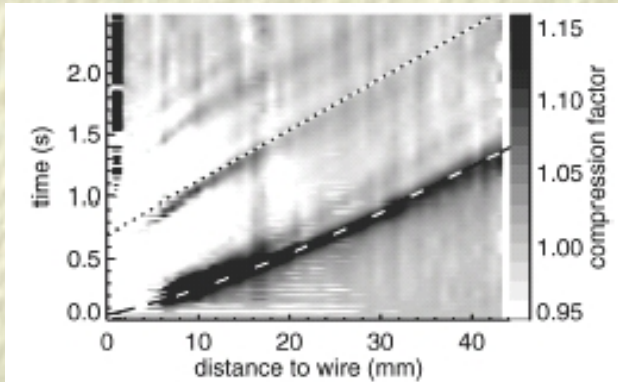


FIG. 2. Compression factor  $n/n_0$  as a function of time and distance to the wire. Darker regions correspond to higher compression. The lower dashed curve is a fit to the trajectory of the soliton. The upper dotted line was drawn above a weak secondary pulse with Mach number  $M \approx 1$ . Its slope is determined by the dust lattice wave speed  $C_{DL} = 23$  mm/s in the middle of the lattice.

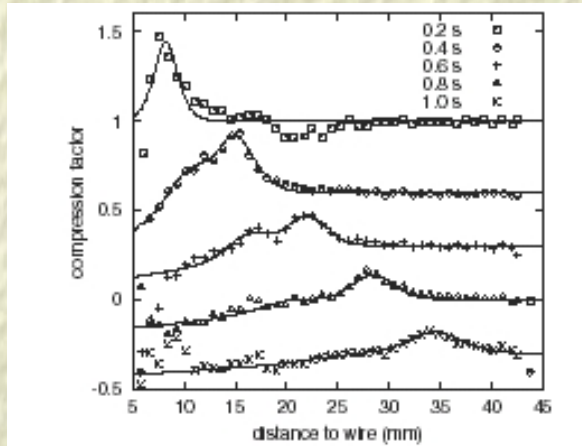


FIG. 3. Compression factor  $n/n_0$  versus distance to the wire at different times. The solid lines show the theoretical fits to the experimental data. Two solitons are present. The fits and experimental points at later times are offset down (by 0.4, 0.7, 1.0, 1.3, respectively).

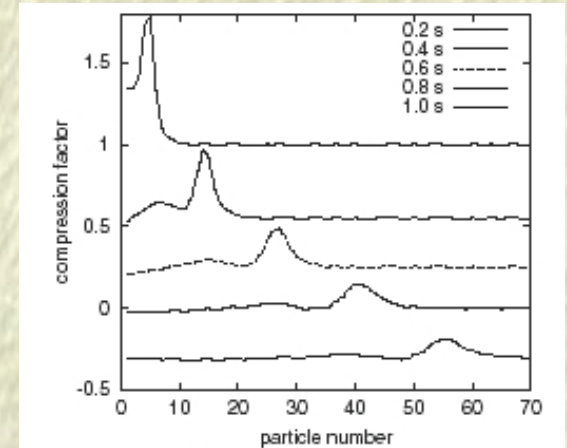


FIG. 5. Compression factor versus particle number for a simulated linear chain model. It describes the formation of two (or more) solitons by a single excitation pulse and qualitatively agrees with the experiment. The curves at later times are offset down (by 0.45, 0.75, 1.0, and 1.3, respectively).

[Samsonov *et al.*, PRL 2002].

- *Only compressive* solitons predicted by KdV theory;
- *Only compressive* solitons experimentally anticipated and, hence, reported;
- What about *rarefactive* longitudinal solitons?

