Nonlinear excitations in dusty plasma (Debye) crystals:

What plasma physics can learn from nonlinear lattice theories

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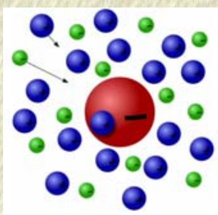
www.tp4.rub.de/~ioannis/conf/200804-IoP-oral.pdf

Outline

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites.
   (i) Focus: 1d dust crystals in lab.
   (ii) Nonlinearity in 1d DP crystals: Origin and modelling.
2. *Transverse* dust-lattice (*TDL*) excitations:
   amplitude modulation, transverse envelope structures.
3. *Longitudinal* dust-lattice (*LDL*) excitations:
   amplitude modulation, envelope structures, solitons.
4. 1d *Discrete Breather excitations* (*intrinsic localized modes*)
5. Conclusions.

1. Dusty Plasmas (or *Complex Plasmas*): prerequisites

Dusty Plasmas (DP): 

→ *electrons* e^- (charge $-e$, mass m_e),

→ *ions* i^+ (charge $+Z_i e$, mass m_i),

→ charged particulates ≡ *dust grains* $d^\pm$ (most often d^-):
charge $Q = sZ_d e \sim \pm(10^3 - 10^4)\, e$, $(s = \pm 1)$
mass $M \sim 10^9\, m_p \sim 10^{13}\, m_e$,
radius $r \sim 10^{-2}\, \mu m$ up to $10^2\, \mu$m.

Where/how do dusty plasmas occur?

- *Space: cosmic debris* (silicates, graphite, amorphous carbon), comet tails, man-made pollution (Shuttle exhaust, satellites), ...
- *Earth's atmosphere:* volcanic eruptions, *extraterrestrial* origin (*meteorites*) ($\geq 2 \cdot 10^4$ tons/yr!), *pollution*, *aerosols*, ...;
- *Fusion devices:* plasma-surface interaction in the divertor region (graphite, CFCs), UFOs, ITER safety concern, ...;
- *Technology:* Semiconductor industry, Si microchip, dust contamination, solar cell stabilization ...;
- *Laboratory:* (man-injected) melamine–formaldehyde particulates injected in *rf* or *dc* discharges.

Sources: P. K. Shukla & A. Mamun, book (IoP, 2002), G. E. Morfill *et al.*, 1998, etc.

Dusty Plasma physics: unique mesoscopic features

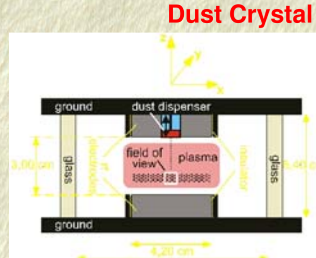
- Studies in *slow motion* are possible due to high M i.e. *low* Q/M *ratio* (e.g. *dust plasma frequency*: $\omega_{p,d} \approx 10 - 100\,\mathrm{Hz}$);
- The (large) microparticles can be *visualised* individually and studied at the kinetic level (with a digital camera!);
- Contrary to *weakly-coupled* $e - i$ plasmas ($\Gamma \ll 1$), Complex Plasmas can be *strongly coupled* and exist in *"liquid"* ($1 < \Gamma < 170$) and *"crystalline"* ($\Gamma > 170$ [Ikezi 1986]) states, depending on the value of:

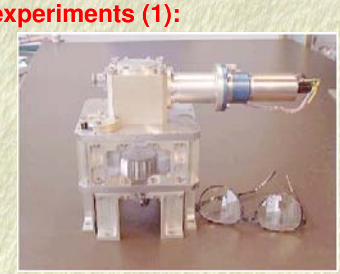
$$\Gamma_{eff} = \frac{< E_{potential} >}{< E_{kinetic} >} \approx \frac{\frac{Q^2}{r}\, e^{-r/\lambda_D}}{k_B T}$$

(r: inter-particle distance, T: temperature, λ_D: Debye length, k_B: Boltzmann's constant).

