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Colóquio, 27 May 2014

Nonlinear Waves in Physics:
from tsunami and freak waves ...
... to optical signal transport

A paradigm across disciplines

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Avant propos ...

- Thanks - Acknowledgments:
  - Hospitality: *UFRGS - Instituto de Física*: Marcia Barbosa, Naira Maria Balzaretti
  - Collaboration: Fernando Haas and group
  - Inspiration & interactions: *UFRGS - IF* members & colleagues
  - Funding: CNPq - *Science Without Borders*

- *Basic questions to be addressed:*
  - *What are* **nonlinear waves**?
  - *How* are they formed?
  - *Where* do we find NWs?
  - *Why* study NWs?
  - Are these of any use, e.g. in applications?
  - *Focus* on NWs in plasma physics

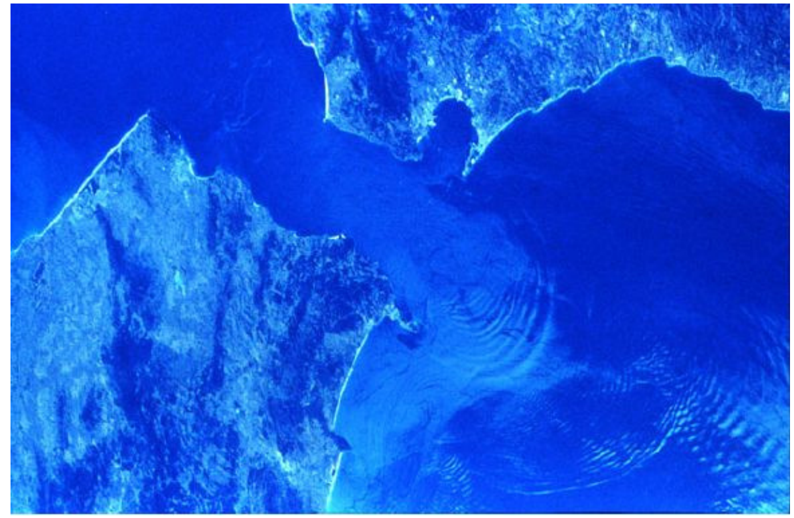
Solitary waves crossing on a beach

Tidal wave on river Severn (England):
goddess Sabrina (later Noadu) rode on the crest of the Severn bore according to an ancient (Gaellic) myth

Tidal wave on river Severn (2): surfer's paradise

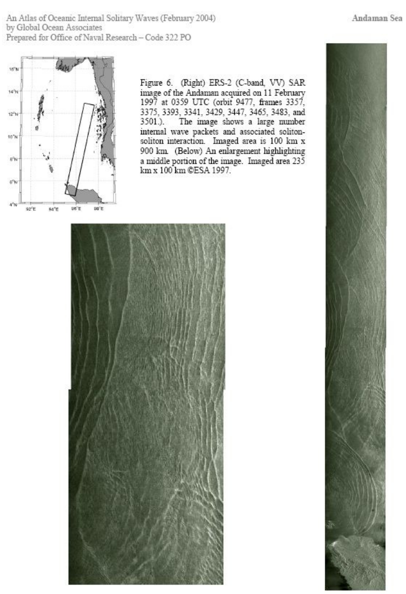
[M. A. Porter, N. J. Zabusky, B. Hu, D. K. Campbell, American Scientist **97** (3), 214 (2009)]

Ocean solitons east of the Strait of Gibraltar

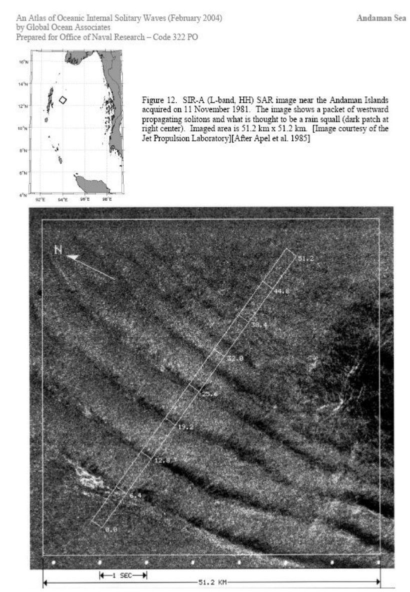


[Credit: Frank Verheest.]

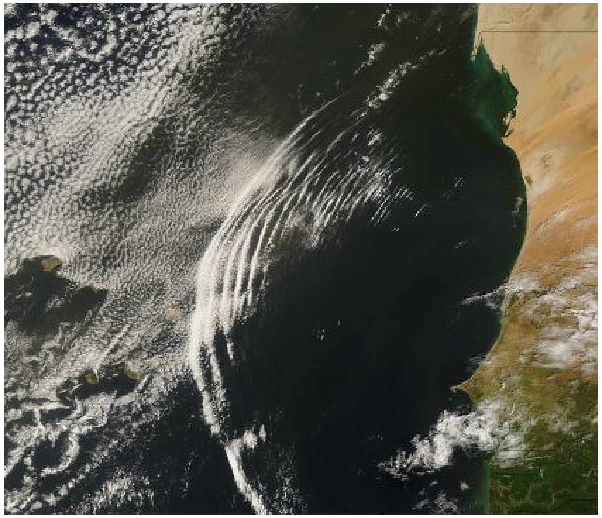
Internal solitons in the Andaman sea (1)



Internal solitons in the Andaman sea (2)



Solitary clouds west of Africa



[Credit: Frank Verheest.]

“Morning Glory” cloud on Gulf of Carpentaria (Australia)

[Credit: Frank Verheest.]

“Morning Glory” cloud on Gulf of Carpentaria (Australia) (2)

Laser plasma experiments on electrostatic waves

PRL **101,** 025004 (2008)   PHYSICAL REVIEW LETTERS   week ending 11 JULY 2008

Observation of Collisionless Shocks in Laser-Plasma Experiments

L. Romagnani,[1,*] S. V. Bulanov,[2,3] M. Borghesi,[1] P. Audebert,[4] J. C. Gauthier,[5] K. Löwenbrück,[6] A. J. Mackinnon,[7] P. Patel,[7] G. Pretzler,[6] T. Toncian,[6] and O. Willi[6]
[1]*School of Mathematics and Physics, The Queen's University of Belfast, Belfast, Northern Ireland, United Kingdom*

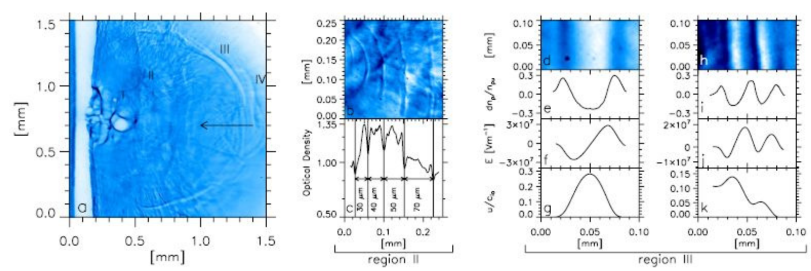


FIG. 1 (color online). (a) Typical proton imaging data taken at the peak of the interaction pulse with protons of 7 MeV energy. Note the strong modulation associated with the ablating plasma in the region I and the modulated pattern ahead of the shock front possibly associated with a reflected ion bunch in the region IV. The arrow indicates the laser beam direction. (b)–(c) Detail and RCF optical density lineout corresponding to the region II showing modulations associated with a train of solitons. (d)–(k) Details of the region III and correspondent lineouts of the probe proton density $\delta n_p/n_{p0}$, reconstructed electric field E, and reconstructed normalized ion velocity u/c_{ia} in the case of an ion acoustic soliton (d)–(g) and of a collisionless shock wave (h)–(k) (the collisionless shock detail corresponds to a different shot not shown here for brevity).

Laser plasma experiments on electrostatic waves (2)

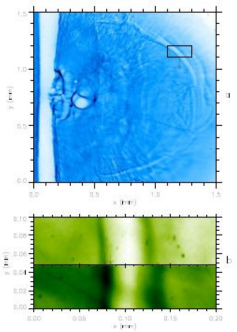


Figure 3.9: a. Proton image showing the ion-acoustic soliton. b Detail of the ion-acoustic soliton at two different times. The relative time between the two frames is ~ 25 ps, and the soliton has moved by ~ 5 ÷ 10 μm.

