Nonlinear excitations in pair-ion and e-p-i plasmas

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1. Prerequisites: *NL excitations & pair plasmas*

The purpose of this study is to investigate the occurrence of *nonlinear ES/EM modes* in *pair plasmas*.

Pair plasmas (*p.p.*) are plasmas consisting of two particle species (say, 1^+ and 2^-) of *equal mass* and *opposite charge* (i.e., $q_1 = -q_2 = +Ze$ and $m_1 = m_2 = m$).

No plasma or cyclotron frequency separation occurs in *p.p.*; furthermore, a variety of novel physical phenomena (e.g. absence of Faraday rotation) characterizes electrostatic (ES) and electromagnetic (EM) wave propagation in such plasmas [1].

Although this simple description of pair plasmas was originally introduced to model (for $Z = 1$, here) *electron-positron (*e-p*) plasmas* (yet overseeing *e-p* annihilation-recombination processes, here neglected throughout), it may claim to provide a consistent model of fullerene-ion pair plasma configurations which were recently successfully produced in experiments [2].

Significant research effort has recently focused on the properties of linear and nonlinear wave propagation in such plasmas [1, 2, 3, 4].

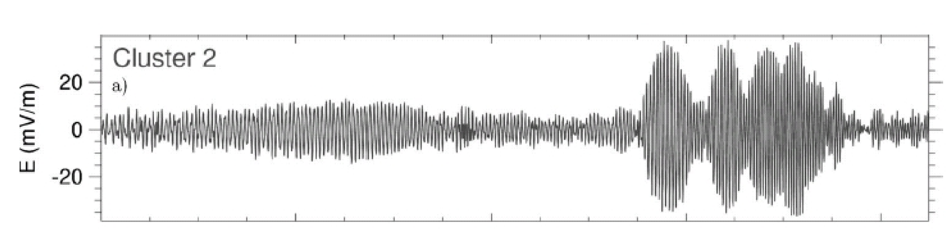
"3-component" p.p.: The pair species' densities at equilibrium, although equal in a symmetric ("pure") *p.p.* configuration, may differ if the charge balance is affected by a 3rd population, e.g. a massive charged defect species $3^{\pm}$ (e.g. dust [5]), assumed present as a stationary background. We shall refer to such p.p. as "3-component" or "doped" p.p.

This picture also refers to the presence of heavy ions in electron-positron-ion (*e-p-i*) plasmas.

Nonlinear structures (solitary waves) are ubiquitous in plasmas. These may either have the form of *localised pulses*, or of *envelope solitons*. The latter generated via *modulational instability* (MI), a well-known mechanism of energy localization dominating wave propagation in nonlinear dispersive media. Modulated envelope structures are generally modelled by means of the nonlinear Schrödinger theory.

Amplitude modulation is a mechanism well known to be associated with *harmonic generation* and the formation of *localized envelope modulated wave packets*, such as the ones abundantly observed during laboratory experiments and satellite observations, e.g. *in the Earth's magnetosphere*:

(a)



(b)