→ *Dusty Plasma (Debye/Yukawa) Crystals!!! (DPCs)*

Dust Crystal experiments (1):

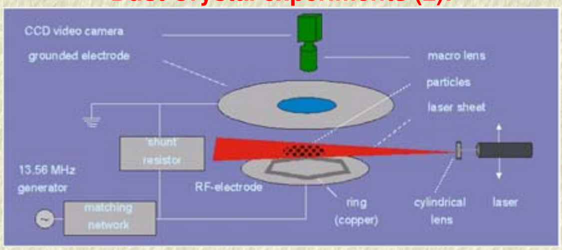




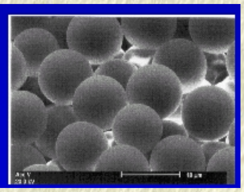
- Theoretical prediction: 1986 [H. Ikezi, Phys. Fluids **29**, 1764 (1986)];
- Experimental realization: 1994

[H. Thomas, A. Melzer *et al.* PRL **73**, 652 (1994); Chu & Lin I J. Phys. D **27** 296 (1994), Hayashi & Tachibana, Jap. J. Appl. Phys. **33** L804 (1994)];

Dust Crystal experiments (2):



Particles:
Melamine-Formaldehyde
diameter: few µm



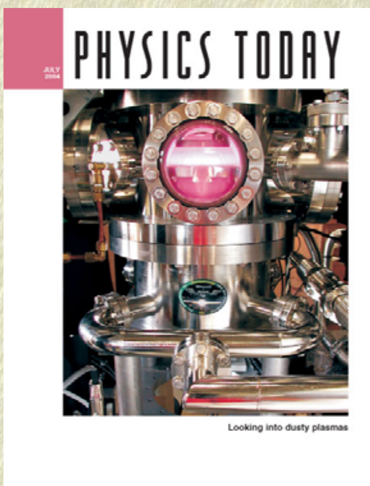
Gas:
noble gas (argon, krypton)
pressure: few Pa … 100 Pa (=1 mbar)

Ionisation fraction: 10^{-6} - 10^{-7}

Temperatures:
kT_e ~2-4 eV (electrons)
kT_i ~0.03 eV (ions)
kT_d ~0.025 eV (microparticles)
in crystalline state

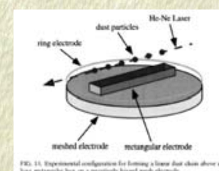
[Source: H. Thomas, A. Melzer *et al.*, PRL 1994].

- Today, various experimental groups active worldwide: *G E Morfill* (MPIeP Garching, Germany), *A Piel* (Kiel, Germany), *A Melzer*(Greifswald, Germany), *J Goree* (Iowa, US), *V Fortov* (Moscow, Russia), *Lin I* (Taiwan), *S Vladimirov* (Sydney, AUS), *S Takamura* (Nagoya, Japan) ...
- Experiments aboard the *International Space Station (ISS)*;
- Mesoscopic analog of micro-structures; research focus:
  – phase transitions, crystallization processes,
  – relaxation times, diffusion effects,
  – phase space distribution (visually observable!),
  – L & NL waves: *harmonic generation, solitons, vortices*, ...
- 3D, 2D (hexagonal, mostly), 1d lattice configurations possible *(→ video)*; *cf. Talk by D. Samsonov.*



Focus on 1d DP crystals: Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt}\right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(\mathbf{r}_n)$$



Terms include:

– *Kinetic energy*;

– $\Phi_{ext}(\mathbf{r}_n)$ accounts for *'external' force fields*: may account for *confinement potentials* and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$.

– Coupling: $U_{int}(r_{nm})$ is the *interaction potential energy*;

Q.: Nonlinearity: *Origin: where from ?*
Effect: which consequence(s) ?