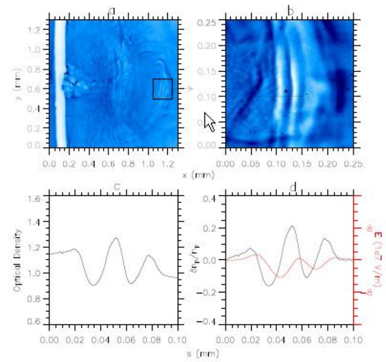


Figure 3.14: a. Proton image taken at the peak of the interaction pulse. b. Detail of the collisionless shock wave. c. Profile of the optical density in the RCF across the collisionless shock. d. Proton density modulation across the shock (black

(Credit: Lorenzo Romagnani & Marco Borghesi, Queen's University Belfast, UK)

Electron-holes observed via proton imaging diagnostics

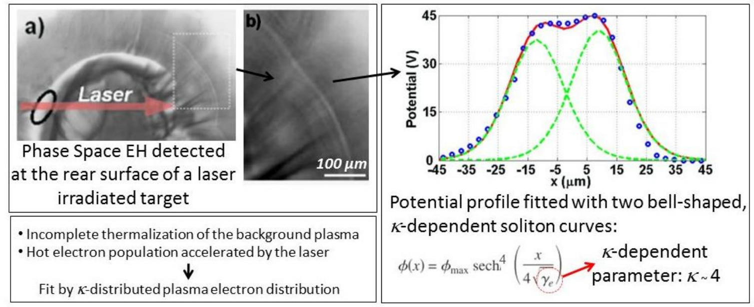
PHYSICS OF PLASMAS **17**, 010701 (2010)

Observation and characterization of laser-driven phase space electron holes

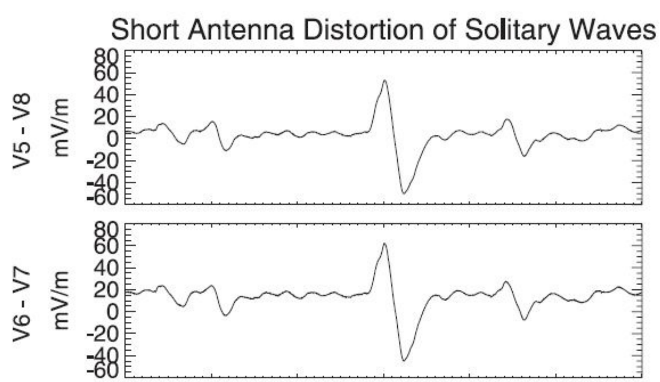
G. Sarri,[1] M. E. Dieckmann,[2] C. R. D. Brown,[3] C. A. Cecchetti,[1] D. J. Hoarty,[3] S. F. James,[3] R. Jung,[4] I. Kourakis,[1] H. Schamel,[5] O. Willi,[4] and M. Borghesi[1]
[1]*School of Mathematics and Physics, The Queen's University of Belfast, Belfast BT7 1NN, United Kingdom*
[2]*ITN, Linkoping University, 60174 Norrkoping, Sweden*
[3]*AWE, Aldermaston, Reading, Berkshire RG7 4PR, United Kingdom*
[4]*Institute for Laser and Plasma Physics, Heinrich-Heine-University, 40225 Düsseldorf, Germany*
[5]*Physikalisches Institut, Universität Bayreuth, D-95440 Bayreuth, Germany*

(Received 12 November 2009; accepted 15 December 2009; published online 7 January 2010)

The direct observation and full characterization of a phase space electron hole (EH) generated during laser-matter interaction is presented. This structure, propagating in a tenuous, nonmagnetized plasma, has been detected via proton radiography during the irradiation with a ns laser pulse ($I\lambda^2 \approx 10^{14}$ W/cm^2) of a gold *hohlraum*. This technique has allowed the simultaneous detection of propagation velocity, potential, and electron density spatial profile across the EH with fine spatial and temporal resolution allowing a detailed comparison with theoretical and numerical models.



FAST satellite observations of large solitary spikes in the Earth's *auroral* region



FAST auroral observations (2)

Nonlinear Processes in Geophysics (1999) 6: 187–194

Properties of fast solitary structures

R. E. Ergun, C. W. Carlson, L. Muschietti, I. Roth, and J. P. McFadden
Space Sciences Laboratory, University of California, Berkeley, CA, 94720, USA

Received: 15 June 1999 – Revised: 6 September 1999 – Accepted: 13 September 1999

Abstract. We present detailed observations of electromagnetic waves and particle distributions from the Fast Auroral SnapshoT (FAST) satellite which reveal many important properties of large-amplitude, spatially-coherent plasma structures known as "fast solitary structures" or "electron phase space holes". Similar structures have been observed in

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Ergun, et al.: Properties of fast solitary structures

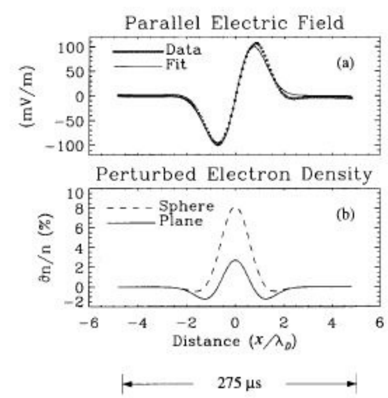
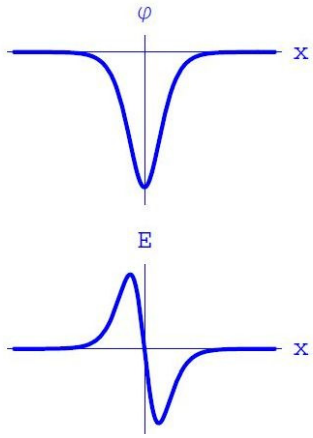
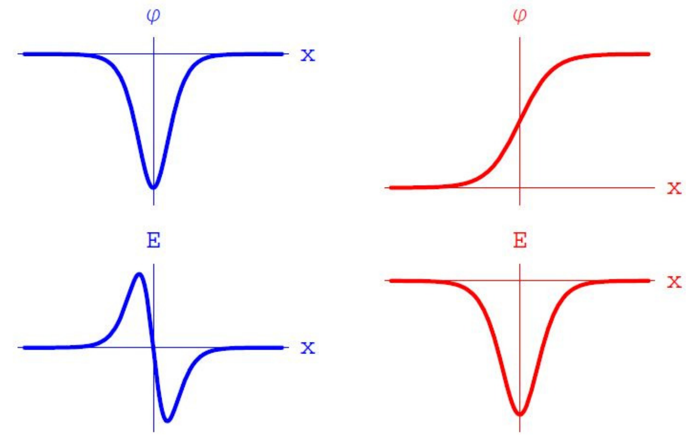


Fig. 5. (From Ergun et al., 1998c) (a) $\Delta E_\parallel$. The dots are the data at 0.5 μs resolution translated into Debye lengths assuming a constant parallel velocity, v_{sol} = 3.2x10^6. The smooth trace is the fit to Eq. (2). (b) Calculated charge densities assuming spherical and planar geometry. The plasma conditions were n_o = 5.7 ± 2.0 cm^{-3}, $Te_\parallel$ = 704 ± 145 eV, v_{sol} = 3.2x10^6 ± 1.1x10^6 m/s, $Ti_\perp$ = 370 ± 74 eV, $|B_o|$ = 11481 ± 10 nT, λ_D = 82 ± 30 m, and ρ_{H+} = 241 ± 24 m.

Electrostatic potential and electric field bipolar structures (1)



ES potential and E-field: *bipolar* vs *monopolar* structures (2)



An advanced topic: "Rogue waves" (*freak waves*) – an emerging concept

- *Rogue waves* are localized excitations (events) of extreme amplitude, exceeding twice the average strength of background turbulence level;

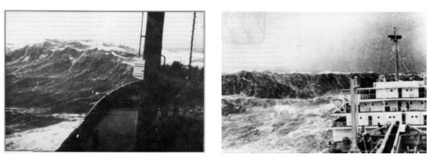
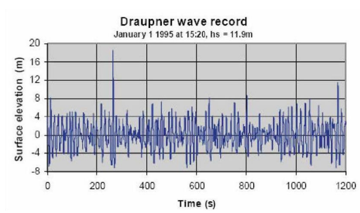


Fig. 2. Various photos of rogue waves.



Data from the Draupner platform event in Norway (Jan. 1995).

Credit: Kharif & Pelinovsky, Eur. Journal of Mechanics B/Fluids **22**, 603 (2003).

Rogue wave measurements at Campos Basin, Brazil

GEOFIZIKA VOL. 21 2004

Original scientific paper
UDC 551.465

Freak Waves at Campos Basin, Brazil

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Received 20 July 2004, in final form 25 November 2004

1.2 Instrumental Registrations and Related Problems 21

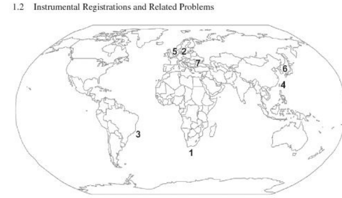

Fig. 1.3 Some instrumental registrations of freak waves (ordered by the number of reported freak waves). **1**) Offshore from Mossel Bay (1563 events, 100 m depth, gas-drilling platform) (Liu and MacHutchon 2006). **2**) The Baltic Sea (414 events, 7–20 m depth, buoys) (Paprota et al. 2003). **3**) Campos Basin near Rio de Janeiro (276 events, 1050 m and 1250 m depth, buoys) (Pinho et al. 2004). **4**) Off the eastern coast of Taiwan (175 events, 43 m depth, buoys) (Chien et al. 2002). **5**) The North Sea (at least 107 events, 126 m and 85 m depth, platforms) (Stansell 2004, 2005, Haver and Andersen 2000). **6**) Sea of Japan (14 events, 43 m depth, ultrasonic submerged gauges) (Mori et al. 2002). **7**) The Black Sea (3 events, 85 m depth, buoy) (Lopatoukhin et al. 2003, Divinsky et al. 2004)

Annales Geophysicae (2004) 22: 1839–1842
SRef-ID: 1432-0576/ag/2004-22-1839

Freak waves - more frequent than rare!