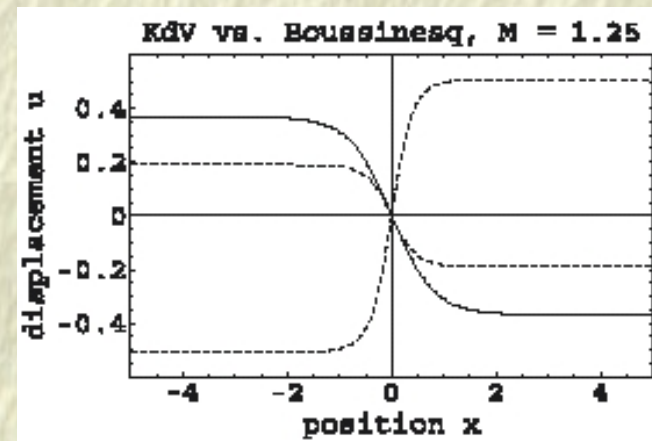
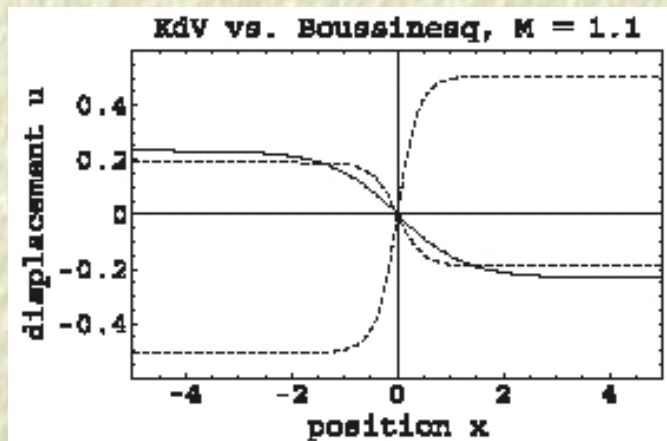


## Extended description: the Boussinesq theory

The *Generalized Boussinesq* (Bq) Equation (for  $w = u_x$ ):

$$\ddot{w} - c_L^2 w_{xx} = \frac{c_L^2 r_0^2}{12} w_{xxxx} - \frac{p_0}{2} (w^2)_{xx} + \frac{q_0}{2} (w^3)_{xx}$$

- predicts *both compressive and rarefactive* excitations;
- reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ... ); *and, ...*
- relaxes the velocity assumption, i.e. is valid  $\forall v > c_L$ .



[from: I Kourakis & P K Shukla, *European Phys. J. D*, **29**, 247 (2004)].



## Menu

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites.
  - (i) Focus: 1d dust crystals in lab.
  - (ii) Nonlinearity in 1d DP crystals: Origin and modelling.
2. *Transverse dust-lattice (TDL) excitations*: amplitude modulation, transverse envelope structures.
3. *Longitudinal dust-lattice (LDL) excitations*: amplitude modulation, envelope structures, solitons.
4. *1d Discrete Breather excitations (intrinsic localized modes)*
5. Conclusions.



## 4. Transverse Discrete Breathers (DBs)

- Eq. of motion in the *transverse* direction:

$$\frac{d^2 u_n}{dt^2} + \omega_{T,0}^2 (u_{n+1} + u_{n-1} - 2u_n) + \omega_g^2 u_n + \alpha u_n^2 + \beta u_n^3 = 0$$

- Damping may be neglected (for low plasma density and/or pressure):  $\nu/\omega_g \simeq 0.00154$  ([Misawa et al., PRL 2001]).
- 1d DPCs are *highly discrete* lattice configurations:  
 $\epsilon = \omega_0^2/\omega_g^2 \simeq 0.016$  ([Misawa et al., 2001]);  $\epsilon \simeq 0.181$  ([Liu et al., 2003]).
- One may seek *discrete breather* solutions (localized modes):

$$u_n(t) = \sum_m A_n(m) \exp(im\omega t)$$

where only few ( $m \simeq 1 - 3$ ) sites are excited.

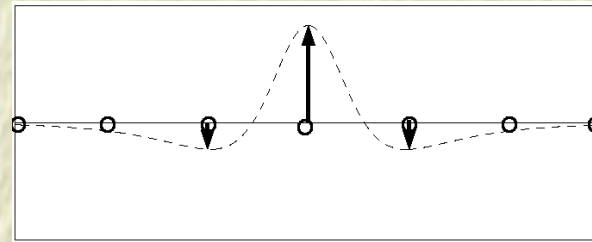
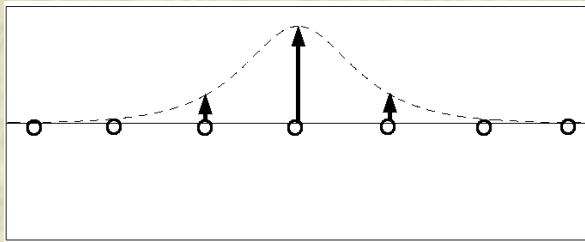