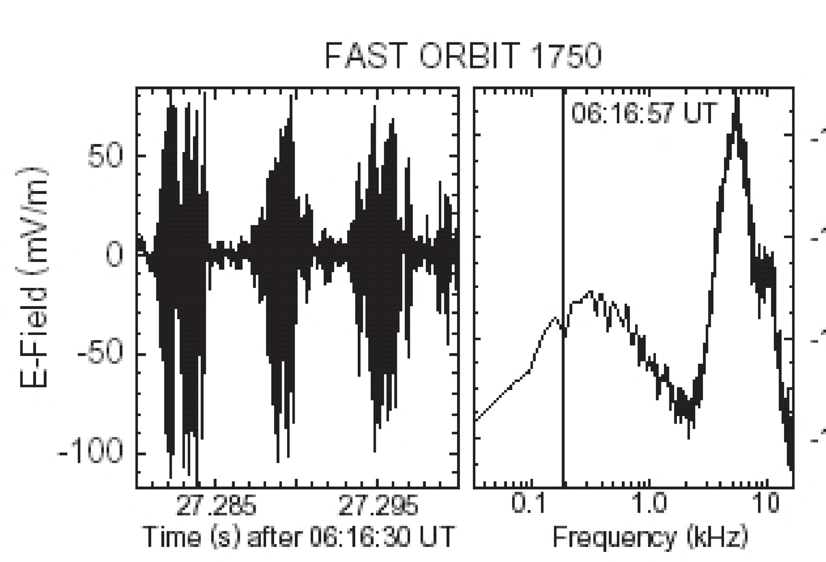
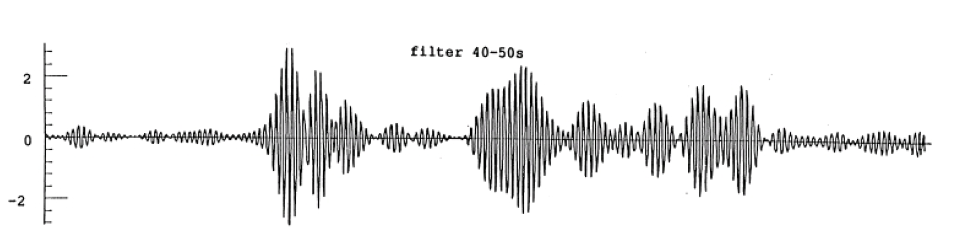
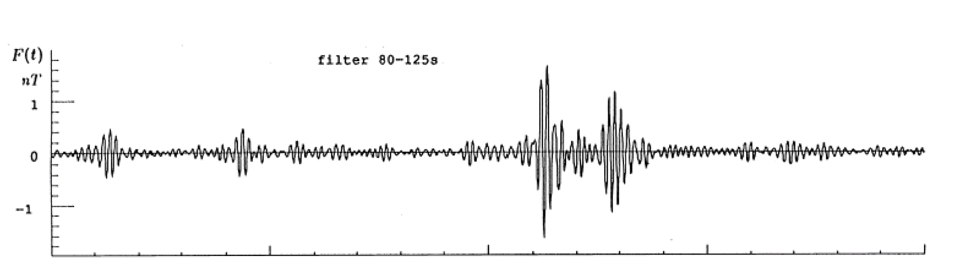


Figure 2. *Left*: Wave form of broadband noise at base of AKR source. The signal consists of highly coherent (nearly monochromatic frequency of trapped wave) wave packets. *Right*: Frequency spectrum of broadband noise showing the electron acoustic wave (at ~ 5 kHz) and total plasma frequency (at ~ 12 kHz) peaks. The broad LF maximum near 300 Hz belongs to the ion acoustic wave spectrum participating in the 3 ms modulation of the electron acoustic waves.

(c)



(d)



Caption: Satellite observations of modulation phenomena:
(a) Cluster data, from O. Santolik *et al.*, *J. Geophys. Res.* **108**, 1278 (2003);
(b) FAST data, from R. Pottelette *et al.*, *Geophys. Res. Lett.* **26** (16) 2629 (1999);
(c), (d) from Ya. Alpert, *Phys. Reports* 339, 323 (2001).

2. A (2+1)-fluid model for ES modes in pair-plasmas

We consider a multi-component collisionless plasma, composed of:

* (species 1) positive ions, or *positrons* (mass m_1, charge $q_1 = s_1 Z_1 e = +Ze$) and

* (species 2) negative ions, or *electrons* (mass $m_2 = m$, charge $q_2 = s_2 Z_2 e = -Ze$).

* (species 3) massive, immobile particles (e.g. dust, or ions in e-p-i plasmas), charge $q_3 = s_3 Z_3 e$ (where $s_3 = \pm 1$), mass $m_3 \gg m_{1/2}$.

* We have defined the charge state(s) Z_j ($j = 1, 2, 3$), the charge sign $s_j = q_j/|q_j| = \pm 1$ and the absolute electron charge e; we shall denote the respective equilibrium number densities by $n_{j,0}$.

* Application 1: *Pair i^+-i^- plasmas (*dusty *or* "pure"*):* Z, Z_3 arbitrary;

* Application 2: *electron-positron-ion (e-p-i)* plasmas: $Z_1 = Z_2 = Z_3 = 1$, $m_1 = m_2 \ll m_3 = m_i$.

