On the Existence of Rarefactive Solitons in Dusty Plasma Lattices

Ioannis Kourakis

Centre for Plasma Physics, School of Mathematics and Physics

Queen’s University of Belfast, BT7 1NN, Belfast, Northern Ireland

35th IEEE International Conference on Plasma Science
Karlsruhe, June 15 - 19, 2008

www.tp4.rub.de/~ioannis/conf/200806-ICOPS-oral2.pdf

Motivation - History

- 1996: Theoretical prediction of longitudinal dust-lattice (DL) waves (acoustic mode) and LDL solitons (compressive, only)
- 2002: Experimental confirmation of compressive LDL solitons
- 2004: Theoretical modelling of compressive *and* rarefactive LDL solitons
- 2008: Experimental confirmation of *rarefactive* LDL solitons

Outline

1. Prerequisites
2. The Melandsø theory (Melandsø, 1996)
3. Experimental confirmation (Samsonov, 2002)
4. An extended nonlinear theory (I Kourakis & P Shukla, 2004)
5. Conclusions

1. Preliminaries - Model Hamiltonian

$$H = \sum_n \frac{1}{2} M \left(\frac{d\mathbf{r}_n}{dt} \right)^2 + \sum_{m \neq n} U_{int}(r_{nm}) + \Phi_{ext}(z_n)$$

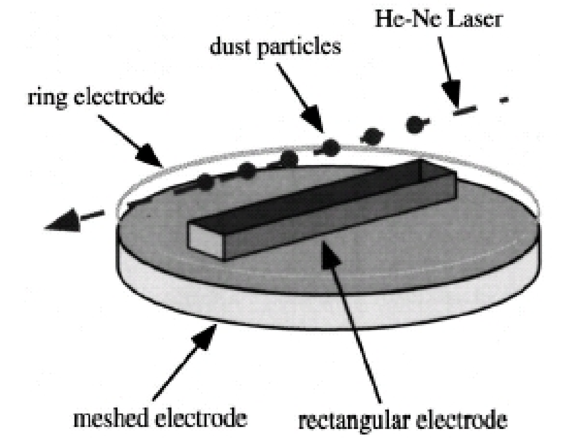


FIG. 10. Experimental configuration for forming a linear dust chain above a long rectangular box on a negatively biased mesh electrode.

Terms include:

– *Kinetic energy*

– *External force fields:* $\Phi_{ext}(z_n)$; may account for *confinement potentials* and/or *sheath electric* forces, i.e. $F_{sheath}(z) = -\frac{\partial \Phi}{\partial z}$

– ES Coupling: $U_{int}(r_{nm})$ is the *interaction potential energy*.

(*Nonlinear*) *longitudinal* excitations.

The *nonlinear* equation of longitudinal motion reads:

$$\frac{d^2(\delta x_n)}{dt^2} = \omega_{0,L}^2 \, (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$- a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2 \right]$$
$$+ a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3 \right]$$

– $\delta x_n = x_n - n r_0$: longitudinal dust grain displacements

– Cf. *Fermi-Pasta-Ulam (FPU) problem*:
anharmonic spring chain model.

Coefficients (for Debye interactions)

In general:

$$a_{20} = -\frac{1}{2M} U'''(r_0)\,, \qquad a_{30} = \frac{1}{6M} U''''(r_0)\,,$$

$$\omega_{0,L}^2 = U''(r_0)/M\,,$$

For a Debye (Yukawa) potential $U_D(r) = Q^2\, e^{-r/\lambda_D}/r$:

$$\omega_{0,L}^2 = \frac{2Q^2}{M\lambda_D^3}\, e^{-\kappa}\, \frac{1+\kappa+\kappa^2/2}{\kappa^3} \equiv c_L^2/(\kappa^2\lambda_D^2)\,,$$

$$p_0 \equiv 2a_{20}\kappa^3\lambda_D^3 = \frac{6Q^2}{M\lambda_D}\, e^{-\kappa} \left(\frac{1}{\kappa} + 1 + \frac{\kappa}{2} + \frac{\kappa^2}{6}\right),$$

$$q_0 \equiv 3a_{30}\kappa^4\lambda_D^4 = \frac{Q^2}{2M\lambda_D}\, e^{-\kappa}\, \frac{1}{\kappa}\Big(\kappa^4 + 4\kappa^3 + 12\kappa^2 + 24\kappa + 24\Big)\,.$$

2. The Melandsø (1996) theory

Q.: *A link to soliton theories: the Korteweg-deVries Equation.*

– *Continuum approximation*, viz. $\delta x_n(t) \to u(x,t)$.