## 4. Transverse Discrete Breathers (DBs) (*cont.*)

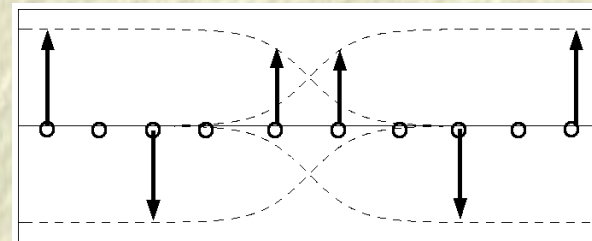
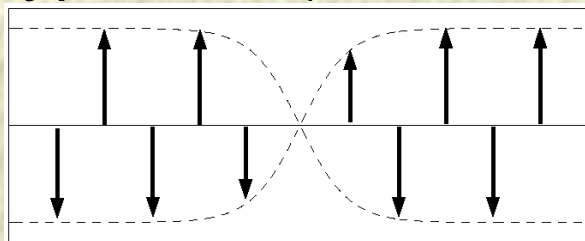
- \* The well-known *non - resonance condition* is recovered:

$$m\omega_B \neq \omega_T(k) \quad \forall k, m = 1, 2, \dots$$

- \* “*Bright-type*” few-site DB solutions (localized pulses):



- \* +: *Dark-type* DBs (holes; *dark* modes [Yu. Kivshar, *PLA* **173**, 172 (1993)]):



- \* Existence & stability profile established:

[V Koukoulouyannis & I Kourakis, *subm. PRE*; [arxiv.org/pdf/nlin.PS/0703020](http://arxiv.org/pdf/nlin.PS/0703020)].

- \* **DB modes may also occur in the  $x$ – direction ( $\sim$  FPU).**



## Conclusions - State of Art

- *Dust crystals* provide an excellent test-field for NL theories;
- *Observations are possible at the kinetic level*: unique possibility for physical data processing & real-time analysis;
- *Technology for experiment: cheap and readily available*;
- Link between *Plasma Phys.*, *Solid State Physics*, *Stat. Mech.*;
- *Theory (1d): Envelope solitons, (non-topological) solitons, Discrete Breathers*: predicted;
- *Theory (2d): Discrete Breathers, vortices, ...*: predicted;
- *Experiment*: Harmonic generation, compressive solitons, NL TDL oscillations, backward wave: observed (*Urge for more :-)*);
- Future scope: *dissipation, mode coupling, ... “(Realism!)”*.



## Overview of existing results

1. **1d: *Transverse dust-lattice (TDL) motion*** ( $\sim$  NL KG, inv. disp.):
  - Envelope (NLS) solitons [IK & P K Shukla, *Phys. Plasmas* **11**, 1384 (2004)]
  - DBs (ILMs) [V. Koukoulouyannis & IK, *PRE* **76**, 016402 (2007)]
2. **1d: *Longitudinal dust-lattice (LDL) motion*** ( $\sim$  FPU):
  - *Asymmetric* envelope structures (coupled 0th/1st harmonics) [IK & P K Shukla, *Phys. Plasmas* **11**, 3665 (2004)]
  - KdV vs. eKdV / Bq solitons [IK & PKS, *Eur. Phys. J. D* **29**, 247 (2004)]  
Rem.: experimentally observed (compressive case only)
3. **2D: *In-plane* (“LDL”) motion in *hexagonal DP crystals***:
  - Envelope structures [Farokhi, IK & PKS, *Phys. Plasmas* **13**, 122304 (2006)]
4. **2D: *Out-of-plane* (TDL) motion in *hexagonal DP crystals***:
  - DBs, vortices (in preparation).



## Future considerations & perspectives

1. *LDL-DBs* ? ( $\sim$  FPU);
2. *Damping* (dissipative system), ion drag, wake potentials, ...
3. *Mixed T-L Mode*: coupled FPU-NLKG Eqs. (ongoing work);
4. *2D hexagonal dust lattices*: vortices ? (seen experimentally);
5. *Experimental feedback*:
  - establish & pursue contacts,
  - seek confirmation of results, motivate experiments ...

→ *A lot remaining to be done!*



**Thank You !**

Ioannis Kourakis

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**V. Basios** (ULB, Brussels, Belgium), **T Bountis** (Patras, Greece).

*Material from:*

I Kourakis & P K Shukla, *Phys. Plasmas*, **11**, 2322 (2004);  
*idem*, *Phys. Plasmas*, **11**, 3665 (2004).  
*idem*, *Phys. Plasmas*, **11**, 1384 (2004).  
*idem*, *European Phys. J. D*, **29**, 247 (2004).

V Koukoulouyannis & IK, *PRE* **76**, 016402 (2007).

*Slides available at:* [www.kourakis.eu](http://www.kourakis.eu)