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Nonlinear Dynamics of Pair Plasmas and e-p-i Plasmas

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Special thanks to: F Verheest (U. Gent, Belgium), M Hellberg (UKZN, Durban, S. Africa)

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Layout

1. Preliminaries on pair-plasmas (p.p.)
2. Nonlinear ES modes in p.p. - state of the art
3. NL modelling of rotating magnetized p.p.
4. An extended nonlinear theory.

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1. Preliminaries – *p.p.* occurrence & motivation
Electron-positron plasmas, *e-p-i* plasmas

- Electron-positron (*e-p*) plasmas occur in Space:
  - in pulsar magnetospheres
    [Ginzburg 1971, Weatherall ApJ 1997, Michel RMP 1982]
  - in bipolar outflows (jets) in active galactic nuclei (AGN)
    [Takahara 1986, Miller 1987, Begelman RMP 1984]
  - at the center of our own galaxy [Burns 1983]
  - in the early universe [Hawking 1983]
- … and on Earth:
  - in inertial confinement fusion [Liang et al. PRL 1998]
  - in laboratory experiments
    [Greaves, Surko *et al* PoP 1994, Zhao *et al* 1996]

Preliminaries – theory
Early theoretical studies on *e-p* plasmas

- Linear waves, dispersion characteristics, collective modes

  [V. Tsytovich & C. B. Wharton CPPCF 1978, Iwamoto 1993,

  Stewart & Laing, JPP 1992, Zank & Greaves PRE 1995, …]

- Nonlinear processes, photon-particle interactions,

  [Tajima & Taniuti PRA 1990, Pelletier 1998, …]

- Nonlinear ES & EM modes, solitons, double layers, …

  [Bharuthram ASSci 1991, Pillay & B 1992, Verheest & Hellberg 1996,

  Farina & Bulanov 2001, Lontano & Bulanov 2001, Melrose 2005...]

- Relativistic EM solitons

  [Berezhiani *et al* JPP 1992, PRE 1993, Shukla, Rao, Yu, Tsintsadze 1992…]

- *Modelling challenge: e-p annihilation (sink term), …*

Theory basics: particular features of pair plasmas

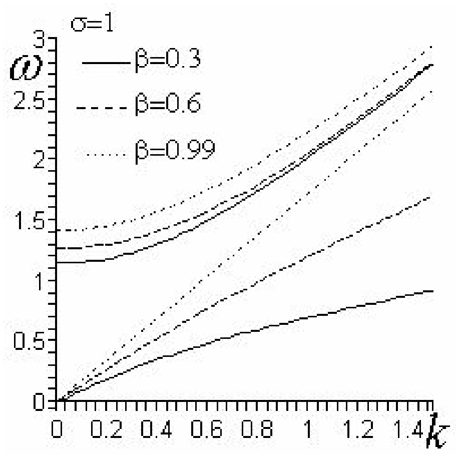
- Equal masses, opposite charges

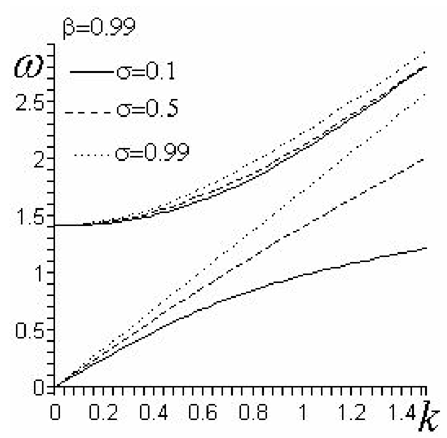
$$m_+ = m_- = m, \quad q_+ = -q_- = q$$

- Equal plasma and Larmor frequencies:

$$\omega_{p,+} = \omega_{p,-}, \quad \omega_{c,+} = \omega_{c,-}$$

- Fundamentally different ES dispersion law
  - ➢ Acoustic low-*f* mode
  - ➢ Langmuir-like upper mode
- Different EM wave characteristics
  - ➢ Linear polarization of //***B*** EM waves
  - ➢ Faraday rotation absent





Experiments on fullerene pair-plasmas

2003: Experimental realization of pair plasmas via the creation of identical preparations of C_{60}^{+} and C_{60}^{-} ions: no pair-ion annihilation

VOLUME 91, NUMBER 20 PHYSICAL REVIEW LETTERS week ending 14 NOVEMBER 2003

Pair-Ion Plasma Generation using Fullerenes

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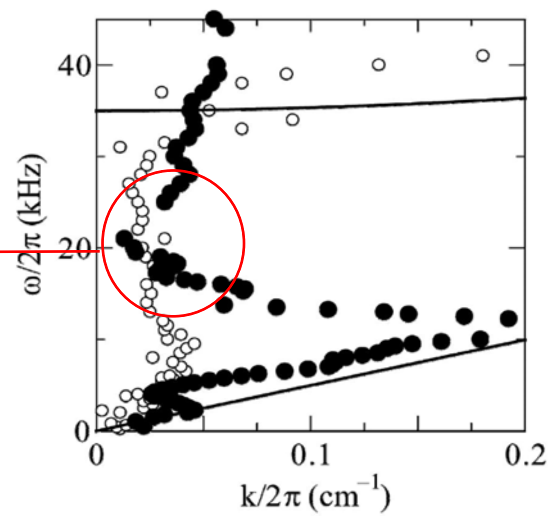
Department of Electronic Engineering, Tohoku University, Sendai 980-8579, Japan
(Received 17 April 2003; published 14 November 2003)

3 ES modes observed:
an acoustic mode,
a Langmuir-like upper mode
and an intermediate f mode



2. Nonlinear modes in pair-ion plasmas
State of the art

- Solitons: arbitrary-amplitude (Sagdeev) & KdV theory
  - ES modes [Verheest 2006, Dubinov 2006, Lazarus *et al* 2008, …]
  - EM modes [Verheest & Cattaert 2005++]
- Self-modulation, MI: ES [Kourakis, Cattaert, Esfandyari 2006, 2007, ++]