Nonlinearity in 1d DPCs: Where does it come from? (1)

→ *Sheath* environment (*anharmonic* vertical potential):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2} M \omega_g^2 (\delta z_n)^2 + \frac{1}{3} M \alpha \, (\delta z_n)^3 + \frac{1}{4} M \beta \, (\delta z_n)^4 + \ldots$$

cf. experiments [Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)];

$\delta z_n = z_n - z_{(0)}$; α, β, ω_g are defined experimentally

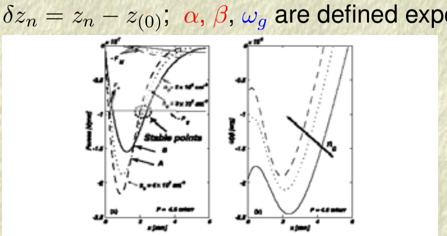


Figure 3: (a) Forces and (b) trapping potential profiles $U(z)$ as function of distance from the electrode for: $n_0 = 2\times10^8 cm^{-3}$ (solid line), $n_0 = 3\times10^8 cm^{-3}$ (dashed line), $n_0 = 4\times10^8 cm^{-3}$ (dotted line). The parameters are: P = 4.6 mtorr, $T_e = 1$ eV, $T_i = T_n = 0.05$ eV, $R = 2.5\ \mu m$, $\rho_d = 1.5\ g\,cm^{-3}$, $\phi_w = 6$ V.

Source: Sorasio *et al.* (2002).

Nonlinearity: Where from? (2)

→ *Interactions* between grains: Electrostatic character (e.g. repulsive, Debye), long-range (yet charge screened: $r_0/\lambda_D \approx 1$), *anharmonic*; typically: $U_{Debye}(r) = \frac{q^2}{r} \exp{(-r/\lambda_D)}$.

Expanding $U_{int}(r_{nm}) = U_{int}(\sqrt{(\Delta x_{nm})^2 + (\Delta z_{nm})^2})$ near equilibrium:

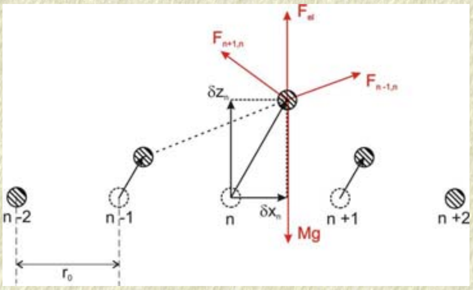
$\Delta x_{nm} = x_n - x_{n-m} \approx m r_0$, $\Delta z_{nm} = z_n - z_{n-m} \approx 0$,

one obtains:

$$U_{nm}(r) \approx \frac{1}{2} M \omega_{L,0}^2 (\Delta x_{nm})^2 + \frac{1}{2} M \omega_{T,0}^2 (\Delta z_{nm})^2 + \frac{1}{3} u_{30} (\Delta x_{nm})^3 + \frac{1}{4} u_{40} (\Delta x_{nm})^4 + \ldots + \frac{1}{4} u_{04} (\Delta z_{nm})^4 + \frac{1}{2} u_{12} (\Delta x_{nm})(\Delta z_{nm})^2 + \frac{1}{4} u_{22} (\Delta x_{nm})^2 (\Delta z_{nm})^2 + \ldots$$

Nonlinearity: Where from? (3)

→ *Coupling* among degrees of freedom induces nonlinearity: anisotropic motion, *not* confined along principal axes ($\sim \hat{x}$, $\hat{z}$).