We consider the (two-) fluid density and momentum equations:

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j\, \mathbf{u}_j) = 0 \qquad (1)$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = -s_j \frac{Ze}{m} \nabla \phi - \frac{1}{m n_j} \nabla p_j \,, \qquad (2)$$

for the species j $(= 1, 2)$. Also, the adiabatic equation of state

$$p_j = C n_j^{\gamma} \,, \qquad p_{j,0} = n_{j,0} k_B T_j \,, \qquad \gamma = 1 + 2/f \,,$$

for f degrees of freedom. Finally, *Poisson's* equation reads

$$\nabla^2 \Phi = -4\pi \sum_s q_s n_s = 4\pi e\, (Z\, n_- - Z\, n_+ - s_3 Z_3 n_3) \qquad (3)$$

Here, n_3 =constant. Cases covered include:

* *"Pure" p.p.* (or e-p): $n_3 = 0$, i.e. $n_{+,0} = n_{-,0}$, whereas

* in $e^- p^+ i^+$ or $X^+ X^- d^{\pm}$: $n_3 \neq 0$.

The RHS of Poisson's Eq. (3) cancels at equilibrium (*only*):

$$Z_1 n_{1,0} - Z_2 n_{2,0} + s_3 n_3 Z_3 = 0 \,, \qquad (4)$$

However, we underline the fact that *no a priori* assumption is made on the (conservation of) charge neutrality (or density balance; ***aka*** the plasma approximation) during dynamical evolution in time.

3. Linear ES wave dispersion properties

The dispersion relation of ES modes reads:

$$\frac{1}{\omega^2 - 3k^2} + \frac{\beta}{\omega^2 - 3\sigma\beta^2 k^2} = 1 \,, \qquad (5)$$

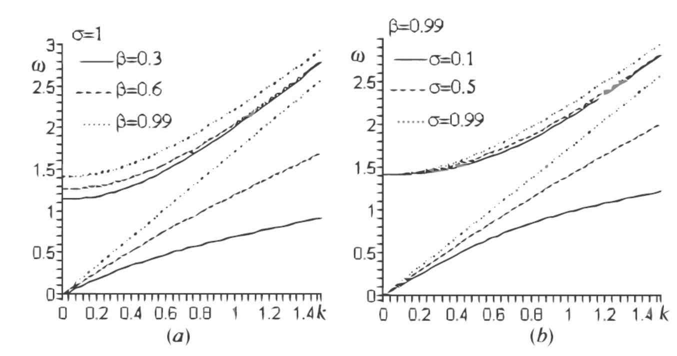
where $\beta = n_{+,0}/n_{-,0}$ is the density ratio and $\sigma = T_+/T_-$ is the temperature ratio, among the pair components. Two branches exist, say $\omega = \omega_L(k)$ and $\omega = \omega_U(k)$. The exact expressions are presented and analyzed in Ref. [3]. The lower branch ω_L describes an acoustic mode, as $\omega_L(0) = 0$, while the upper one bears a Langmuir-type curve, featuring a cutoff frequency $\omega_U(0) = (1+\beta)^{1/2}$ (in units of $\omega_{p,-}$). Interestingly, *no* acoustic mode in principle exists for perfectly symmetric ("pure" p.p.) configurations; to see this, set $\beta = \sigma = 1$ in (5) above[a], to obtain $\omega^2 = 2 + 3k^2$ (cf. literature[2]). *Asymmetric p.p.* are henceforth implicitly assumed everywhere, here. Near $k = 0$, we obtain

$$\omega_L(k) \approx c_s^2 k^2 \,, \qquad \omega_U(k) \approx (\omega_c^2 + c_s^2 k^2)^{1/2} \,. \qquad (6)$$

where

$$\omega_c = (1+\beta)^{1/2} \,, \qquad c_s = [3\,\beta(1+\sigma\beta)/(1+\beta)]^{1/2} \,.$$

Note the dependence on the background (third) species (via β), and also on the pair species' T asymmetry (via σ).



4. Nonlinear analysis-*reductive perturbation* [6]

* Consider small deviations from the equilibrium state $\mathbf{S_0}$

* Assume, for all state variables S_l ($l = 1, ..., 5$):

$$S_l(x,t) = S_{0,l} + \sum_{m=1}^{\infty} \epsilon^n \sum_{L=-\infty}^{\infty} S_{L,l}^{(n)}(\zeta,\tau) \exp[iL(kx - \omega t)]$$

where $\epsilon \ll 1$ is a small real; $S_{L,l}^{(n)} = (S_{-L,l}^{(n)})^*$ is implied.