  EM [Kourakis Verheest & Cramer PoP 2006, …]
- Kinetic theory, ion holes [Schamel & Luque 2005+]
- Ponderomotive effects [Shukla & coworkers PoP 2005+, …]
- Ion surface waves [Hasegawa & Shukla 2005]
- Kinetic + FP theory, instabilities [Vranjes & Poedts 2005, Zhao 2007]
- Interpretation of the IFW

[Verheest 2006, Saleem 2007, Schamel 2007, Vranjes 2008, …*to be continued*…]

3. Rotating pair-plasmas: a 2-fluid ES model

Fluid Eqs. (for $j = 1^+, 2^-$):

Coriolis force

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = -s_j \frac{Ze}{m} \nabla \phi - \frac{1}{m n_j} \nabla p_j + s_j \frac{Ze}{mc} (\mathbf{u}_j \times B_0 \hat{x}) + 2(\mathbf{u}_j \times \Omega_0 \hat{x})$$

$$p_j = C n_j^\gamma \,, \qquad p_{j,0} = n_{j,0} k_B T_j \,, \qquad \gamma = 1 + 2/f \,, \qquad s_j = q_j/|q_j| = \pm 1$$

Poisson's eq.

$$\nabla^2 \Phi = -4\pi \sum_s q_s n_s = 4\pi e \left(Z n_- - Z n_+ - s_3 Z_3 n_3 \right)$$

Neutrality hypothesis: $\quad Z n_{+,0} - Z n_{-,0} + s_3 Z_3 n_3 = 0 \qquad$ (n_3 =cst.).

(cf. Uberoi & Das PP 1970, Verheest 1974, Das & Nag PoP 2007)

3. Rotating pair-plasmas: a 2-fluid ES model

Fluid Eqs. (for $j = 1^+, 2^-$):

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \, \mathbf{u}_j) = 0$$

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j \; = \; -s_j \frac{Ze}{m} \nabla \phi - \frac{1}{m n_j} \nabla p_j + s_j \frac{Ze}{mc} (\mathbf{u}_j \times B_0 \hat{x}) + 2(\mathbf{u}_j \times \Omega_0 \hat{x})$$

$$p_j = C n_j^\gamma \, , \qquad p_{j,0} = n_{j,0} k_B T_j \, , \qquad \gamma = 1 + 2/f \, , \qquad s_j = q_j / |q_j| = \pm 1$$

Poisson's eq.

$$\nabla^2 \Phi \; = \; -4\pi \sum_s q_s \, n_s \; = \; 4\pi \, e \, (Z \, n_- - Z \, n_+ - s_3 Z_3 \, n_3)$$

Neutrality hypothesis: $\quad Z n_{+,0} - Z n_{-,0} + s_3 Z_3 \, n_3 = 0 \qquad (n_3 =$cst.$)$.

- Species $3^{+/-}$: a massive charged background species in stationary state
- 3= i^+ (ions) in *e-p-i* plasmas; 3= $d^{+/-}$ (defects, dust) in pair-ion plasmas

3. Rotating pair-plasmas: a 2-fluid ES model

- Species $3^{+/-}$: a massive charged background species in stationary state
  - Case 1: 3= i^{+} (ions) in *e-p-i* plasmas (*s*=+1)
  - Case 2: 3= $d^{+/-}$ (defects, dust) in pair-ion plasmas (*s*= +/-1)
- Charge balance equation is *not* pair-ion symmetric:

$$\delta = \frac{n_{+,0}}{n_{-,0}} = 1 - s\frac{Z_3}{Z}\frac{n_3}{n_{-,0}} \equiv 1 - \beta$$

- Potential pair-ion-temperature asymmetry: $\frac{T_+}{T_-} \equiv \sigma$

- Symmetric “pure” pair-plasma recovered for $\delta = \sigma = 1$

(Multi-*d*) Multiple scales perturbation technique

- Space and time “stretching”:

$$X = \epsilon^{1/2}(x - \lambda t)\,,\, Y = \epsilon^{1/2} y\,,\, \tau = \epsilon^{3/2} t$$

- Density, potential, velocity // **B** assumed slow:

$$S = S_{+}^{(0)} + \epsilon S^{(1)} + \epsilon^2 S^{(2)} + \ldots$$

where

$$S \in \{n_+, n_-, u_{+,x}, u_{-,x}, \phi\}$$

- Perp. **B** velocity assumed ***very*** slow:

$$u_{\pm,y/z} = \epsilon^{3/2} u^{(1)}_{\pm,y/z} + \epsilon^2 u^{(2)}_{\pm,y/z} + \epsilon^{5/2} u^{(3)}_{\pm,y/z} + \ldots$$

Lowest-order (~ε) state variable corrections

- Density: $$n_+^{(1)} = \frac{\delta}{\lambda^2 - 2\delta\sigma}\phi^{(1)} \qquad n_-^{(1)} = \frac{-1}{\lambda^2 - 2}\phi^{(1)}$$
- Velocity: $$u_{+,x}^{(1)} = \frac{\lambda}{\lambda^2 - 2\delta\sigma}\phi^{(1)} \qquad u_{-,x}^{(1)} = \frac{-\lambda}{\lambda^2 - 2}\phi^{(1)}$$

$$u_{+,z}^{(1)} = \frac{\lambda^2}{\Omega_+(\lambda^2 - 2\delta\sigma)}\frac{\partial\phi^{(1)}}{\partial Y} \qquad u_{-,z}^{(1)} = \frac{-\lambda^2}{\Omega_-(\lambda^2 - 2)}\frac{\partial\phi^{(1)}}{\partial Y}$$