[cf. A. Ivlev *et al.*, PRE **68**, 066402 (2003); I. Kourakis & P. K. Shukla, Phys. Scr. (2004)]

Discrete coupled equations of motion

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2\,(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$-a_{20}\left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right] + a_{30}\left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]$$
$$+ a_{02}\left[(\delta z_{n+1} - \delta z_n)^2 - (\delta z_n - \delta z_{n-1})^2\right]$$
$$-a_{12}\left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n)^2 - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})^2\right],$$

$$\frac{d^2(\delta z_n)}{dt^2} = \omega_{0,T}^2\,(2\delta z_n - \delta z_{n+1} - \delta z_{n-1}) - \omega_g^2\,\delta z_n$$
$$- K_1\,(\delta z_n)^2 - K_2\,(\delta z_n)^3 + \frac{a_{02}}{r_0}\left[(\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3\right]$$
$$+ 2\,a_{02}\left[(\delta x_{n+1} - \delta x_n)(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})(\delta z_n - \delta z_{n-1})\right]$$
$$- a_{12}\left[(\delta x_{n+1} - \delta x_n)^2(\delta z_{n+1} - \delta z_n) - (\delta x_n - \delta x_{n-1})^2(\delta z_n - \delta z_{n-1})\right].$$

Continuum coupled equations of motion

$$\ddot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12} r_0^2\,u_{xxxx} =$$
$$-2\,a_{20}\,r_0^3\,u_x\,u_{xx} + 3\,a_{30}\,r_0^4\,(u_x)^2\,u_{xx}$$
$$-a_{12}\,r_0^4\,[(w_x)^2\,u_{xx} + 2 w_x w_{xx} u_x] + 2\,a_{02}\,r_0^3\,w_x\,w_{xx}\,,$$

$$\ddot{w} + c_T^2\,w_{xx} + \frac{c_T^2}{12} r_0^2\,w_{xxxx} + \omega_g^2\,w =$$
$$-K_1\,w^2 - K_2\,w^3 + 3\,a_{02}\,r_0^3\,(w_x)^2\,w_{xx}$$
$$+2\,a_{02}\,r_0^3\,(u_x\,w_{xx} + w_x\,u_{xx}) - a_{12}\,r_0^4\,[(u_x)^2\,w_{xx} + 2 u_x u_{xx} w_x]\,,$$

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2. *Transverse* DP-lattice excitations

The vertical $n-$th grain displacement $\delta z_n = z_n - z_{(0)}$ obeys

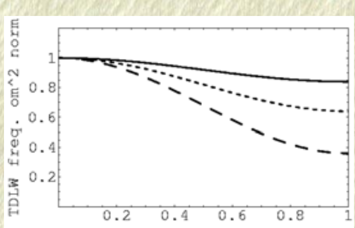
$$\frac{d^2(\delta z_n)}{dt^2} + \omega_{T,0}^2\,(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n = 0$$

* $\omega_{T,0} = \left[-qU'(r_0)/(Mr_0)\right]^{1/2} = \omega_{DL}^2\,\exp(-\kappa)\,(1+\kappa)/\kappa^3$ (†)

* $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$; λ_D is the *Debye length*;

* *Optical* dispersion relation (*backward* wave, $v_g < 0$):

$\omega^2 = \omega_g^2 - 4\omega_{T,0}^2\,\sin^2(kr_0/2)$



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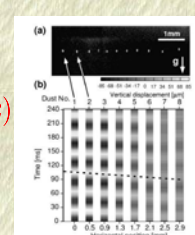

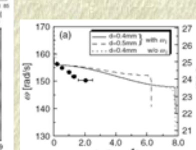

† Cf. experiment:
T. Misawa *et al.*, *PRL* **86**, 1219 (2001)
(Nagoya, Japan).