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Received: 21 August 2003 – Revised: 20 November 2003 – Accepted: 8 January 2004 – Published: 8 April 2004

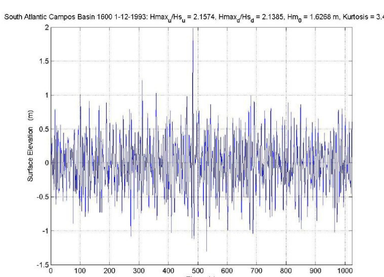


Fig. 2. An example of Draupner-like freak wave time series data recorded in Campos Basin, South Atlantic Ocean, by a heave-pitch-roll buoy moored in over 1000 m depth.

[Credit: Pinho *et al.*, Geofizika **21** (2004); Liu and Pinho, *Annales Geophysicae*, **22**, 1839 (2004)]

nature Vol 450|13 December 2007|doi:10.1038/nature06402

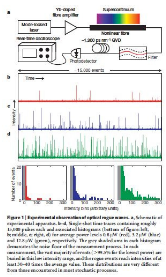
LETTERS

Optical rogue waves

D. R. Solli[1], C. Ropers[1,2], P. Koonath[1] & B. Jalali[1]

Recent observations show that the probability of encountering an extremely large rogue wave in the open ocean is much larger than expected from ordinary wave-amplitude statistics[1–3]. Although considerable effort has been directed towards understanding the physics behind these mysterious and potentially destructive events, the complete picture remains uncertain. Furthermore, rogue waves have not yet been observed in other physical systems. Here, we introduce the concept of optical rogue waves, a counterpart of

Although the physics behind rogue waves is still under investigation, observations indicate that they have unusually steep, solitary or tightly grouped profiles, which appear like "walls of water"[10]. These features imply that rogue waves have relatively broadband frequency content compared with normal waves, and also suggest a possible connection with solitons—solitary waves, first observed by J. S. Russell in the nineteenth century, that propagate without spreading in water because of a balance between dispersion and nonlinearity. As



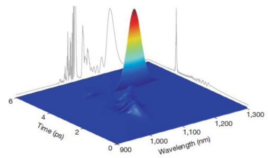


Figure 3 | Time–wavelength profile of an optical rogue wave obtained from a short-time Fourier transform. The optical wave has broad bandwidth and has extremely steep slopes in the time domain compared with the typical events. It appears as a 'wall of light' analogous to the 'wall of water' description of oceanic rogue waves. The rogue wave travels a curved path in time–wavelength space because of the Raman self-frequency shift and group velocity dispersion, separating from non-solitonic fragments and remnants of the seed pulse at shorter wavelengths. The grey traces show the full time structure and spectrum of the rogue wave. The spectrum contains sharp spectral features that are temporally broad and, thus, do not reach large peak power levels and do not appear prominently in the short-time Fourier transform.

Credit: D.R. Solli, C. Ropers, P. Koonath, B. Jalali, Nature **450**, 1054 (2007).

IOP Publishing Plasma Physics and Controlled Fusion
Plasma Phys. Control. Fusion **56** (2014) 035007 (17pp) doi:10.1088/0741-3335/56/3/035007

Freak waves and electrostatic wavepacket modulation in a quantum electron–positron–ion plasma

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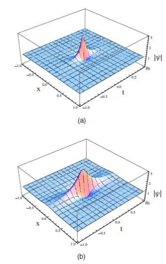


Figure 17. Peregrine's solution is depicted for two values of β. The first figure has $k = 2.5$, $T_p/T_e = 1$ and $\beta = 0$, whereas $\beta = 0.1$ in the second, which is of greater duration and spatial extension (i.e. less localized).

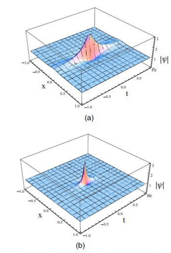


Figure 18. The duration in both space and time of Peregrine's solution is observed for two values of T_p/T_e. The first figure has $k = 3.5$, $T_p/T_e = 0.5$ and $\beta = 1$, whereas $T_p/T_e = 1.5$ in the second, which leads to a shorter duration.

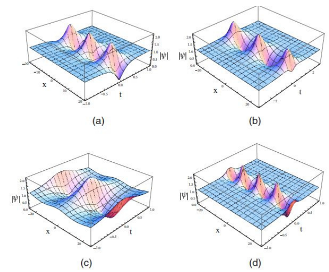


Figure 19. Plots of Akhmediev's breather for different values of β and T_e/T_p: (*a*), (*b*) $k = 2.5$, $T_p/T_e = 1$, $\beta = 0,\ 0.1$; (*c*), (*d*) $k = 3.5$, $\beta = 1$, $T_p/T_e = 0.5,\ 1.5$.

Outline

- Introduction: *Nonlinear Nature*, *Nonlinear World*: Prerequisites
- *Solitary Waves* – a paradigm across scientific disciplines: first principles, modelling, conditions for occurrence
- Focus on charged many-particle systems (*plasmas*)
- Part A: *Korteweg - de Vries soliton theory*: history and applications
- Part B: *Energy localization* and *nonlinear Schrödinger theory* for envelope pulses: from first principles to observables
- Conclusions and Summary

Nonlinear excitations – preliminaries

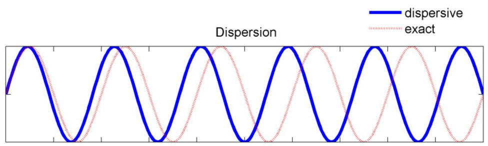
Solitary waves (SWs):

- ... occur in abundance in Nature, in various physical contexts
- ... are ***localized coherent structures,*** bearing remarkable properties: preserve their shape (stationary profile), are robust, i.e. persist against perturbations and collisions with one another, ...
- ... represent localized lumps of energy, whose manifestation may be constructive (e.g. signal transmission) or destructive (*tsunami*)
- ... bear various generic forms and names: ***pulses, kinks, holes, shocks, double layers*** or ***potential dips*** (in plasmas), ...
- may either be non-periodic forms (e.g., pulses) or may possess a quasi-periodic internal structure (e.g., oscillons, envelope pulses, breathers)

Solitary waves require a balance between:

– ***Dispersion***, manifested via:

- wave spreading in Fourier space: different modes ($\mathbf{k}$) travel at different speeds:

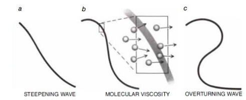


(Source: http://www.scholarpedia.org)

- Chromatic dispersion effect in Optics (*rainbow!*)
- Curvature in the dispersion curve $\omega = \omega(k)$, in solid state physics.
- *The phase speed* $v_{ph} = \frac{\omega}{k} = f(\mathbf{k})$ *is a function of the wavenumber* $\mathbf{k}$.

... and ***Nonlinearity***, manifested as:

- Amplitude-dependence of the phase speed: larger amplitudes travel faster!
- This results in wave steepening, and eventually wave-breaking:



Waves steepen because the higher-amplitude part of a wave moves faster. As time passes, a wave (*a*) grows steeper and more abrupt. In a sound wave, collisions between individual air molecules (molecular viscosity) transfer momentum forward (*b*) and prevent the wave from overtaking the slow-moving material that lies upstream. In strong collisionless shocks, this process can lead to overturning of the wave (*c*).

... a physical phenomenon well-know to seafarers (or *surfers*):

Further tracers of nonlinearity include:

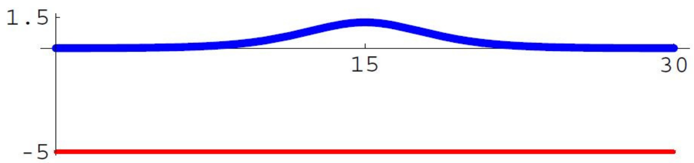
- *Secondary harmonic generation*
- *No superposition principle*: different normal (Fourier) modes do not sum up
- *Sidebands* appear in the Fourier spectrum
- ***Energy localization***

Part A – KdV theory: *Saga* of an equation

Soliton history begins on a Scottish canal 180 years ago

when a young engineer named **John Scott Russell** was hired for a summer job, to investigate how to improve the efficiency of barges in canals.
One day, he *"was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped; the mass of water which it had put in motion ... rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel without change of form or diminution of speed.*
I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel." (JSR, 1834)

Russell's soliton



(Original values in feet.)

Russell's soliton recreated in 1995

(Credit: Heriot-Watt University, Edinburgh, Scotland.)

On shallow water solitary waves ...

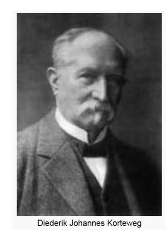
- 1834-1844: **John Scott Russell** experimentally observed his ***"great solitary wave of translation"*** in 1834 and reported it during the 1844 Meeting of the British Association for the advancement of science.

- It was not immediately believed that Scott Russell's solitary wave was of importance. Theoretical descriptions had to wait until ...

- 1871: French mathematician **Joseph Valentin Boussinesq** proposed his dispersive-nonlinear model for water surface waves, inspired by Russell's observation. 