- The compatibility condition: determines the excitation speed λ $$\frac{1}{\lambda^2 - 2} + \frac{\delta}{\lambda^2 - 2\delta\sigma} = 0$$
- Definitions: $\tilde{\Omega}_\pm = 2\tilde{\Omega}_0 \pm \tilde{\omega}_c$ (frequencies scaled by $\omega_{p,-}$)

Zakharov-Kuznetsov equation (ZKE) for $\phi^{(1)}$

$$\frac{\partial\psi}{\partial\tau} + A\psi\frac{\partial\psi}{\partial X} + \frac{\partial}{\partial X}\left(B\frac{\partial^2\psi}{\partial X^2} + C\frac{\partial^2\psi}{\partial Y^2}\right) = 0$$

- Nonlinearity coeffient:

$$A = B\left[\frac{3\delta\lambda^2}{(\lambda^2 - 2\delta\sigma)^3} - \frac{3\lambda^2}{(\lambda^2 - 2)^3}\right]$$

- Dispersion coefficients:

$$B = \left[\frac{2\lambda}{(\lambda^2 - 2)^2} + \frac{2\delta\lambda}{(\lambda^2 - 2\delta\sigma)^2}\right]^{-1}$$

$$C = B\left[1 + \frac{1}{\Omega_-^2}\frac{\lambda^4}{(\lambda^2 - 2)^2} + \frac{\delta\lambda^4}{\Omega_+^2(\lambda^2 - 2\delta\sigma)^2}\right]$$

Travelling wave ansatz for the ZK equation: definitions

- Moving coordinate (2D): $\zeta = L_x X + L_y Y - MT$
- Directional cosines: $L_x\,, \quad L_y\,, \quad L_x^2 + L_y^2 = 1$
- Pseudo-energy-balance equation

$$\frac{1}{2}\left(\frac{d\psi}{d\zeta}\right)^2 + \frac{1}{B_0}\left(\frac{-M}{2}\psi^2 + \frac{A_0}{6}\psi^3\right) = 0$$

- Definitions:

$$A_0 = AL_x\,, \qquad B_0 = L_x R\,, \qquad R = BL_x^2 + CL_y^2$$

Travelling wave ansatz for the ZK equation: definitions and solution

- Pulse soliton solution:

$$\psi = \phi_0 \,\mathrm{sech}^2(\zeta/\mathrm{W})$$

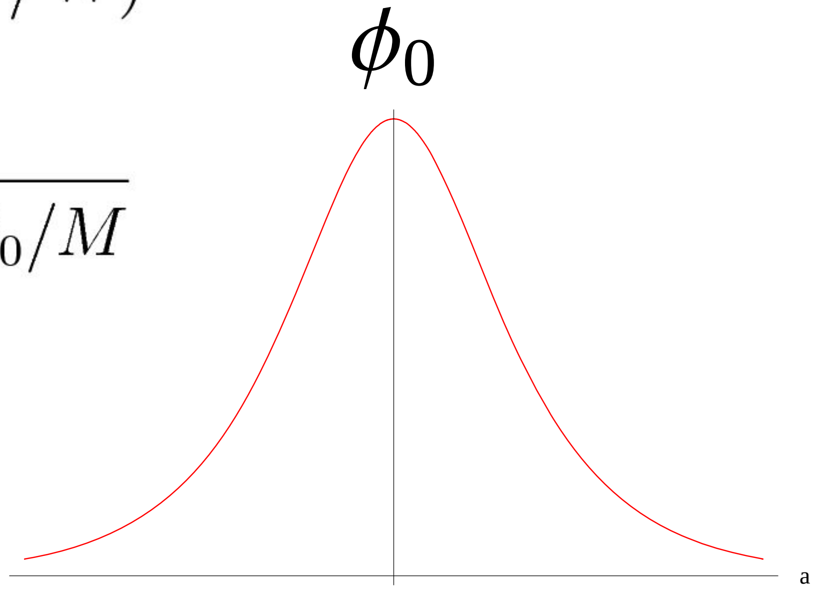
- Pulse characteristics:

$$\phi_0 = 3M/A_0\,, \quad W = \sqrt{4B_0/M}$$

- Functions of $\frac{T_+}{T_-} \equiv \sigma$

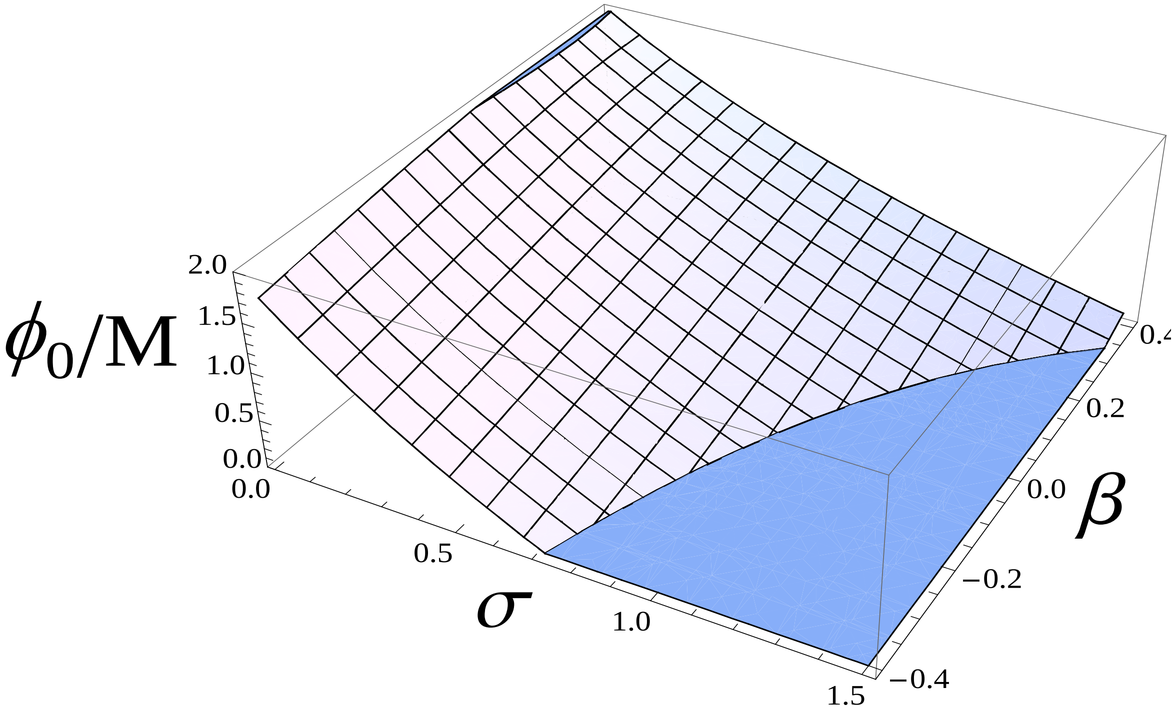
  and

$$\delta = \frac{n_{+,0}}{n_{-,0}} = 1 - s\frac{Z_3}{Z}\frac{n_3}{n_{-,0}} \equiv 1 - \beta$$



Parametric investigation:
pulse characteristics vs. σ, β

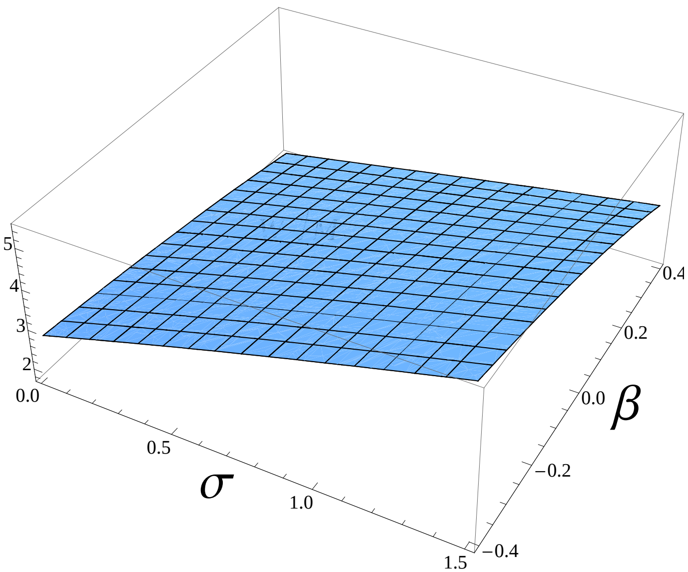
- Pulse amplitude ϕ_0 (and pulse polarity/sign +/-) :



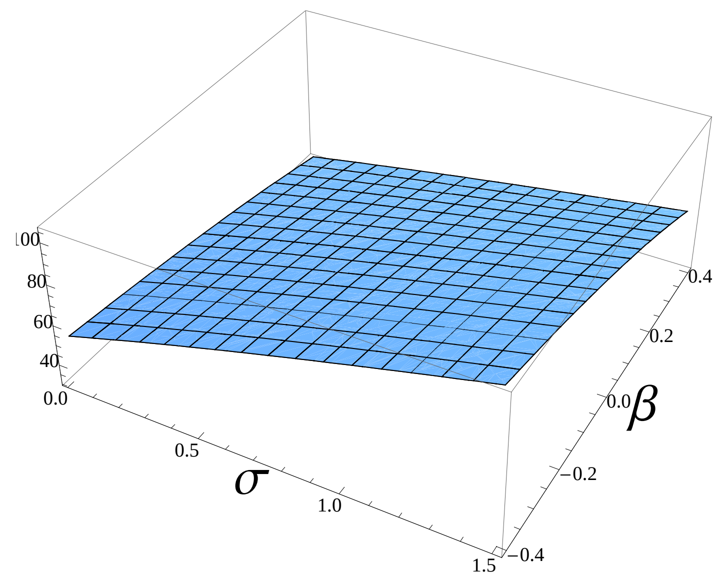
- Dependence on *pair-plasma* composition via σ and β

Parametric investigation: pulse width vs. *σ, β : role of ω_c , Ω_0*

- Pulse width $W\sqrt{M}$
  L_x=0.2
  ω_c =0.4, Ω_0 =0.1:



- Pulse width
  L_x=0.2
  ω_c =0.21, Ω_0 =0.1:



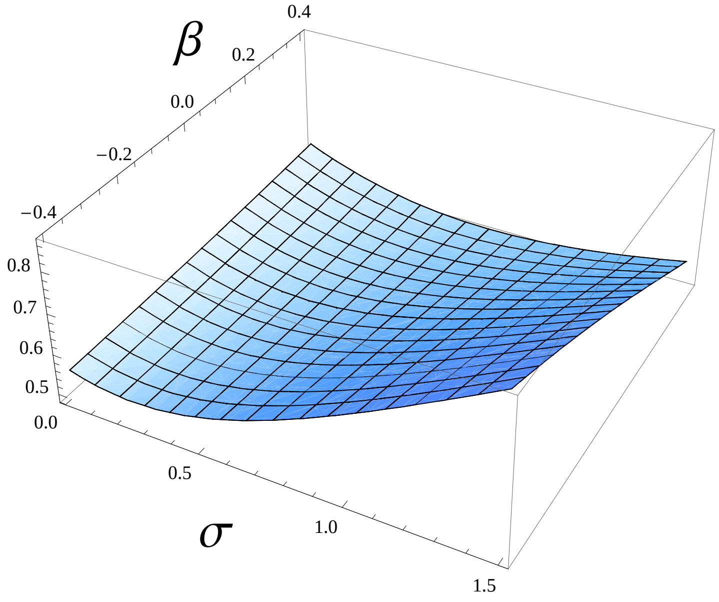
- Weak density (*β*) dependence, strong T effect (*σ*) on width
- ω_c vs. Ω_0 : divergence near vanishing $\tilde{\Omega}_{\pm} = 2\tilde{\Omega}_0 \pm \tilde{\omega}_c$