* allow for a ***weak*** modulation of the amplitude(s) A_j via

$$\zeta = \epsilon\,(x - v_g\, t) \qquad \text{and} \qquad \tau = \epsilon^2 t,$$

where $\lambda = v_g = d\omega/dk$ is the group velocity [6].

[a] This suggests an asymmetry among the pair ion species in the experiment(s) of Oohara and Hatakeyama [2] where an acoustic mode (?) *was* reported.

The tedious algebra leads, to order ϵ^2, to the result

$$\phi \approx \epsilon\, \psi\, \cos\theta_c + \epsilon^2 \left[\phi_0^{(2)} + \phi_1^{(2)} \cos\theta_c + \phi_2^{(2)} \cos 2\theta_c\right] + \mathcal{O}(\epsilon^3) \,, \quad (7)$$

for the electric potential ϕ; ψ represents the (linear) carrier wave (unperturbed phase $\theta_c = kx - \omega t$); similar expressions are obtained for $n_{+/-}$ and $\mathbf{u}_{+/-}$.

We anticipate a solution in the form $\psi = \psi_0 \exp i\Theta$, where ψ_0 and Θ represent the potential (wavepacket) amplitude and a (small) phase correction, leading to a weakly varying total phase

$$\theta = \theta_c + \epsilon^2 \Theta + \mathcal{O}(\epsilon^3) \,.$$

5. Nonlinear Schrödinger (NLS) equation for $\psi = \phi_1^{(1)}$:

In order $n = 3$, we obtain the compatibility equation:

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0 \,. \qquad (8)$$

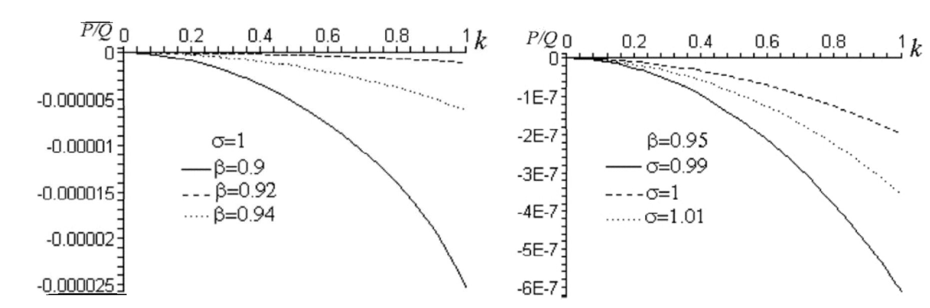
- Dispersion coefficient: $P = \frac{1}{2}\omega''(k) = ...$
- Nonlinearity coefficient: $Q = Q(\{\omega; \beta, \sigma, ...\}) = ...$ ($\rightarrow$ a lengthy expression, omitted here).

6. Modulational (in)stability of ES wavepackets

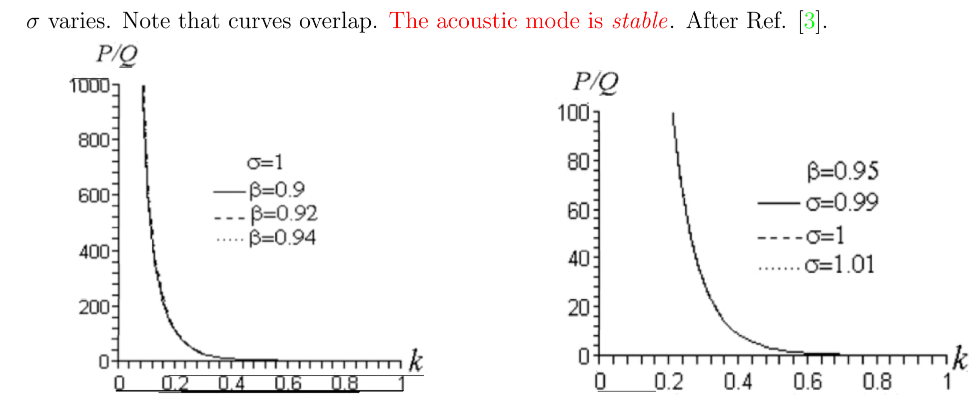
- Plane wave solution of (8): $\psi = \psi_0 \exp(iQ|\psi_0|^2\tau)$;
- Linear analysis: set $\hat\psi = \hat\psi_0 + \epsilon\, \hat\psi_{1,0} \cos(\tilde k \zeta - \tilde\omega \tau)$;
- (Perturbation) dispersion relation:

$$\tilde\omega^2 = P\,\tilde k^2\,(P\tilde k^2 - 2Q|\hat\psi_{1,0}|^2) \,; \qquad (9)$$