– *"Standard"* description: keeping *lowest order* nonlinearity,

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} \;=\; -\,p_0\,u_x\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$ and p_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), slow profile evolution in time τ and defining the *relative displacement* $w = u_\zeta$, one obtains

$$w_\tau \;-\; a\,w\,w_\zeta + b\,w_{\zeta\zeta\zeta} \;=\; 0$$

(for $\nu = 0$); $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$.

The KdV description

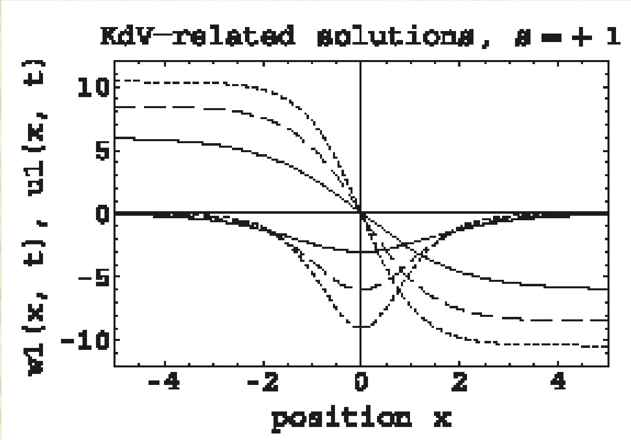
The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, here) solutions, in the form (here):

$$w_1(\zeta, \tau) = -w_{1,m} sech^2 \Big[(\zeta - v\tau - \zeta_0)/L_0\Big]$$

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$.



The KdV description

The Korteweg-deVries (KdV) Equation

$$w_\tau - a\, w\, w_\zeta + b\, w_{\zeta\zeta\zeta} = 0$$

yields *compressive* (*only*, here) solutions, in the form (here):

$$w_1(\zeta, \tau) = -w_{1,m} sech^2\left[(\zeta - v\tau - \zeta_0)/L_0\right]$$

– Pulse amplitude: $w_{1,m} = 3v/a = 6vv_0/|p_0|$;

– Pulse width: $L_0 = (4b/v)^{1/2} = [2v_1^2 r_0^2/(vv_0)]^{1/2}$;

– Note that: $w_{1,m}L_0^2 = \text{constant}$ (cf. experiments)†.

– This solution is a negative pulse for $w = u_x$,
describing a *compressive* excitation for the *displacement* $\delta x = u$,
i.e. a localized increase of *density* $n \sim -u_x$.

† F Melandsø 1996; S Zhdanov *et al.* 2002; K Avinash *et al.* 2003; V Fortov *et al.* 2004.

Characteristics of the KdV theory

The *Korteweg - deVries* theory:

– provides a *qualitative description* of *compressive* excitations;

– benefits from the KdV *"artillery"* of analytical know-how: *integrability*, *multi-soliton* solutions, *conservation laws*, ... ;

but possesses certain drawbacks:

– *approximate* derivation:
– propagation velocity v near (longitudinal) sound velocity c_L,
– time evolution terms omitted *'by hand'*,
– higher order nonlinear contributions omitted;

– *only* accounts for *compressive* solitary excitations (for Debye interactions).

3. Experimental observation of LDL solitons

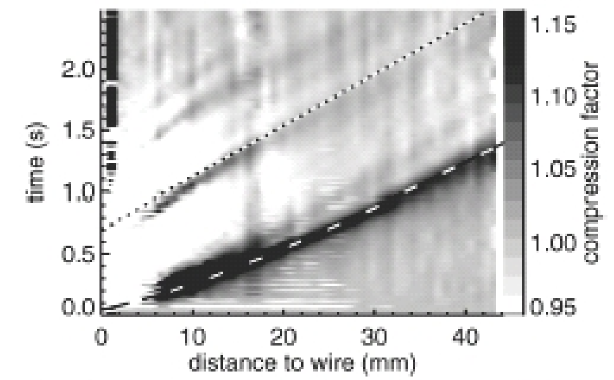


FIG. 2. Compression factor n/n_0 as a function of time and distance to the wire. Darker regions correspond to higher compression. The lower dashed curve is a fit to the trajectory of the soliton. The upper dotted line was drawn above a weak secondary pulse with Mach number $M \simeq 1$. Its slope is determined by the dust lattice wave speed $C_{DL} = 23$ mm/s in the middle of the lattice.