- *In 1876*, Lord Rayleigh published his mathematical theory to support Russell's experimental observation.

- 1895: 20 years later, Dutch mathematician **Diederik Johannes Korteweg** and his doctoral student **Gustav de Vries**



developed their theory for shallow water waves (*the KdV theory*).

The *Korteweg-de Vries (KdV) equation* reads:

$$\frac{\partial \phi}{\partial t} + a\,\phi\,\frac{\partial \phi}{\partial x} + b\,\frac{\partial^3 \phi}{\partial x^3} = 0$$

where a is the nonlinearity coefficient, and b is the dispersion coefficient.

More history: the FPU paradox

- 1955: Nobel prize winner **Enrico Fermi**, computer expert and physicist **John Pasta** and mathematician **Stan Ulam**, along with their talented student **Mary Tsingou**, in a classified scientific report with the title *''Studies of nonlinear problems"* (Fermi et al., Los Alamos report, 1955), proposed a chain model for anharmonic particle interactions in solids.

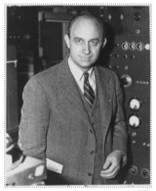


Figure 1: Enrico Fermi


Figure 2: Stan Ulam


Figure 3: Mary Tsingou

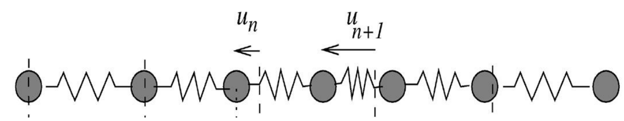
- Their aim was to use the power of the freshly developed MANIAC computer at Los Alamos (USA), to investigate the relaxation of a particle chain towards thermal equilibrium, under the effect of anharmonic interparticle forces.

- The FPU chain is a mechanical model that consists of identical atoms exerting forces on their nearest neighbors, according to the second order ordinary differential equations (ODEs):

$$\frac{d^2 q_j}{dt^2} = F(q_{j+1} - q_j) - F(q_j - q_{j-1}) \; , \; j = \ldots, -1, 0, 1, \ldots \; .$$

where the inter-particle forces are anharmonic:

$$F(x) = x + \alpha x^2 + \beta x^3 + \ldots \; .$$



- The expectation was that energy launched into the system via an initial mode would spread along all modes, according to the *equipartition theorem*.

The FPU-paradox

- The original expectation of Fermi, Pasta and Ulam was never confirmed!
- After an initial flow of energy towards other modes, the total energy was observed to almost fully return to the initial mode (*FPU recurrence*).

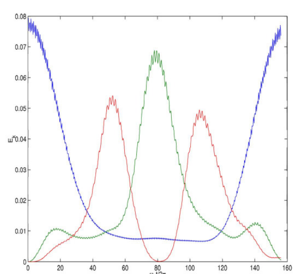


Figure 2: FPU recurrence for a FPU-α model with $N = 32$ masses and fixed ends. The plot shows the time evolution of the energy (kinetic + potential) $E_k = (\dot{A}_k^2 + \omega_k^2 A_k^2)/2$ of each of the three lowest normal modes, related to the displacements through $A_k = \sqrt{2/(N+1)} \sum_{n=1}^{N} u_n \sin(nk\pi/(N+1))$ with the frequencies $\omega_k^2 = 4\sin^2(k\pi/(2N+2))$ .Initially, only mode $k = 1$ (blue) is excited. After flowing to other modes, $k = 2$ (green), $k = 3$ (red), etc., the energy almost fully returns to mode $k = 1$: this was a surprise! This picture might be easily reproduced using the MATLAB code provided below.

[Figure from Thierry Dauxois and Stefano Ruffo, Scholarpedia, **3**(8):5538 (2008)]

Zabusky & Kruskal

- 1965: Zabusky and Kruskal explained the **Fermi-Pasta-Ulam (FPU) paradox** by introducing the KdV equation in a "plasma" lattice:

VOLUME 15, NUMBER 6 PHYSICAL REVIEW LETTERS 9 AUGUST 1965

INTERACTION OF "SOLITONS" IN A COLLISIONLESS PLASMA AND THE RECURRENCE OF INITIAL STATES

N. J. Zabusky
Bell Telephone Laboratories, Whippany, New Jersey
and
M. D. Kruskal
Princeton University Plasma Physics Laboratory, Princeton, New Jersey
(Received 3 May 1965)

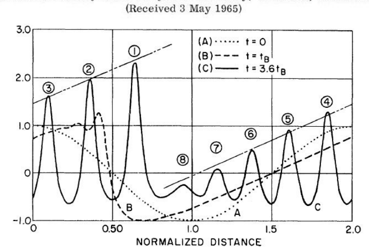


FIG. 1. The temporal development of the wave form $u(x)$.

- Since 1965, this has led to explosion of applications, including many in plasma and solid state physics
- Ingredients of nonlinear evolution equations like KdV are:
  - ⋆ slow time variation
  - ⋆ dispersion
  - ⋆ nonlinearity
- Solutions in terms of solitons rely on extraordinary stability, ...
- ... constrained by infinite series of first integrals (constants of motion).
- KdV equation is an integrable dynamical system $\rightarrow$ conservation laws.

Plasmas – preliminaries

- Plasmas are ***large ensembles of charged particles***
- Particles interact with one another ("Coulomb"/Debye interactions, "collisions")
- They interact with external electric/magnetic fields
- Many-body statistical effects (long-range correlations)
- Rigorous analytical approach:
  N-body statistics $\Rightarrow$ 1-body pdf $\Rightarrow$ kinetic theory
- Reduced description: equations for "moments" of the pdf

  $\Rightarrow$ *Plasma-Fluid Theory*

Korteweg de Vries (KdV) theory for electrostatic (ES) waves

Taniuti and Wei [J. Phys. Soc. Jpn. **24**, 941 (1968)] propose their *reductive perturbation technique*, for long-wavelength ES *acoustic modes* in plasmas.

We review the basic qualitative aspects of this technique below.

- Dispersion relation (*acoustic mode*):

$$\omega \simeq v_{ph} k + A k^3 + \dots ,$$

(where A is to be determined, for a given plasma composition), thus

$$kx - \omega t \simeq k(x - v_{ph} t) - A k^3 t + \dots .$$

- Appropriate space/time stretching

$$\xi = \epsilon^{1/2}(x - Vt) , \qquad \tau = \epsilon^{3/2} t \qquad (V \in \Re)$$

- $n \simeq n_0 + \epsilon n_1 + \epsilon^2 n_2 + ...; \quad u \simeq \epsilon u_1 + \epsilon^2 u_2 + ...; \quad \phi \simeq \epsilon \phi_1 + \epsilon \phi_2 + ...$

Plasma fluid *toy-model* for electrostatic waves (1D)

Continuity (for plasma species s, e.g. *ions*):

$$\frac{\partial n_s}{\partial t} + \frac{\partial \phi}{\partial x}(n_s u_s) = 0$$

Mean velocity u_s equation:

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} = -\frac{q_s}{m_s} \frac{\partial \phi}{\partial x}$$

The potential Φ obeys *Poisson's* eq.:

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_{species\ s'} q_{s'} n_{s'} = 4\pi e (n_e - Z_i n_i + ...)$$

– At a given dynamical scale for species s $(= e, i, d)$, the state of other species may be prescribed by simplifying assumptions;

– Typical paradigm: for ion-acoustic waves $(s = i)$, *ions* are inertial, so *electrons* are assumed at equilibrium (e.g. Maxwellian: $n_e \sim e^{e\phi/k_B T_e}$).

- The method is rather tedious yet straigfhtforward; details are omitted here.
- *Korteweg-de Vries (KdV) equation*:

$$\frac{\partial \psi}{\partial \tau} + A\,\psi\,\frac{\partial \psi}{\partial \xi} + B\frac{\partial^3 \psi}{\partial \xi^3} = 0\,.$$

  - ⋆ $\psi = \phi_1$ denotes a small ($\sim \epsilon \ll 1$) correction to the electric potential,
  - ⋆ Constraint: $V = c_s \quad \rightarrow$ propagation at (or slightly above) the sound speed.
  - ⋆ The coefficients A and B incorporate the physics of the particular problem considered, as they contain the dependence on relevant plasma parameters (lengthy expressions omitted here).
  - ⋆ The *dispersion coefficient* B is positive;
  - ⋆ The *nonlinearity coefficient* A determines the soliton polarity, i.e., the sign (positive/negative) of the soliton pulse ($\rightarrow$ *next slide*).

- The soliton solution of the KdV equation above reads:

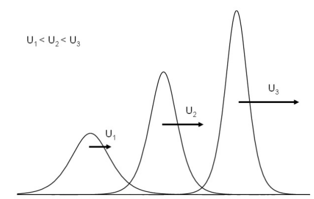
$$\psi = \psi_0\,\mathrm{sech}^2\left(\frac{\xi - U\tau}{L}\right)$$

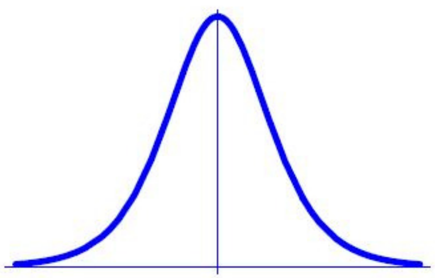
which represents a propagating pulse.

