Localized excitations of charged dust grains in dusty plasma lattices

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Abstract. The nonlinear aspects of charged dust grain motion in a one-dimensional dusty plasma (DP) monolayer are discussed. Both horizontal (longitudinal, acoustic mode) and vertical (transverse, optic mode) displacements are considered, and various types of localized excitations are reviewed, in a continuum approximation. Dust crystals are shown to support nonlinear kink-shaped supersonic longitudinal solitary excitations, as well as modulated envelope (either longitudinal or transverse) localized modes. The possibility for Discrete Breather (DB-) type excitations (Intrinsic Localized Modes, ILMs) to occur is investigated, from first principles. These highly localized excitations owe their existence to lattice discreteness, in combination with the interaction and/or substrate (sheath) potential nonlinearity. This possibility may open new directions in DP- related research. The relation to previous results on atomic chains as well as to experimental results on strongly-coupled dust layers in gas discharge plasmas is discussed.

1. Introduction. A number of recent theoretical studies have been devoted to collective processes in dusty plasmas (DP), in relevance with experimental observations. Dust (quasi-)lattices (DL) are typically formed in the sheath region above the negative electrode in discharge experiments, horizontally suspended at a levitated equilibrium position, at $z = z_0$, where gravity and electric (and/or magnetic) forces balance. The linear regime of low-frequency oscillations in DP crystals, in the longitudinal (acoustic mode) and transverse (in-plane, shear acoustic mode and vertical, off-plane optical mode) direction(s), is now quite well understood. However, the *nonlinear* behaviour of DP crystals is little explored, and has lately attracted experimental [1-3] and theoretical [1-8] interest.

Recently [4], we considered the coupling between the horizontal ($\sim \hat{x}$) and vertical (off-plane, $\sim \hat{z}$) degrees of freedom in a dust mono-layer; a set of nonlinear equations for longitudinal and transverse dust lattice waves (LDLWs, TDLWs) was thus rigorously derived [4]. Here, we review the nofnlinear dust grain excitations which may occur in a DP crystal (assumed quasi-one-dimensional and infinite, composed from identical grains, of equilibrium charge q and mass M, located at $x_n = n\,r_0,\ n \in \mathscr{N}$). Ion-wake and ion-neutral interactions (collisions) are omitted, for simplicity. This study complements recent experimental investigations [1-3] and may hopefully motivate future ones.

2. Transverse envelope structures (continuum) & discrete breathers. The vertical (off-plane) n−th grain displacement $\delta z_n = z_n - z_0$ in a dust crystal obeys the equation[1,2]

$$\frac{d^2\delta z_n}{dt^2} + \nu\,\frac{d(\delta z_n)}{dt} + \omega_{T,0}^2\,(\,\delta z_{n+1} + \delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,. \quad (1)$$

The characteristic frequency $\omega_{T,0} = \big[-qU'(r_0)/(Mr_0)\big]^{1/2}$ is related to the interaction potential[3] $U(r)$. The *gap frequency* ω_g and the nonlinearity coefficients α, β are defined via the potential $\Phi(z) \approx \Phi(z_0) + M[\omega_g^2\delta z_n^2/2 + \alpha\,(\delta z_n)^3/3 + \beta\,(\delta z_n)^4/4] + \mathscr{O}[(\delta z_n)^5]$ (expanded near z_0, in account of the electric and/or magnetic field inhomogeneity and charge variations[4]), related to the overall vertical force $F(z) = F_{el/m}(z) - Mg \equiv -\partial\Phi(z)/\partial z$ [recall that $F(z_0) = 0$]. Linear excitations, viz. $\delta z_n \sim \cos\phi_n$ (here $\phi_n = nkr_0 - \omega t$; k and ω are the wavenumber and frequency) obey the *optic*-like *discrete* dispersion relation[5]: $\omega^2 = \omega_g^2 - 4\omega_{T,0}^2\sin^2\!\big(kr_0/2\big) \equiv \omega_T^2$. Transverse vibrations propagate as a *backward wave* [see that $v_{g,T} = \omega_T'(k) < 0$], for any form of $U(r)$ in agreement with recent experiments [2]. Notice the lower cutoff $\omega_{T,min} = (\omega_g^2 - 4\omega_{T,0}^2)^{1/2}$ (at the edge of the Brillouin zone, at $k = \pi/r_0$), which is *absent in the continuum limit*.