- If $PQ < 0$, the amplitude ψ is stable;
- If $PQ > 0$, the amplitude ψ is *unstable* for $\tilde k < \sqrt{2Q/P}|\hat\psi_{1,0}|$.
- Here: The acoustic mode is *stable* ($PQ < 0$) (see Fig. below)
- The upper (optic) mode is *unstable* ($PQ > 0$) (see Fig. below).



The NLSE coefficient ratio P/Q corresponding to the *lower* (acoustic) dispersion branch ω_L is depicted against the (reduced) wavenumber k. (a) $\sigma = 1$ and different values of β are considered; (b) $\beta = 0.95$, and σ varies. Note that curves overlap. The acoustic mode is *stable*. After Ref. [3].



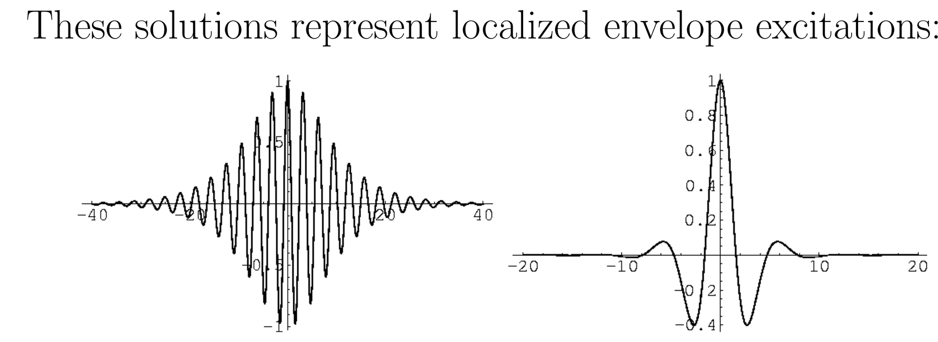
The NLSE coefficient ratio P/Q corresponding to the *upper* (optic) dispersion branch ω_U is depicted against the (reduced) wavenumber k. (a) $\sigma = 1$ and different values of β are considered; (b) $\beta = 0.95$, and σ varies. The upper (optic) mode is *unstable*. After Ref. [3].

7. Envelope soliton solutions of the NLS Equation

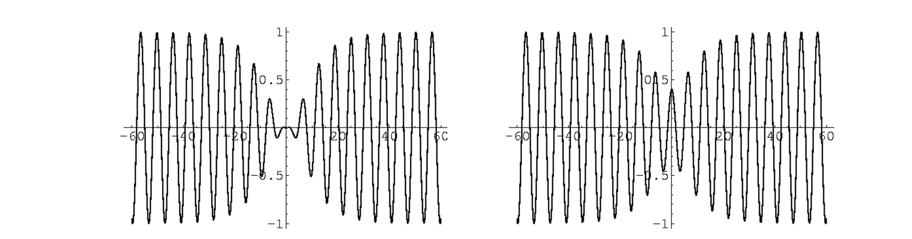
Modulated wave-form:

$$\psi = \epsilon \hat\psi_0\, \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta) + \mathcal{O}(\epsilon^2) \,.$$

The amplitude ψ_0 and phase correction Θ are functions of $\{\zeta, \tau\}$.
These are given by exact expressions (here omitted).
These solutions represent localized envelope excitations:



Bright-type envelope solitons (for $PQ > 0$).



Dark-/grey-type envelope solitons (for $PQ < 0$).

8. Electromagnetic wavepackets in pair plasmas

The analogous investigation for EM waves propagating in p.p. has been carried out, for the ordinary mode (*O-mode*), in [4].

References

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