Here:

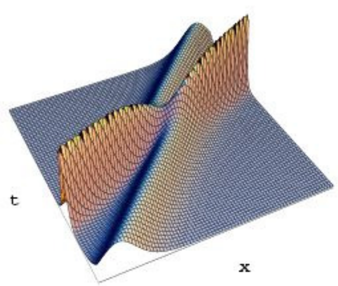
  - ⋆ U is the soliton velocity increment (total soliton speed $= c_s + \epsilon U$)
  - ⋆ $\psi_0 = \frac{3U}{A}$ is the maximum *soliton amplitude*, and
  - ⋆ $L = 2\sqrt{B/U}$ is the *soliton width*.

- Width-amplitude relation: $\psi_0 L^2 = 12B/A = \mathrm{constant}$, thus *faster* solitons are *taller* and *narrower*:



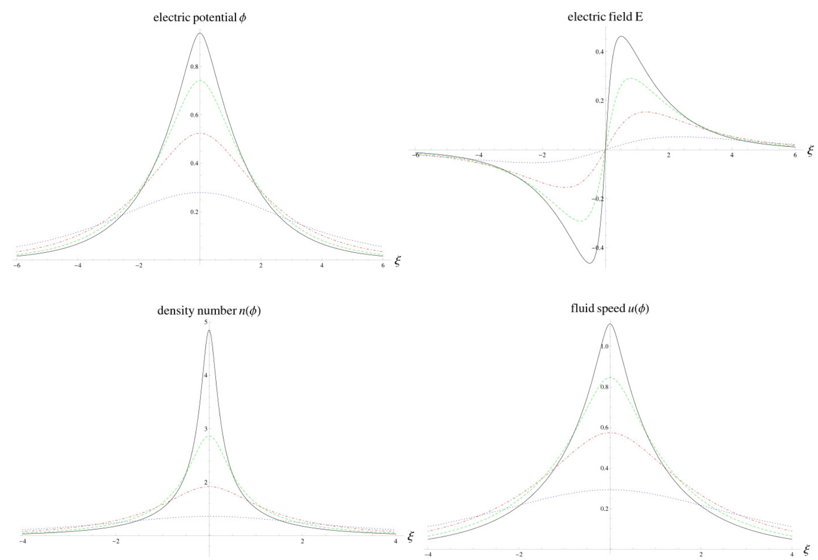


Typical shape of positive potential KdV soliton (in arbitrary units)



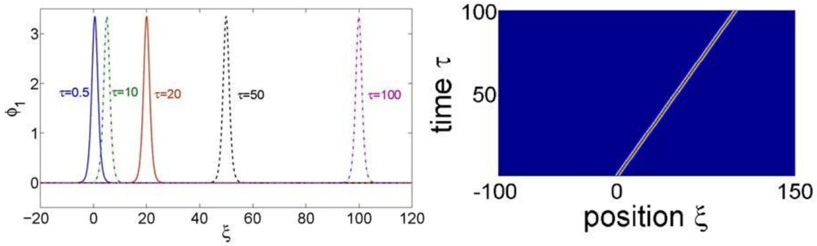
Typical interaction between two positive potential KdV solitons

Solution in terms of ϕ + ambipolar field $E = -\nabla\phi$ and fluid variables n, u



Solitons in action (1a)

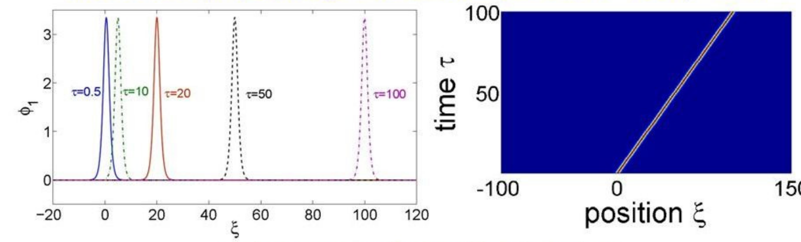
KdV soliton (propagating in Maxwellian plasma; κ=100, μ =0.1)



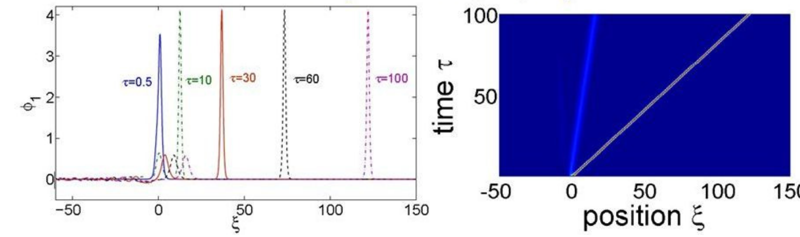
[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

Solitons in action (1b): non-Maxwellian electron background

KdV soliton (propagating in Maxwellian plasma; κ=100, μ =0.1)



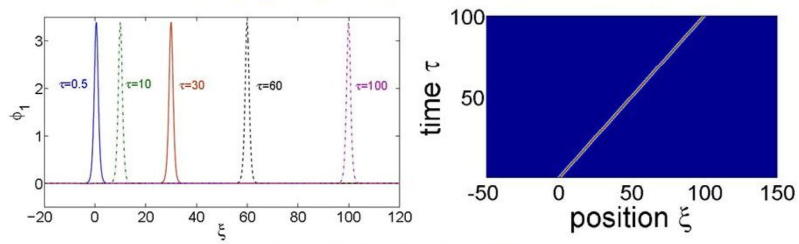
... enters a superthermal (κ=3) region:



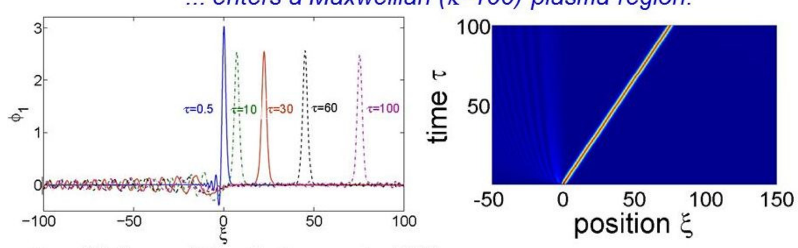
[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

Solitons in action (2): non-Maxwellian electron background (cont.)

KdV soliton (propagating in superthermal plasma; κ=3, μ =0.1)



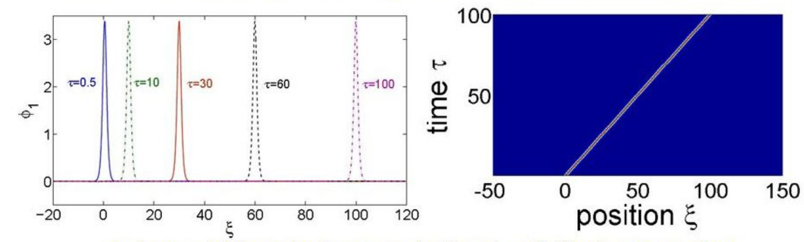
... enters a Maxwellian (κ=100) plasma region:



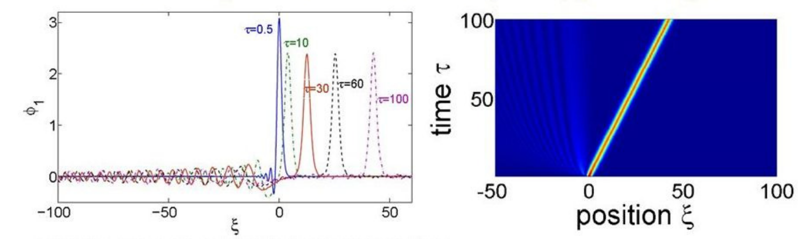
[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

Solitons in action (3): the influence of dust (defects)

KdV soliton (propagating in superthermal plasma; κ=3, μ =0.1)



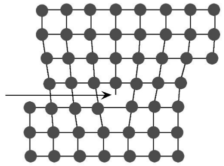
... enters a higher dust concentration (μ =0.3) plasma region:



[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

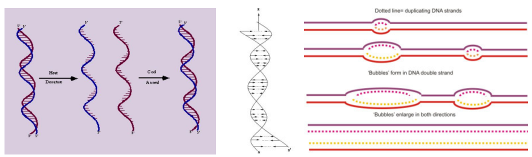
The *Soliton* paradigm is now used across all fields of Science

- *Plasmas*: KdV model for superacoustic *acoustic* electrostatic pulses;
- *Solid state physics*: longitudinal pulses in lattices; dislocations in crystals;

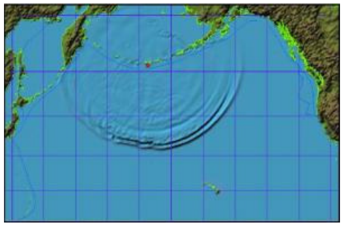


commensurability effects [Aubry, 1978];

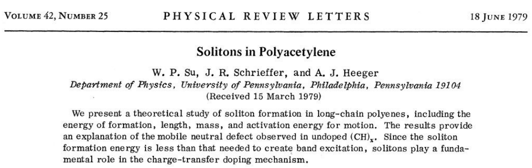
- *Biology*: signal propagation across membranes, or along nerves; DNA bubbles (denaturation) [Peyrard-Dauxois-Bishop, 1993; L.V. Yakushevich, *Nonlinear physics of DNA* (Wiley-VCH, 2004)]