Parametric investigation: pulse width vs. σ, β : role of ω_c , Ω_0

- Pulse width $W\sqrt{M}$

L_x=0.2

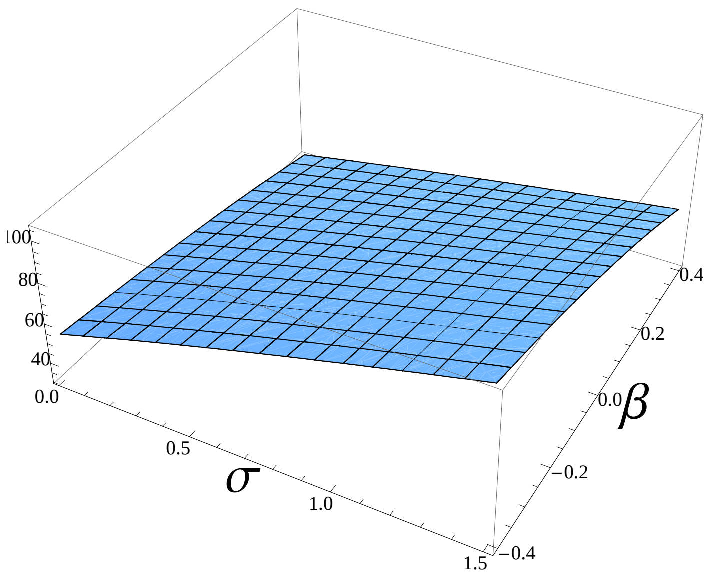
ω_c =0.21, Ω_0 =0.9:



- Pulse width $W\sqrt{M}$

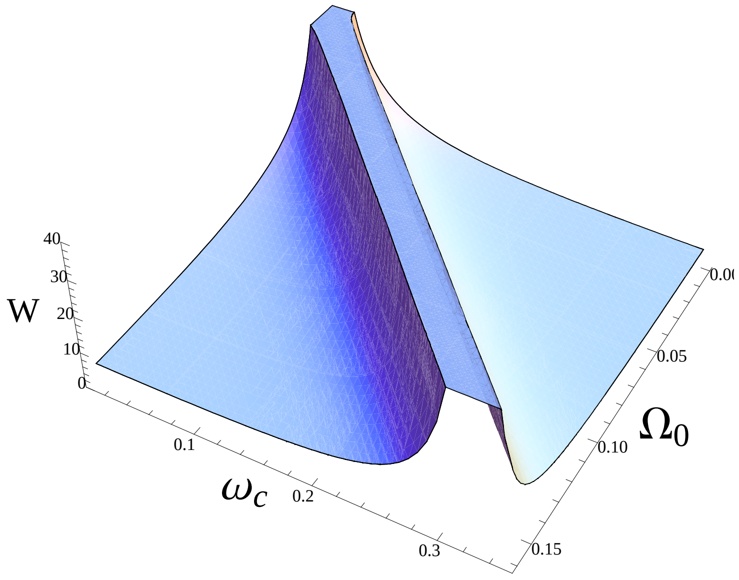
L_x=0.2

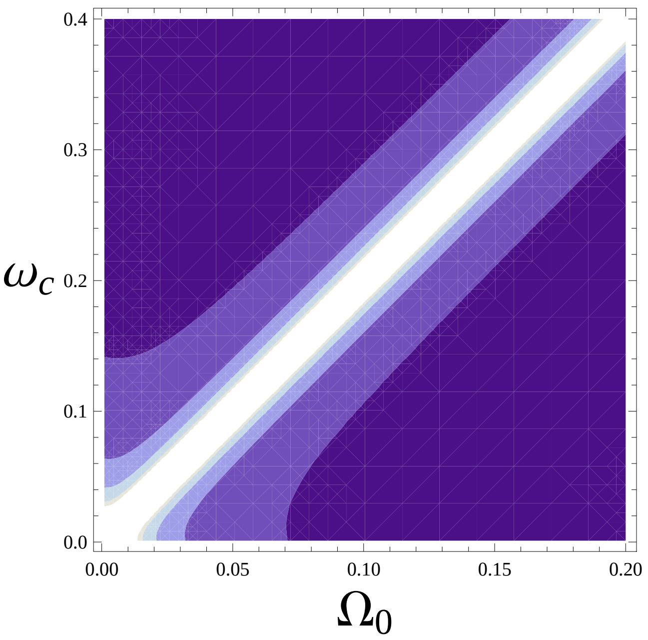
ω_c =0.21, Ω_0 =0.1:



ω_c vs. Ω_0 : Ω_0 suppresses *dispersion* divergence : narrower pulses

Parametric investigation:
pulse width vs. σ, β: *role of* ω_c , Ω_0

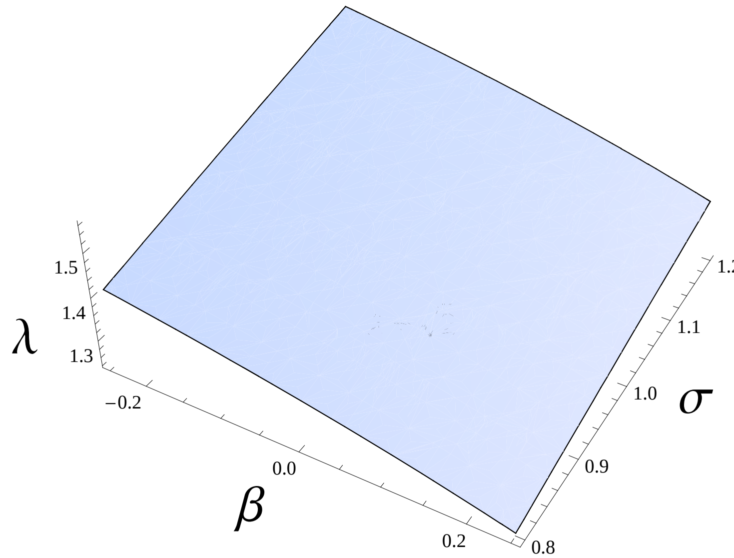


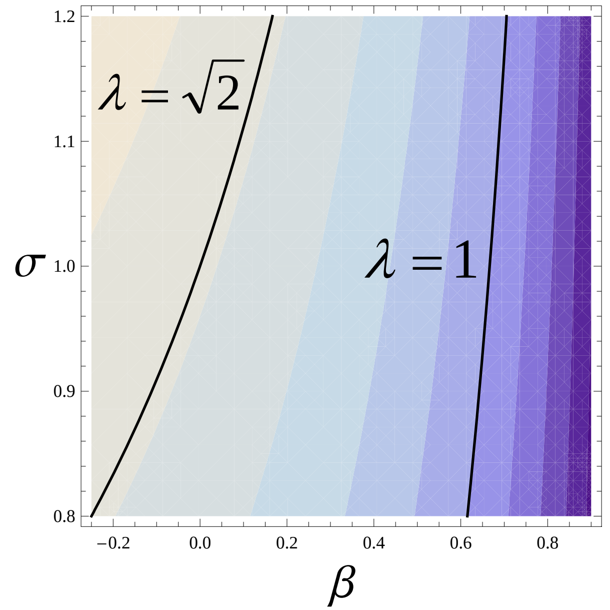


- ω_c vs. Ω_0 : divergence near vanishing
- Ω_0 , ω_c suppress(es) divergence, lead(s) to narrower pulses

Parametric investigation: pulse characteristics

- Propagation speed λ vs. *temperature ratio* σ and *density ratio* β:





- Supersonic pulses for *small* “background defect” density β
- Propagation speed can be controlled by tuning σ and β

Conclusions on the ZK description

- Positive ES pulses tuned by σ, for any *species 3 charge*
- Negative pulses for high $\sigma > 1$, for a *negative S 3 ($\beta < 0$)* mainly
- Pulse polarity controlled by *T*- and *n*- asymmetry among pair-ions
- Pulse features controlled by rotation/gyration (Larmor vs Coriolis)
- No double-layers predicted
- A modified Tanh method (details omitted) predicts nonlinear periodic excitations and singular excitations (blow-up pulses).

4. Alternative analytical approach near critical nonlinearity

- Aim: model dynamics near vanishing nonlinearity *(A<< 1)*

- lower" space and time "stretching":

$$X = \epsilon(x - \lambda t) , \qquad Y = \epsilon y , \qquad \tau = \epsilon^3 t$$

- Density, potential, velocity // **B** as previously:

$$S = S_{+}^{(0)} + \epsilon S^{(1)} + \epsilon^2 S^{(2)} + \dots$$

- Perp. **B** velocity also assumed slower:

$$u_{\pm,y/z} = \epsilon^2 u^{(1)}_{\pm,y/z} + \epsilon^3 u^{(2)}_{\pm,y/z} + \epsilon^4 u^{(3)}_{\pm,y/z} + \dots$$

Extended Zakharov-Kuznetsov equation (eZKE)

- Lowest-order (~ε) contributions remain as previously;

$$\frac{\partial \psi}{\partial \tau} + A\psi\frac{\partial \psi}{\partial X} + D\psi^2\frac{\partial \psi}{\partial X} + \frac{\partial}{\partial X}\left(B\frac{\partial^2 \psi}{\partial X^2} + C\frac{\partial^2 \psi}{\partial Y^2}\right) = 0$$

- Quadratic nonlinearity coeffient *A* as in ZK Equation

- Dispersion coefficients *B* and *C* as in ZK Equation

- Cubic nonlinearity coeffient *D*:

$$D = \frac{3}{2}B\left[\frac{\delta\lambda^2(5\lambda^2 + 8\delta\sigma)}{(\lambda^2 - 2\delta\sigma)^5} + \frac{\lambda^2(5\lambda^2 + 8)}{(\lambda^2 - 2)^5}\right]$$

Travelling wave ansatz for the eZK equation: definitions

- Moving coordinate (2D): $\zeta = L_x X + L_y Y - MT$
- Directional cosines: $L_x\,, \quad L_y\,, \quad L_x^2 + L_y^2 = 1$
- *Quartic* polynomial pseudo-energy-balance equation

$$S(\psi) = \frac{1}{B_0}\left(\frac{-M}{2}\psi^2 + \frac{A_0}{6}\psi^3 + \frac{DL_x}{12}\psi^4\right)$$

- Definitions:

$$A_0 = AL_x\,, \qquad B_0 = L_x R\,, \qquad R = BL_x^2 + CL_y^2$$

Travelling wave ansatz for the eZK equation: definitions and pulse soliton solution

- Pulse soliton solution: instead of the KdV soliton, we now obtain [Wadati JPSJ 1975]:

$$\psi = \frac{6M}{D}\left[\varphi_{1/2}\sinh^2\left(\frac{1}{2}\sqrt{\frac{M}{R}}\zeta\right) - \varphi_{2/1}\cosh^2\left(\frac{1}{2}\sqrt{\frac{M}{R}}\zeta\right)\right]^{-1}$$