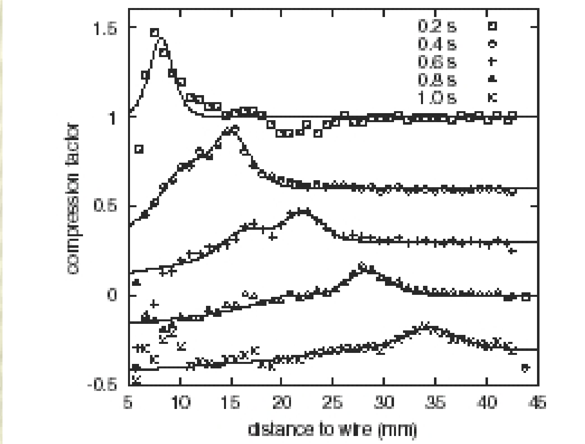


FIG. 3. Compression factor n/n_0 versus distance to the wire at different times. The solid lines show the theoretical fits to the experimental data. Two solitons are present. The fits and experimental points at later times are offset down (by 0.4, 0.7, 1.0, 1.3, respectively).

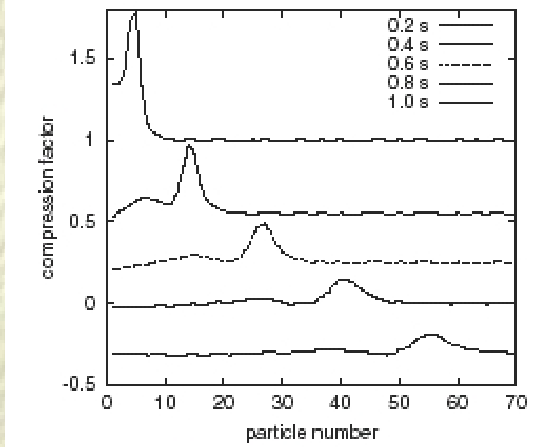


FIG. 5. Compression factor versus particle number for a simulated linear chain model. It describes the formation of two (or more) solitons by a single excitation pulse and qualitatively agrees with the experiment. The curves at later times are offset down (by 0.45, 0.75, 1.0, and 1.3, respectively).

[Samsonov *et al.*, PRL 2002].

- *Only compressive* solitons predicted by KdV theory
- *Only compressive* solitons anticipated (and thus reported)
- What about *rarefactive longitudinal solitons*?

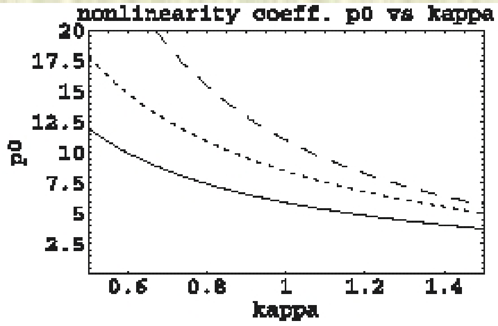
4. *Extended* longitudinal soliton formalism

Q.: *What if we also kept the next order in nonlinearity ?*

– *"Extended"* description: :

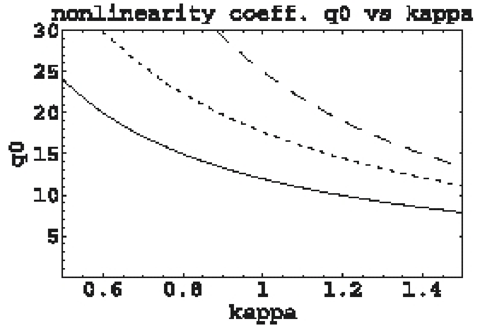
$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx} + q_0\,(u_x)^2\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$, $p_0 \sim -U'''(r)$ and $q_0 \sim U''''(r)$ (cf. above).



(b)

Fig. 4. (a) The nonlinearity coefficient p_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.