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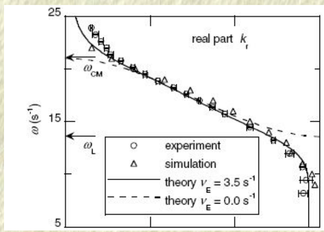
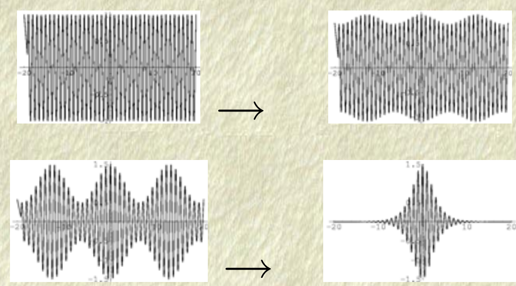


† Cf. experiment: B. Liu *et al.*, *PRL* **91**, 255003 (2003) (Iowa, USA).

What if *nonlinearity* is taken into account?

$$\frac{d^2\delta z_n}{dt^2} + \omega_{T,0}^2 \left(\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n \right) + \omega_g^2 \,\delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,.$$

* *Intermezzo: NL wave amplitude modulation → localisation!*



Large amplitude oscillations - envelope structures

A reductive perturbation (multiple scale) technique, viz.
$t \to \{t_0, t_1 = \epsilon t, t_2 = \epsilon^2 t, \dots\}$, $x \to \{x_0, x_1 = \epsilon x, x_2 = \epsilon^2 x, \dots\}$
yields ($\epsilon \ll 1$; damping omitted):

$$\delta z_n \approx \epsilon\,(A\,e^{i\phi_n} + \mathrm{c.c.}) + \epsilon^2\,\alpha \left[-\frac{2|A|^2}{\omega_g^2} + \left(\frac{A^2}{3\omega_g^2} e^{2i\phi_n} + \mathrm{c.c.} \right) \right] + \dots$$

($\phi_n = nkr_0 - \omega t$); the harmonic amplitude $A(X,T)$:
– depends on the *slow* variables $\{X, T\} = \{\epsilon(x - v_g t), \epsilon^2 t\}$;
– obeys the *nonlinear Schrödinger equation* (NLSE):

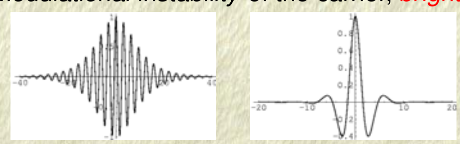
$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2\,A = 0\,, \qquad (1)$$

– *Dispersion coefficient*: $P = \omega''(k)/2$ → see dispersion relation;
– *Nonlinearity coefficient*: $Q = \left[10\alpha^2/(3\omega_g^2) - 3\,\beta\right]/2\omega$.

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); **11**, 3665 (2004).]

Modulational stability analysis & envelope structures

- $PQ > 0$: *Modulational instability* of the carrier, *bright solitons*:

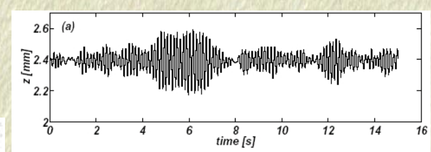


→ *TDLWs*: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

Rem.: $Q > 0$ for *all* known experimental values of α, β.

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)]

Source: G. Sorasio *et al.* (2002).



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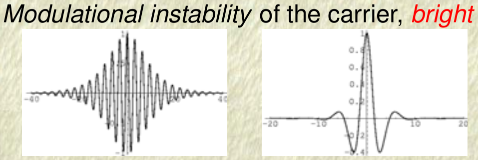
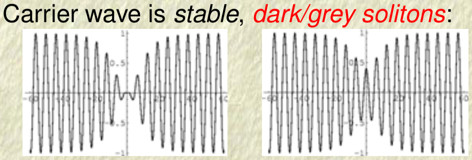


→ *TDLWs*: possible for *short* wavelengths i.e. $k_{cr} < k < \pi/r_0$.

- $PQ < 0$: Carrier wave is *stable*, *dark/grey solitons*:



→ *TDLWs*: possible for *long* wavelengths i.e. $k < k_{cr}$.

Rem.: $Q > 0$ for *all* known experimental values of α, β

[Ivlev *et al.*, PRL **85**, 4060 (2000); Zafiu *et al.*, PRE **63** 066403 (2001)].