- *Oceanography, hydrodynamics*: modeling of *tsunami* waves; modeling of internal waves in the Andaman sea [Osborne, *et al.*, Science 451 (1980)]



- *Chemistry*: hydrogen bonding along macromolecules, proton conductivity;

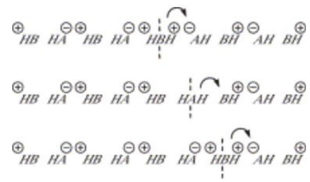


VOLUME 42, NUMBER 25 PHYSICAL REVIEW LETTERS 18 JUNE 1979

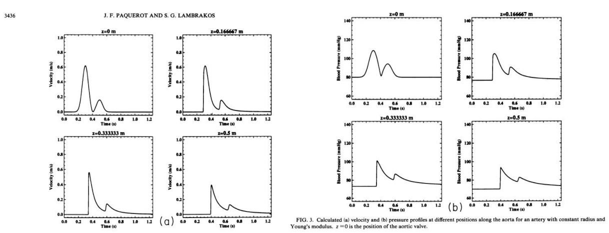
Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 15 March 1979)

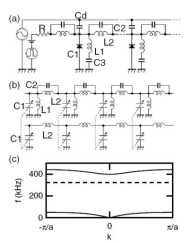
We present a theoretical study of soliton formation in long-chain polyenes, including the energy of formation, length, mass, and activation energy for motion. The results provide an explanation of the mobile neutral defect observed in undoped $(CH)_x$. Since the soliton formation energy is less than that needed to create band excitation, solitons play a fundamental role in the charge-transfer doping mechanism.



- *Blood pressure waves in the human arteriae* [S. Yomosa, J. Phys. Soc. Jpn. **56** 506 (1987); J. F. Paquerot and S. Lambrakos, Phys. Rev. E **49**, 3432 (1994)]

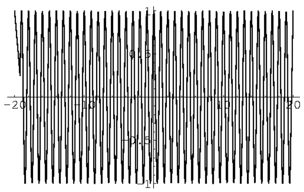


- *Electric transmission lines* [Scott 1970, Lonngren and Scott 1978, Remoissenet 1990]:

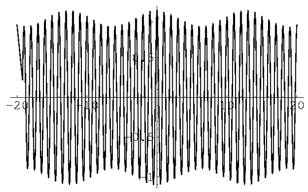


Part B – Nonlinear localization, modulational instability, envelope solitons: prerequisites

The *amplitude* of a harmonic wave may vary in space and time:



→



Part B – Nonlinear localization, modulational instability, envelope solitons: prerequisites

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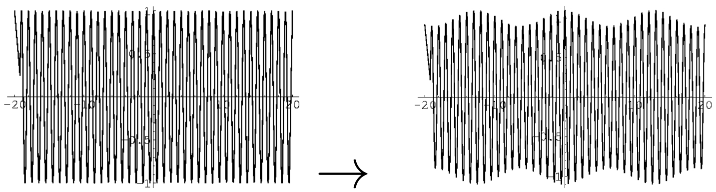
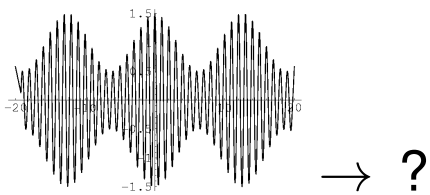
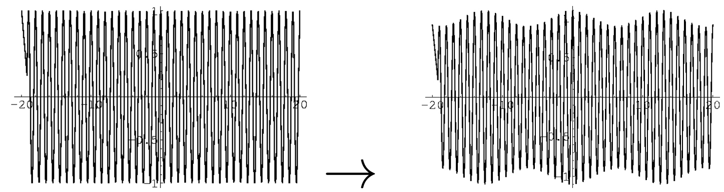


This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* (modulational instability) or ...

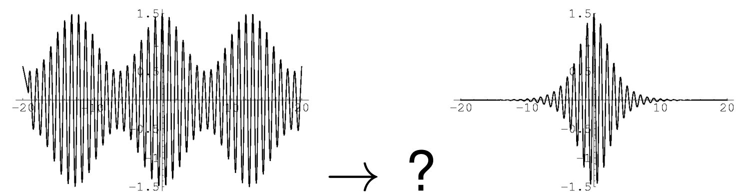


Part B – Nonlinear localization, modulational instability, envelope solitons: prerequisites

The *amplitude* of a harmonic wave may vary in space and time:



This *modulation* (due to nonlinearity) may be *strong* enough to lead to wave *collapse* (modulational instability) or to the formation of *envelope solitons*:



Solitons in optical communications

- *Kerr effect*: in a medium with cubic nonlinearity, the index of refraction is

$$n \simeq n_0(\omega) + n_2\,|E|^2\,.$$

- In 1973, Akira Hasegawa suggested that the ***nonlinear Kerr effect*** might lead to energy localization in the form of envelope solutions of a nonlinear Schrödinger (NLS) equation.

Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion

Akira Hasegawa and Frederick Tappert

Bell Laboratories, Murray Hill, New Jersey 07974
(Received 12 April 1973)

Theoretical calculations supported by numerical simulations show that utilization of the nonlinear dependence of the index of refraction on intensity makes possible the transmission of picosecond optical pulses without distortion in dielectric fiber waveguides with group velocity dispersion. In the case of anomalous dispersion ($\partial^2\omega/\partial k^2 > 0$) discussed here [the case of normal dispersion ($\partial^2\omega/\partial k^2 < 0$) will be discussed in a succeeding letter], the stationary pulse is a "bright" pulse, or envelope soliton. For a typical glass fiber guide, the balancing power required to produce a stationary 1-ps pulse is approximately 1 W. Numerical simulations show that above a certain threshold power level such pulses are stable under the influence of small perturbations, large perturbations, white noise, or absorption.

Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion

Akira Hasegawa and Frederick Tappert

Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 12 April 1973; in final form 23 May 1973)

Theoretical calculations supported by numerical simulations show that utilization of the nonlinear dependence of the index of refraction on intensity makes possible the transmission of picosecond optical pulses without distortion in dielectric fiber waveguides with group velocity dispersion. In the case of normal dispersion ($\partial^2\omega/\partial k^2 < 0$) discussed here [the case of anomalous dispersion ($\partial^2\omega/\partial k^2 > 0$) was discussed in an earlier letter], the stationary pulse is a "dark" pulse or envelope shock. Numerical simulations show that such pulses are stable under the influence of small perturbations, white noise, or absorption. Important considerations relating to the practical applications of both "bright" and "dark" pulses are also discussed.

[Applied Phys. Lett. **23**, 142 & 171 (1973)]

- Hasegawa's scheme relied on a simple idea: if the optical frequency ω depends (not only on the wavenumber k, but also) on the (field) wave amplitude $|\phi|$, viz. $\omega = \omega(k, |\phi|^2)$, then a Taylor expansion leads to:

$$\omega \simeq \omega_0 + \left.\frac{\partial\omega}{\partial k}\right|_0 (k - k_0) + \frac{1}{2}\left.\frac{\partial^2\omega}{\partial k^2}\right|_0 (k - k_0)^2 + \frac{\partial\omega}{\partial|\phi|^2}(|\phi|^2 - |\phi_0|^2)$$

- Setting $\omega - \omega_0 \to i\partial/\partial t$ and $k - k_0 \to -\partial/\partial x$, it is straightforward to find the *nonlinear Schrödinger (NLS) equation*:

$$i\frac{\partial\phi}{\partial t} + P\,\frac{\partial^2\phi}{\partial x^2} + Q\,|\phi|^2\,\phi = 0\,.$$

- P: dispersion coefficient; Q: nonlinearity coefficient;
- Other mechanisms may be present: energy *loss* (dissipation), *gain* (via resonance mechanisms, Raman effect, artificial amplification), *noise*, *turbulence*, *kinetic instabilities* (+ Landau damping), etc...

- Hasegawa's idea would be realized experimentally a few years later:

May 1985 / Vol. 10, No. 5 / OPTICS LETTERS 229

Experimental demonstration of soliton propagation in long fibers: loss compensated by Raman gain

L. F. Mollenauer and R. H. Stolen
AT&T Bell Laboratories, Holmdel, New Jersey 07733

M. N. Islam
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received January 7, 1985; accepted February 22, 1985

With sufficient Raman gain to compensate exactly for net fiber energy loss, we have demonstrated distortionless propagation of 10-psec FWHM fundamental ($N = 1$) soliton pulses ($\lambda = 1.56\ \mu$m) over a 10-km length of single-mode fiber. The implications of this experiment for development of an all-optical, high-bit-rate, long-distance telecommunications system are discussed briefly.