(b)

Fig. 5. (a) The nonlinearity coefficient q_0 (normalized over $Q^2/(M\lambda_D)$) is depicted against the lattice constant κ for $N = 1$ (first-neighbor interactions: —), $N = 2$ (second-neighbor interactions: - - -), $N = \infty$ (infinite-neighbors: – – –), from bottom to top. (b) Detail near $\kappa \approx 1$.

Rq.: q_0 is *not* negligible, compared to p_0! (instead, $q_0 \approx 2p_0$ practically!)

4. *Extended* longitudinal soliton formalism *(continued)*

Q.: *What if we also kept the next order in nonlinearity ?*

– *“Extended”* description: :

$$\ddot{u} + \nu\,\dot{u} - c_L^2\,u_{xx} - \frac{c_L^2}{12}\,r_0^2\,u_{xxxx} = -\,p_0\,u_x\,u_{xx} + q_0\,(u_x)^2\,u_{xx}$$

$c_L = \omega_{L,0}\,r_0$; $\omega_{L,0}$, p_0 and q_0 were defined above.

– For *near-sonic propagation* (i.e. $v \approx c_L$), and defining the *relative displacement* $w = u_\zeta$, one has

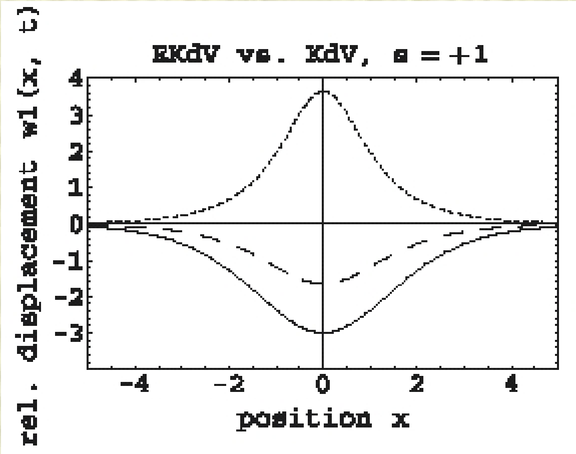
$$w_\tau - a\,w\,w_\zeta + \hat{a}\,w^2\,w_\zeta + b\,w_{\zeta\zeta\zeta} = 0 \qquad (1)$$

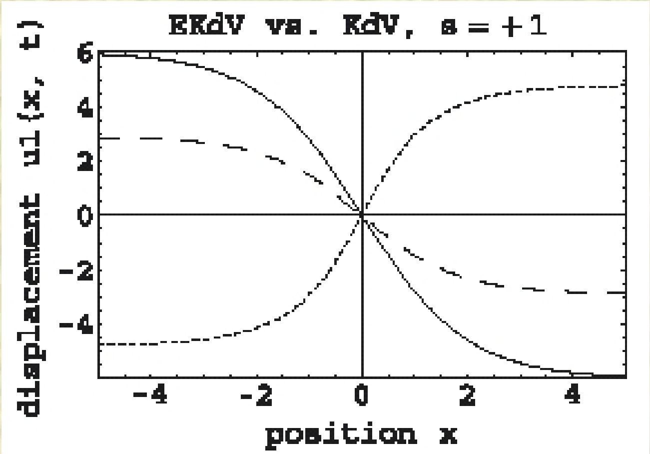
(for $\nu = 0$); $\zeta = x - vt$; $a = p_0/(2c_L) > 0$; $b = c_L r_0^2/24 > 0$;
$\hat{a} = q_0/(2c_L) > 0$.

Characteristics of the EKdV theory

The *extended Korteweg - deVries* Equation:

– accounts for *both compressive and rarefactive* excitations;





(*horizontal grain displacement* $u(x,t)$)

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– is previously widely studied, in literature;

Still, ...

– It was derived under the *assumption*: $v \approx c_L$.

One more alternative: the Boussinesq theory

The *Generalized* *Boussinesq* (Bq) Equation (for $w = u_x$):

$$\ddot{w} - c_L^2 w_{xx} = \frac{c_L^2 r_0^2}{12} w_{xxxx} - \frac{p_0}{2} (w^2)_{xx} + \frac{q_0}{2} (w^3)_{xx}$$

– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– has been widely studied in literature;

and, ...