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3a. (*nonlinear*) *longitudinal* excitations.

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2\,(\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$-a_{20}\left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right]$$
$$+\,a_{30}\left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right]$$

– $\delta x_n = x_n - n r_0$: longitudinal dust grain displacements

– Cf. *Fermi-Pasta-Ulam (FPU) problem*:
anharmonic spring chain model.

Longitudinal Dust-Lattice wave (LDLW) modulation

The reductive perturbation technique (cf. above) now yields:

$\delta x_n \approx \epsilon\left[u_0^{(1)} + (u_1^{(1)}\, e^{i\phi_n} + \mathrm{c.c.})\right] + \epsilon^2\,(u_2^{(2)}\, e^{2i\phi_n} + \mathrm{c.c.}) + \ldots,$

[Harmonic generation; Cf. experiments: K. Avinash PoP 2004].

where the amplitudes now obey the coupled equations:

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L\frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0\,|u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L}\,u_1^{(1)}\frac{\partial u_0^{(1)}}{\partial X} = 0\,,$$
$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - c_L^2}\frac{\partial}{\partial X}\,|u_1^{(1)}|^2 \equiv R(k)\frac{\partial}{\partial X}\,|u_1^{(1)}|^2$$

– $Q_0 = -\frac{k^2}{2\omega}\left(q_0\,k^2 + \frac{2p_0^2}{c_L^2 r_0^2}\right)$; $P_L = \omega_L''(k)/2$;

– $v_{g,L} = \omega_L{}'(k)$; $\{X, T\}$ are *slow* variables;

– $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$, $q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$.

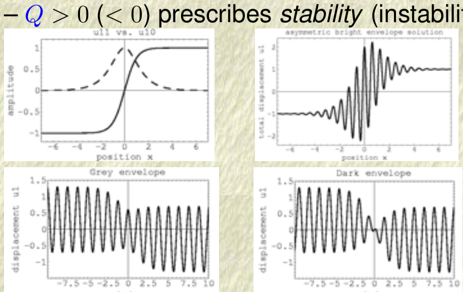
– $R(k) > 0$, since $\forall\, k$ $v_{g,L} < \omega_{L,0}\, r_0 \equiv c_L$ *(sound velocity)*.

Asymmetric *longitudinal* envelope structures.

– The system of Eqs. for $u_1^{(1)}$, $u_0^{(1)}$ may be solved exactly:
→ *asymmetric* envelope solutions.

– $P = P_L = \omega_L''(k)/2 < 0$;

– $Q > 0$ (< 0) prescribes *stability* (instability) at *low* (high) k.



(at high k)

(at low k)

[I. Kourakis & P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).].

3b. *Longitudinal* soliton formalism.

A link to soliton theories:

– *Continuum approximation*, viz. $\delta x_n(t) \to u(x,t)$.

– *"Standard"* description: keeping *lowest order* nonlinearity,

$$\ddot{u} + \nu\,\dot{u} - c_L^2\, u_{xx} - \frac{c_L^2}{12}\, r_0^2\, u_{xxxx} = -\,p_0\, u_x\, u_{xx}$$

$c_L = \omega_{L,0}\, r_0$; $\omega_{L,0}$ and p_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), slow profile evolution in time τ and defining the *relative displacement* $w = u_\zeta$, one obtains (for $\nu = 0$) the Korteweg-deVries Equation:

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

Defs.: $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$.