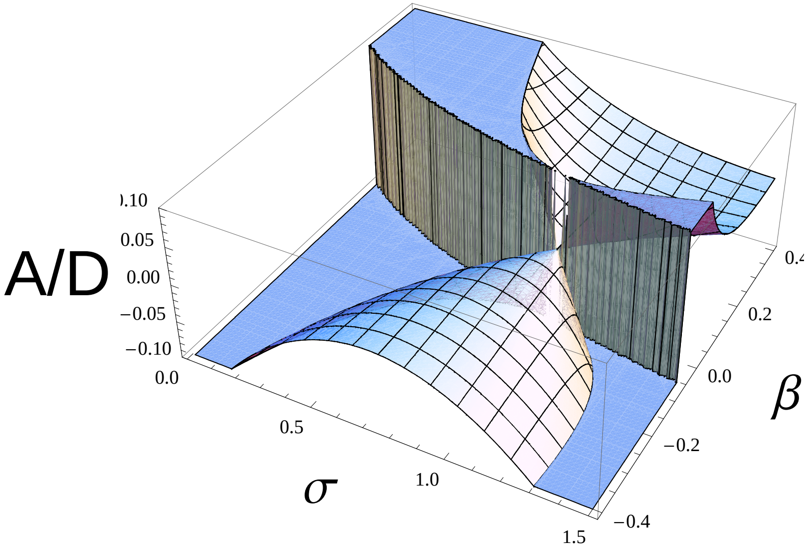
- Pulse characteristics:

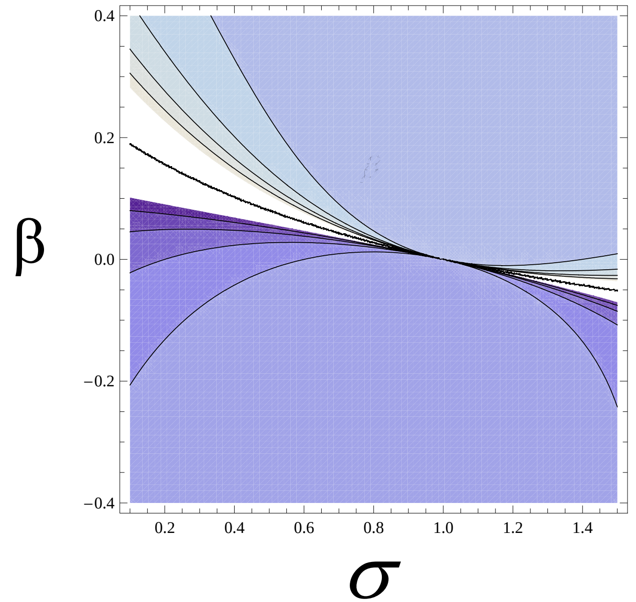
$$\varphi_{1/2} = -\frac{1}{D}\left(A \pm \sqrt{A^2 + \frac{6MD}{L_x}}\right)$$

- Functions of $\frac{T_+}{T_-} \equiv \sigma$ and $\delta = \frac{n_{+,0}}{n_{-,0}} = 1 - s\frac{Z_3}{Z}\frac{n_3}{n_{-,0}} \equiv 1 - \beta$

- Now, *both positive and negative pulses* are possible, $\forall\ A$.

Testing the weak quadratic nonlinearity hypothesis

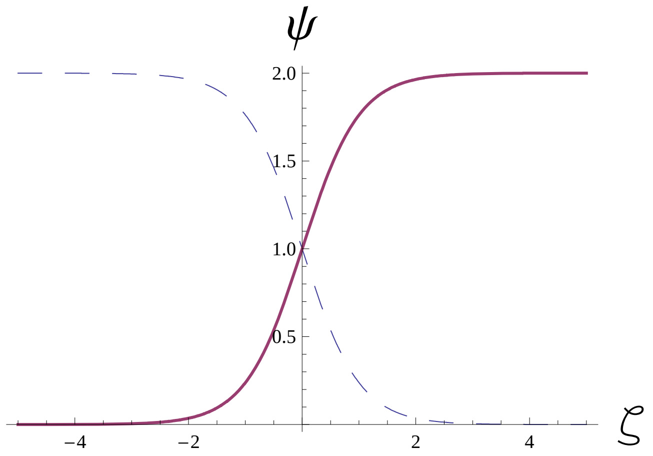




The ratio A/D remains within 10% in the vicinity of 0 (~ε) for most values of β near σ =1:

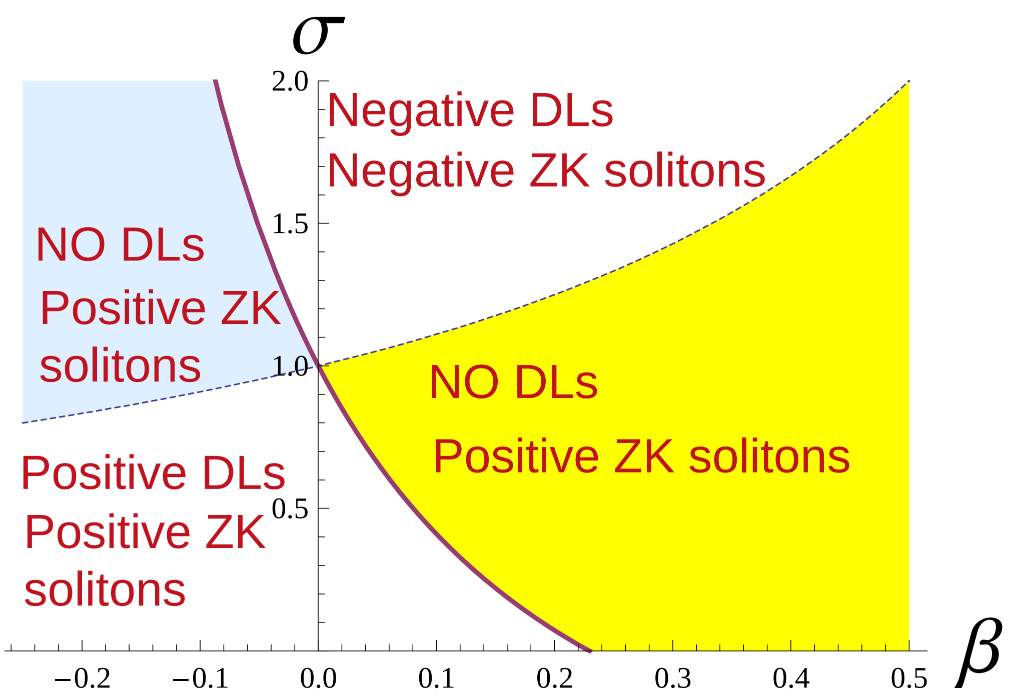
a posteriori confirmation of the eZK theory!