One more alternative: the Boussinesq theory

The *Generalized* Boussinesq (Bq) Equation (for $w = u_x$):

$$\ddot{w} - c_L^2\, w_{xx} = \frac{c_L^2 r_0^2}{12}\, w_{xxxx} - \frac{p_0}{2}\,(w^2)_{xx} + \frac{q_0}{2}\,(w^3)_{xx}$$

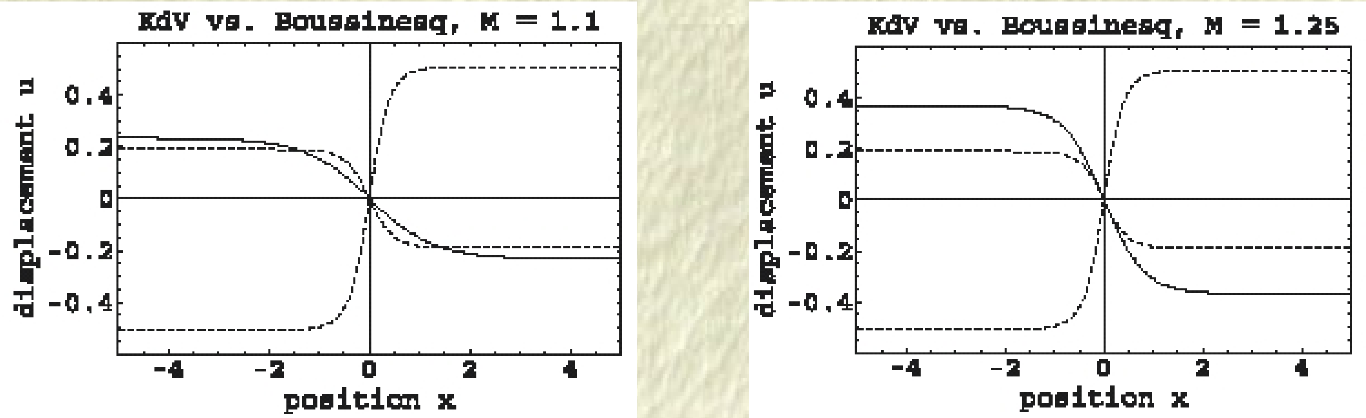
– predicts *both compressive and rarefactive* excitations;

– reproduces the *correct qualitative character* of the KdV solutions (amplitude - velocity dependence, ...);

– has been widely studied in literature;

and, ...

– relaxes the velocity assumption, i.e. is valid $\forall \quad v > c_L$.



"Breaking News": Experimental observation of rarefactive LDL solitons (2008)

Solitary Rarefaction Wave in a Three Dimensional Complex Plasma

Ralf Heidemann, Max-Planck-Institut für extraterrestrische Physik

Abstract
Observation of a solitary rarefaction wave in a three dimensional complex plasma containing monodisperse microparticles will be presented.
The experiments were performed in a capacitively coupled, symmetrically driven RF discharge. The discharge chamber is a slightly modified version of the PK3plus design currently flying on board the ISS.
A gas temperature gradient of 400K/m was applied to balance gravity and to levitate the particles in the plasma bulk. The wave was exited by a short voltage pulse on the electrodes of the rf discharge chamber. It was found that the wave propagates with constant speed and that the amplitude damping factor is significantly lower then Epstein damping.

Ralf Heidemann *et al*,
Int. Conf. Phys. Dusty Plasmas (Azores, Portugal), May 2008.

Conclusions

- Compressive *and/or* rarefactive longitudinal solitons may exist
- Their characteristics are determined by inter-grain interactions
- They are efficiently described by an extended KdV theory for near-sonic velocity, or by an extended Boussinesq theory for arbitrary velocity
- Experiments at preliminary stage, relation/agreement with theory to be investigated.

Conclusions

- Compressive *and/or* rarefactive longitudinal solitons may exist
- Their characteristics are determined by inter-grain interactions
- They are efficiently described by an extended KdV theory for near-sonic velocity, or by an extended Boussinesq theory for arbitrary velocity
- Experiments at preliminary stage, relation/agreement with theory to be investigated.

Thank You!

Material from: I. Kourakis and P. K. Shukla, Eur. Phys. J. D, **29**, 247 (2004).

Slides at: `www.kourakis.eu`.