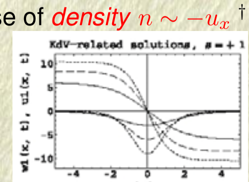
The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, since $a > 0$) solutions, in the form:

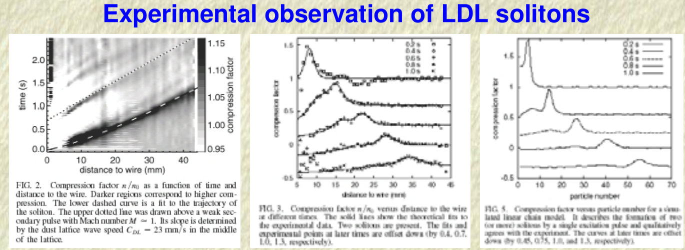
$$w_1(\zeta, \tau) = -\frac{3v}{a}\mathrm{sech}^2\left[(v/4b)^{1/2}(\zeta - v\tau - \zeta_0)\right]$$

– This solution is a negative pulse for $w = u_x$, describing a *compressive* excitation for the *displacement* $\delta x = u$, i.e. a localized increase of *density* $n \sim -u_x$ †.



† F. Melandsø 1996; S. Zhdanov *et al.* 2002; K. Avinash *et al.* 2003; V. Fortov *et al.* 2004.

Experimental observation of LDL solitons



[Samsonov *et al.*, PRL 2002].

- *Only compressive* solitons predicted by KdV theory;
- *Only compressive* solitons experimentally anticipated and, hence, reported;
- What about *rarefactive* longitudinal solitons?

Extended description: the Boussinesq theory

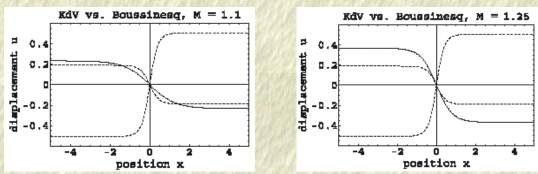
The *Generalized* *Boussinesq* (Bq) Equation (for $w = u_x$):

$$\ddot{w} - c_L^2 \, w_{xx} = \frac{c_L^2 r_0^2}{12} w_{xxxx} - \frac{p_0}{2} (w^2)_{xx} + \frac{q_0}{2} (w^3)_{xx}$$

– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...); *and, ...*

– relaxes the velocity assumption, i.e. is valid $\forall \quad v > c_L$.



[from: I Kourakis & P K Shukla, *European Phys. J. D*, **29**, 247 (2004)].

Menu

1. *Dusty plasmas (DP) & DP crystals (DPCs)*: Prerequisites.
   (i) Focus: 1d dust crystals in lab.
   (ii) Nonlinearity in 1d DP crystals: Origin and modelling.
2. *Transverse* dust-lattice (*TDL*) excitations:
   amplitude modulation, transverse envelope structures.
3. *Longitudinal* dust-lattice (*LDL*) excitations:
   amplitude modulation, envelope structures, solitons.
4. 1d *Discrete Breather excitations* (*intrinsic localized modes*)
5. Conclusions.

4. Transverse Discrete Breathers (DBs)

- Eq. of motion in the *transverse* direction:

$$\frac{d^2 u_n}{dt^2} + \omega_{T,0}^2 \, (u_{n+1} + u_{n-1} - 2 \, u_n) + \omega_g^2 \, u_n + \alpha \, u_n^2 + \beta \, u_n^3 = 0$$

- Damping may be neglected (for low plasma density and/or pressure): $\nu/\omega_g \simeq 0.00154$ ([Misawa et al., PRL 2001]).
- 1d DPCs are *highly discrete* lattice configurations: $\epsilon = \omega_0^2/\omega_g^2 \simeq 0.016$ ([Misawa et al., 2001]); $\epsilon \simeq 0.181$ ([Liu et al., 2003]).
- One may seek *discrete breather* solutions (localized modes):

$$u_n(t) = \sum_m A_n(m) \exp(im\omega t)$$

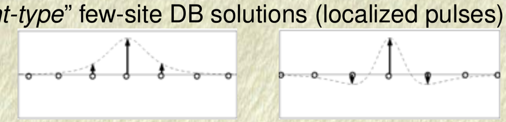
where only few ($m \simeq 1 - 3$) sites are excited.