... *triumphantly* leading to *long-distance signal transmission in fibers*,

August 1988 / Vol. 13, No. 8 / OPTICS LETTERS 675

Demonstration of soliton transmission over more than 4000 km in fiber with loss periodically compensated by Raman gain

L. F. Mollenauer and K. Smith
AT&T Bell Laboratories, Holmdel, New Jersey 07733

Received April 11, 1988; accepted May 20, 1988

By recirculating 55-psec soliton pulses ($\lambda_s \sim 1600$ nm) many times around a closed 42-km loop with loss exactly compensated by Raman gain ($\lambda_p \sim 1497$ nm), we have successfully demonstrated pulse transmission, without electronic regeneration, over distances in excess of 4000 km.

which was meant to *revolutionize telecommunications* as known today!

Optical pulses & *modulational instability*: summary

- *Anomalous group-velocity dispersion* (GVD) (self-focusing waveguide nonlinearity), i.e., $PQ > 0$: a constant amplitude continuous wave is unstable due to the *modulational instability* [see, e.g., Hasegawa (1989)],
- ... and breaks down into a sequence of localized pulses (or beams for the spatial domain). These pulses are *bright solitons*.
- *Normal GVD* (or defocusing nonlinearity), i.e., $PQ < 0$: bright solitons do not exist, instead initial pulses undergo enhanced dispersion- (or diffraction-) induced broadening and chirping.
- In this case a constant amplitude wave is modulationally stable, so localized envelope structures can appear only as *holes*, on a continuous wave (cw) background. These pulses are *dark solitons*.
- Review papers and books: e.g., Hasegawa (1989), Agrawal (1989), Hasegawa and Kodama (1995), Haus and Wong (1996), Kivshar (1998).

Systematic derivation of the NLS equation

- Apart from the heuristic derivation discussed above, the NLS equation may be derived via a rigorous multiple scales technique.
- The idea relies in defining space and time scales, to distinguish the fast carrier wave from the slow envelope dynamics:

$$X_0 = x,\ X_1 = \epsilon\, x,\ X_2 = \epsilon^2\, x, \qquad T_0 = x,\ T_1 = \epsilon\, x,\ T_2 = \epsilon^2\, x,$$

- A lengthy calculation then leads to a solution in the form:

$$\phi = \phi_0 + \sum_{n=1}^{\infty} \epsilon^n \sum_{m=-n}^{n} \hat{\phi}_n^{(m)}(X_{1+}, T_{1+})\, e^{im(kx-\omega t)}$$

i.e.

$$\phi \simeq \phi_0 + \epsilon\, \phi_1^{(1)}\, e^{i(kx-\omega t)} + \epsilon^2\, [\phi_2^{(0)} + \phi_2^{(1)} e^{i(kx-\omega t)} + \phi_2^{(2)} e^{i2(kx-\omega t)}] + \ldots$$

- The method is generic, i.e., although first proposed for plasmas [M. Kako, J. Phys. Soc. Jpn. **33**, 1678 (1972)], it may be applied in *any dynamical problem* supporting excitations in the form of modulated wavepackets, propagating in a nonlinear dispersive medium.
- Plasma derivation for electrostatic modes: see, e.g., M. McKerr, I. Kourakis, F. Haas, Plasma Phys. Cont. Fusion **56**, 035007 (2014).
- The general result for the dynamics of the fundamental-harmonic amplitude $\phi_1^{(1)} = \psi$ bears the form of a *Nonlinear Schrödinger–type Equation* (NLSE) :

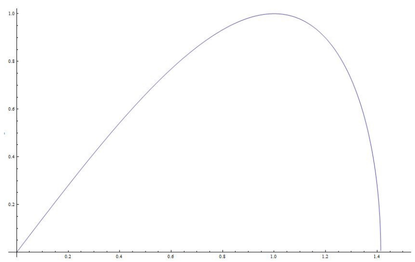
$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0$$

where the (slow) variables are: $\zeta = \epsilon(x - v_g t)$ and $\tau = \epsilon^2\, t$;

- Group velocity: $v_g = \frac{d\omega}{dk}$; *Dispersion coefficient*: $P = \frac{1}{2}\,\frac{d^2\omega}{dk^2}$.
- *Nonlinearity coefficient* Q: ... *(to be determined)*

Modulational (in)stability analysis *(continued)*

- If $PQ > 0$: the amplitude ψ is *unstable* for $\tilde{k} < \sqrt{2\frac{Q}{P}}\,|\psi_{1,0}|$.



- Maximum (instability) growth rate: $\sigma = Q|\psi_{1,0}|^2$, occurs at $\tilde{k}_m < \sqrt{\frac{Q}{P}}\,|\psi_{1,0}|$
- Instability occurs in the "window": $0 < \tilde{k} < \sqrt{2\frac{Q}{P}}\,|\psi_{1,0}|$.
- The wave may either "blow up", or localize its energy towards the formation of (envelope) solitons.

Localized envelope excitations (solitons)

- The NLSE:

$$i\,\frac{\partial \psi}{\partial \tau} + P\,\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0$$

  accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}$;

- The *total* electric potential is then: $\phi \approx \epsilon\,\rho\,\cos(\mathbf{kr} - \omega t + \Theta)$;
- The *amplitude* ρ and *phase correction* Θ depend on ζ, τ.

Localized envelope excitations (solitons)

- The NLSE accepts various solutions in the form: $\psi = \rho\, e^{i\Theta}$;

  the *total* electric potential is then: $\phi \approx \epsilon\,\rho\,\cos(\mathbf{kr} - \omega t + \Theta)$

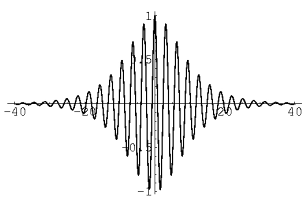
  where the *amplitude* ρ and *phase correction* Θ depend on ζ, τ.

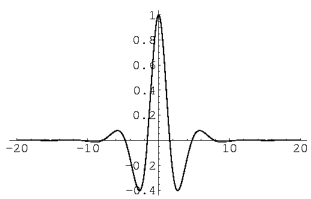
- Bright–type envelope soliton (pulse):

$$\rho = \rho_0\,\mathrm{sech}\left(\frac{\zeta - v\,\tau}{L}\right), \qquad \Theta = \frac{1}{2P}\left[v\,\zeta - (\Omega + \frac{1}{2}v^2)\tau\right].$$

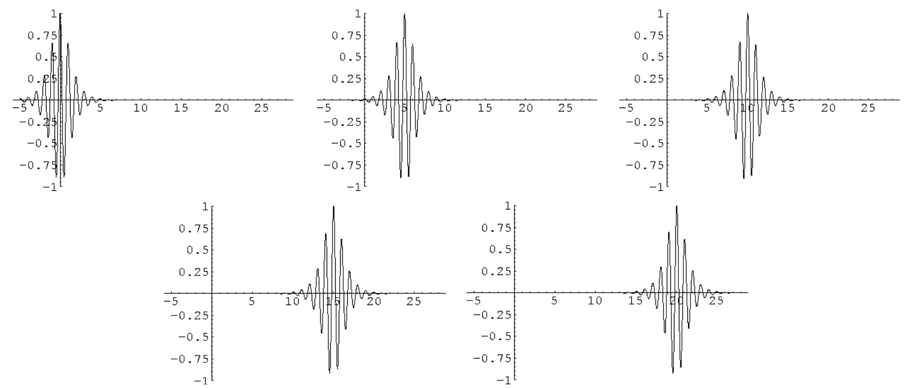
$L = \sqrt{\frac{2P}{Q}\frac{1}{\rho_0}}$

This is a propagating (and *oscillating*) localized pulse:

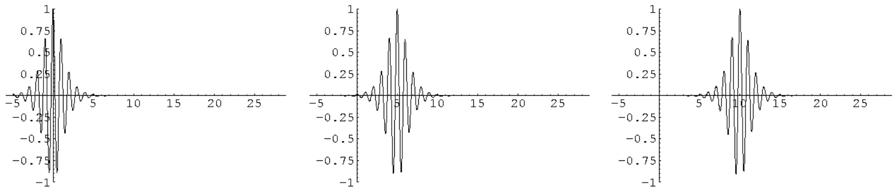




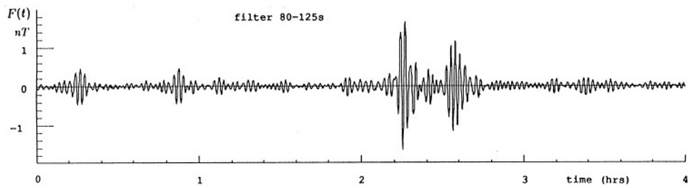
Propagation of a bright envelope soliton (pulse)



Propagation of a bright envelope soliton (pulse)

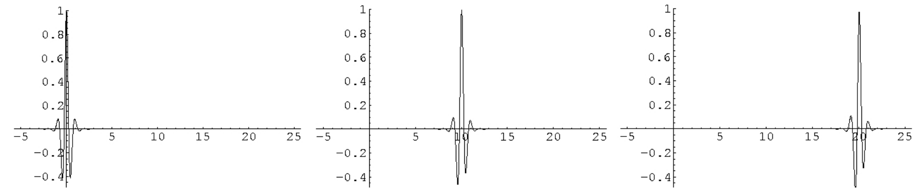


Cf. electrostatic plasma wave data from satellite observations:

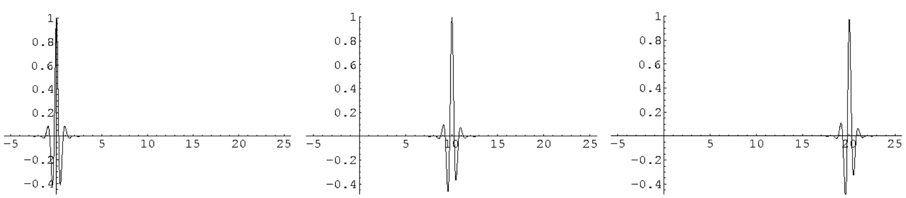


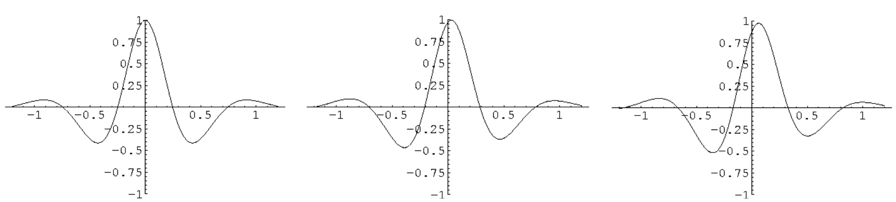
(from: [Ya. Alpert, Phys. Reports **339**, 323 (2001)])

Bright envelope soliton in the discrete limit

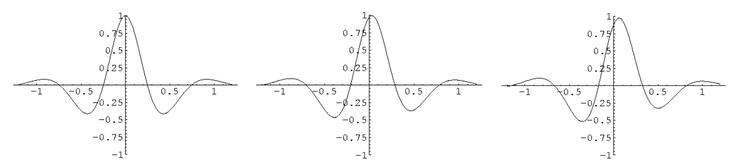


Bright envelope soliton in the discrete limit

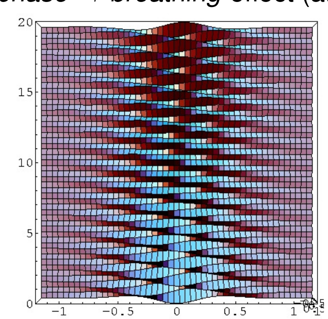




Bright envelope soliton in the discrete limit



Rem.: *Time-dependent phase* → *breathing* effect (at rest frame):



Localized envelope excitations

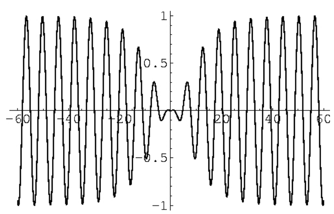
- Dark–type envelope solution (*hole soliton*):

$$\rho = \pm\rho_1 \left[1 - \mathrm{sech}^2\left(\frac{\zeta - v\tau}{L'}\right)\right]^{1/2} = \pm\rho_1 \tanh\left(\frac{\zeta - v\,\tau}{L'}\right),$$

$$\Theta = \frac{1}{2P}\left[v\,\zeta - \left(\frac{1}{2}v^2 - 2PQ\rho_1^2\right)\tau\right]$$

$$L' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{\rho_1}$$

This is a *propagating localized hole* (zero density void):



Localized envelope excitations

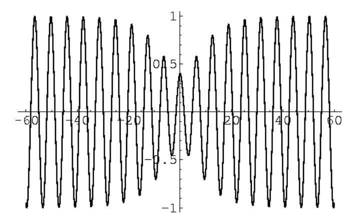
- Grey–type envelope solution (*void soliton*):

$$\rho = \pm\rho_2 \left[1 - a^2\,\mathrm{sech}^2\left(\frac{\zeta - v\,\tau}{L''}\right)\right]^{1/2}$$

$$\Theta = \ldots$$

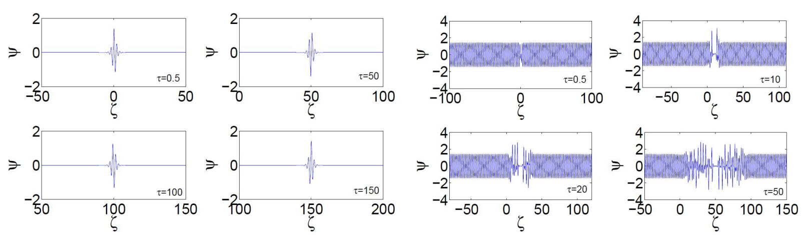
$$L'' = \sqrt{2\left|\frac{P}{Q}\right|}\frac{1}{a\rho_2}$$

This is a propagating *(non zero-density)* void:

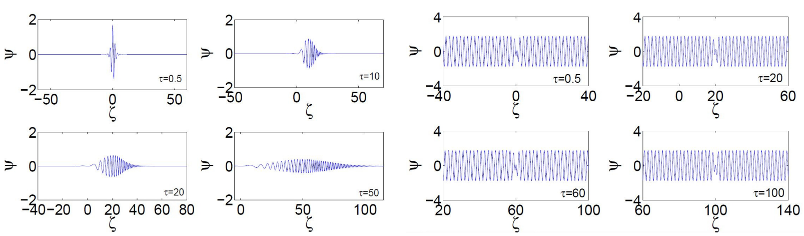


Envelope solitons in action (1): *anomalous* vs. *normal* dispersion

Case $PQ > 0$ (**"Anomalous dispersion"**): stable bright (left plot)/ unstable dark (right plot) envelopes:



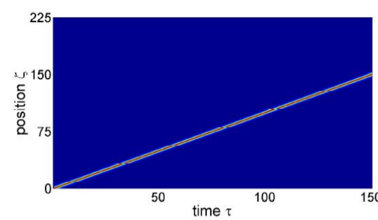
Case $PQ < 0$ (**"Normal dispersion"**): unstable bright (left plot) / stable dark (right plot) envelopes:

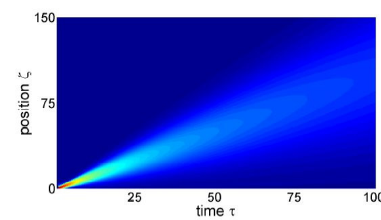


[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

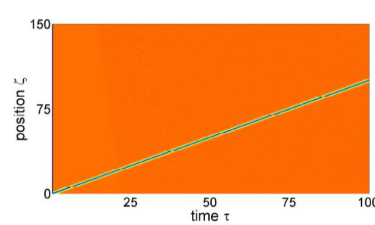
Envelope solitons in action (2): *anomalous* vs. *normal* dispersion

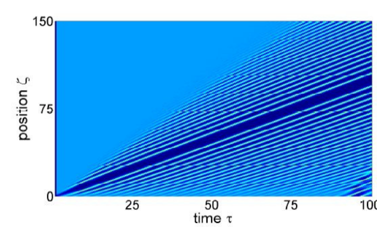
Bright envelope solitons on the space-time plane: stable vs unstable:





Dark-type envelope solitons on the space-time plane: stable vs unstable:





[I. Kourakis, S. Sultana and M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012)]

Epilogue: Bose-Einstein Condensates (BECs)

- BECs: first predicted by Satyendra Nath Bose and Albert Einstein in 1924-25; realised in 1995 [Anderson et al, 1995; Davis et al, 1995; Bradley et al, 1997]; awarded the *Nobel prize in Physics* in 2001.
- The time-dependent GrossPitaevskii eq. describes the evolution of a BEC:

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t)$$

  where Ψ represents a single particle wave function.
- The GPE possesses supports *both bright and dark envelope solitons*, as shown theoretically and confirmed experimentally: an excellent testbed for NLS theory.
- Vast literature available; e.g. [Pitaevskii & Stringari (2003), *Bose - Einstein Condensation*, Oxford: Clarendon Press].

Concluding remarks

- *Solitons & Nonlinear Physics* – a *unifying concept* across disciplines.
- The main two pillars of soliton theory, the *Korteweg - de Vries equation* and the *nonlinear Schrödinger equation* still provide a plethora of works, and may have a lot to give.
- Multidimensional extensions, including the *Zakharov-Kuznetsov* equation, the *Kadomtsev-Petviashvili* equation, and the *Davey-Stewartson* system, still remain partly unexplored (stability, integrability properties).
- Areas of relevance span: plasma physics, materials, condensed matter physics, optics, oceanography, biology, chemistry, and others.
- Frontier areas include: metamaterials / *left-handed materials* (*negative refraction media*), *nonlinear optics*, petawatt scale ultra-short *laser plasma/matter interactions*, *Bose-Einstein condensation*, and *more* ...

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  - – M. McKerr, I. Kourakis, F. Haas, Plasma Phys. Cont. Fusion **56**, 035007 (2014)
- Various sources, acknowledgments:

  R Z Sagdeev & C F Kennel, *Scientific American*, April 1991, p. 106; Thierry Dauxois and Stefano Ruffo, Scholarpedia, **3** (8), 5538 (2008); Norman J. Zabusky and Mason A. Porter, Scholarpedia, **5** (8), 2068 (2010); M. A. Porter, N. J. Zabusky, B. Hu, D. K. Campbell, American Scientist **97** (3), 214 (2009); Wikipedia.org.
  – *"Great Wave off Kanagawa"*, painting by Japanese artist Katsushika Hokusai (c. 1829 - 32).

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