Parametric investigation: Double Layers vs pulses



- Positive ZK solitons (pulses) exist where *A>0*
- Negative ZK solitons exist where *A<0*
- eZK solitons of either polarity exist everywhere

Double Layers (DLs) exist in the white regions:



Conclusions on the eZK description

- Positive and negative ES pulses predicted
- Characteristics determined by σ, possible for *species 3 + or -*
- Pulse features controlled by rotation/gyration (Larmor vs Coriolis)
- Double-layers predicted, of both polarities
- Theory generically relevant in pair plasmas (*A* << *D*).

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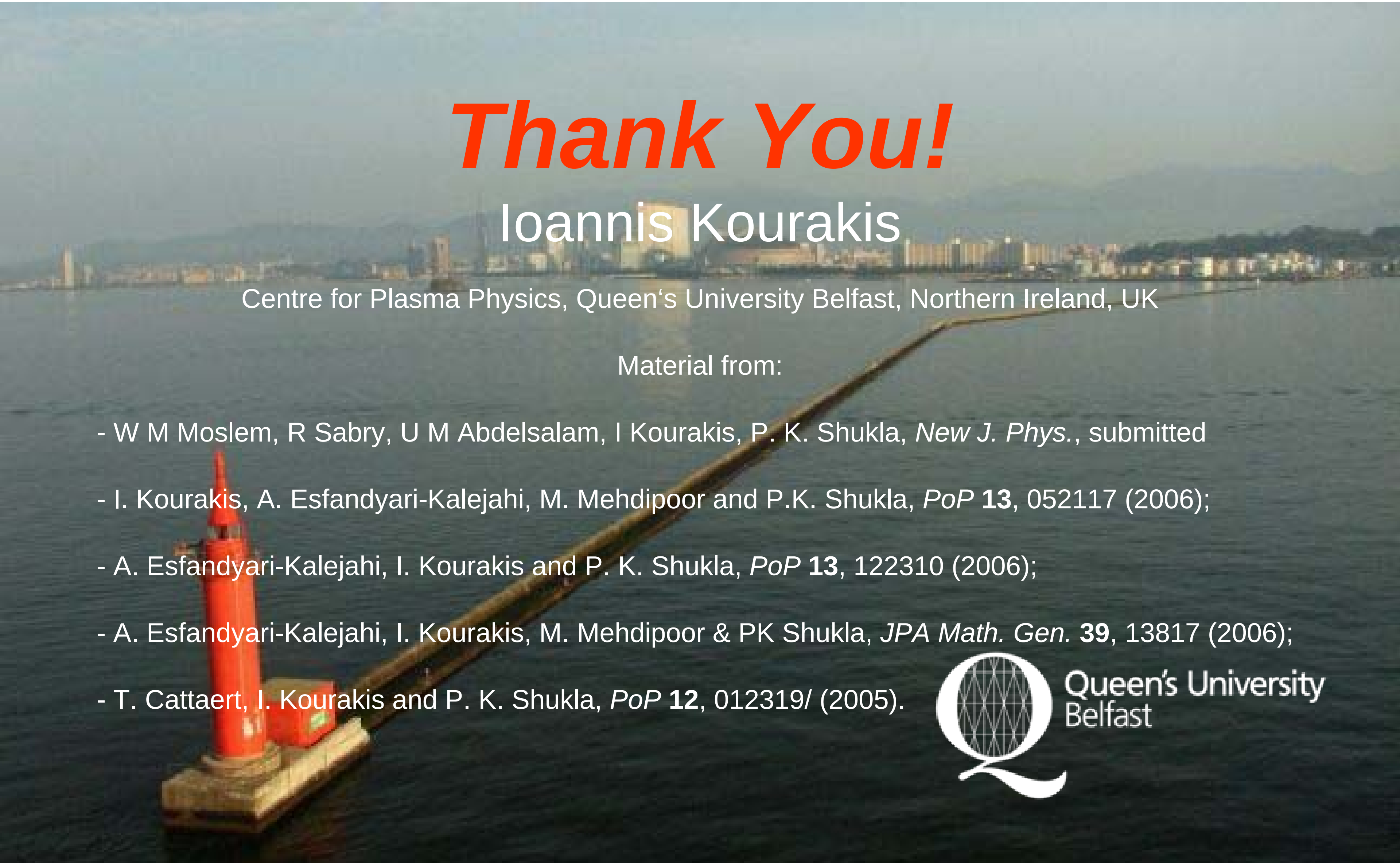

Thank You!

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Material from:

- W M Moslem, R Sabry, U M Abdelsalam, I Kourakis, P. K. Shukla, *New J. Phys.*, submitted
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