4. Transverse Discrete Breathers (DBs) *(cont.)*

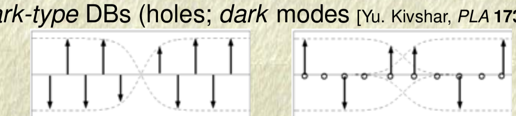
* The well-known *non - resonance condition* is recovered:

$$m\omega_B \neq \omega_T(k) \qquad \forall k, \, m = 1, 2, ...$$

* "*Bright-type*" few-site DB solutions (localized pulses):



* +: *Dark-type* DBs (holes; *dark* modes [Yu. Kivshar, *PLA* **173**, 172 (1993)]):



* Existence & stability profile established:
[V Koukouloyannis & I Kourakis, subm. PRE; arxiv.org/pdf/nlin.PS/0703020].

* DB modes may also occur in the $x-$ direction ($\sim$ FPU).

Conclusions - State of Art

– *Dust crystals* provide an excellent test-field for NL theories;

– *Observations are possible at the kinetic level*: unique possibility for physical data processing & real-time analysis;

– *Technology for experiment*: *cheap* and *readily available*;

– Link between *Plasma Phys., Solid State Physics, Stat. Mech.*;

– *Theory (1d)*: *Envelope solitons, (non-topological) solitons, Discrete Breathers*: predicted;

– *Theory (2d)*: *Discrete Breathers, vortices, ...*: predicted;

– *Experiment:* Harmonic generation, compressive solitons, NL TDL oscillations, backward wave: observed (*Urge for more :-)*);

– Future scope: *dissipation*, *mode coupling*, ... *"(Realism!)"*.

Overview of existing results

1. 1d: *Transverse* dust-lattice (*TDL*) motion ($\sim$ NL KG, inv. disp.):
   $\rightarrow$ Envelope (NLS) solitons [IK & P K Shukla, Phys. Plasmas **11**, 1384 (2004)]
   $\rightarrow$ DBs (ILMs) [V. Koukouloyannis & IK, PRE **76**, 016402 (2007)]
2. 1d: *Longitudinal* dust-lattice (*LDL*) motion) ($\sim$ FPU):
   $\rightarrow$ *Asymmetric* envelope structures (coupled 0th/1st harmonics) [IK & P K Shukla, Phys. Plasmas **11**, 3665 (2004)]
   $\rightarrow$ KdV vs. eKdV / Bq solitons [IK & PKS, Eur. Phys. J. D **29**, 247 (2004)]
   Rem.: experimentally observed (compressive case only)
3. 2D: *In-plane* ("LDL") motion in *hexagonal* DP crystals:
   $\rightarrow$ Envelope structures [Farokhi, IK & PKS, Phys. Plasmas **13**, 122304 (2006)]
4. 2D: *Out-of-plane* (TDL) motion in *hexagonal* DP crystals:
   $\rightarrow$ DBs, vortices (in preparation).

Future considerations & perspectives

1. *LDL-DBs ?* (∼ FPU);
2. *Damping* (dissipative system), ion drag, wake potentials, ...
3. Mixed *T-L Mode*: coupled FPU-NLKG Eqs. (ongoing work);
4. *2D hexagonal dust lattices*: vortices ? (seen experimentally);
5. ***Experimental feedback***:
   - establish & pursue contacts,
   - seek confirmation of results, motivate experiments ...

→ A lot remaining to be done!

Thank You !

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Material from:

I Kourakis & P K Shukla, *Phys. Plasmas*, **11**, 2322 (2004);
idem, *Phys. Plasmas*, **11**, 3665 (2004).
idem, *Phys. Plasmas*, **11**, 1384 (2004).
idem, *European Phys. J. D*, **29**, 247 (2004).
V Koukouloyannis & IK, PRE **76**, 016402 (2007).

Slides available at: www.kourakis.eu