Assuming a weakly nonlinear *continuum* amplitude, one obtains, via a multiple scale technique [5]: $\delta z_n \approx \varepsilon\,(A\,e^{i\phi_n} + \text{c.c.}) + \varepsilon^2\,\big[w_0^{(2)} + (w_2^{(2)}\,e^{2i\phi_n} + \text{c.c.})\big] + \ldots$ (where $w_0^{(2)} \sim |A|^2$, $w_2^{(2)} \sim A^2$); the amplitude A obeys the *nonlinear Schrödinger equation* (NLSE):

$$i\,\frac{\partial A}{\partial T} + P\,\frac{\partial^2 A}{\partial X^2} + Q\,|A|^2 A = 0\,, \quad (2)$$

where $\{X, T\}$ are the *slow* variables $\{\varepsilon(x - v_g t), \varepsilon^2 t\}$. The *dispersion coefficient* $P_T = \omega_T''(k)/2$ takes negative (positive) values for low (high) k. The *nonlinearity coefficient* $Q = \big[10\alpha^2/(3\omega_g^2) - 3\beta\big]/2\omega_T$ is positive for *all* known experimental values of α, β [3]. For small wavenumbers k (where $PQ < 0$), TDLWs will be modulationally stable, and may propagate in the form of dark/grey envelope excitations (*hole* solitons or *voids* [5]. For larger k, *modulational instability* may lead to the formation of bright (*pulse*) envelope solitons. Exact expressions for these excitations can be found in [5].

Intrinsic Localized Modes (ILMs), i.e. highly localized *Discrete Breather* (DB) and *multi-breather*-type few-site vibrations, were also shown to occur in transverse DL motion [6], and are currently being investigated from first principles [7]. These excitations have recently received increased interest among researchers in solid state physics, due to

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[1] Only first neighbor interactions are considered here. See in [4] for details and coefficient definitions.
[2] Coupling anharmonicity, expressed by a term $\sim \big[(\delta z_{n+1} - \delta z_n)^3 - (\delta z_n - \delta z_{n-1})^3\big]$, is omitted here.
[3] No specific form is assumed here for U; for a Debye-Hückel potential: $U_D(r) = (q/r)\,e^{-r/\lambda_D}$, one has $\omega_{0,D}^2 = \omega_{DL}^2\,\exp(-\kappa)\,(1+\kappa)/\kappa^3$; $\omega_{DL} = [q^2/(M\lambda_D^3)]^{1/2}$ is the characteristic dust-lattice frequency; λ_D is the Debye length; $\kappa = r_0/\lambda_D$ is the DP lattice parameter.
[4] We follow exactly the definitions in [4, 5], not reproduced here.
[5] The damping term is neglected by setting $\nu = 0$ here; for $\nu \neq 0$, an imaginary part appears, in account for damping, in both the dispersion relation $\omega(k)$ and the resulting envelope equations.

their omnipresence in periodic lattices and remarkable physical properties [8]. Remarkably, the existence of such DB structures at a frequency ω_{DB}) generally requires the non-resonance condition $n\omega_{DB} \neq \omega(k)$ ($n \in \mathscr{N}$), which *is* indeed satisfied in all known TDLW experiments [2].

3. Longitudinal envelope excitations. The *nonlinear* equation of motion[1,6]:

$$\frac{d^2(\delta x_n)}{dt^2} + \nu \frac{d(\delta x_n)}{dt} = \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n)$$
$$-a_{20} \left[(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2\right] + a_{30} \left[(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3\right] \quad (3)$$

describes the longitudinal dust grain displacements $\delta x_n = x_n - nr_0$. The resulting *acoustic* linear mode[4] obeys: $\omega^2 = 4\omega_{L,0}^2 \sin^2\big(kr_0/2\big) \equiv \omega_L^2$. One now obtains $\delta x_n \approx \varepsilon \left[u_0^{(1)} + (u_1^{(1)} e^{i\phi_n} + \text{c.c.})\right] + \varepsilon^2 (u_2^{(2)} e^{2i\phi_n} + \text{c.c.}) + \ldots$, where $u_{1/0}^{(1)}$ obey [9]

$$i\frac{\partial u_1^{(1)}}{\partial T} + P_L \frac{\partial^2 u_1^{(1)}}{\partial X^2} + Q_0 |u_1^{(1)}|^2 u_1^{(1)} + \frac{p_0 k^2}{2\omega_L} u_1^{(1)} \frac{\partial u_0^{(1)}}{\partial X} = 0\,, \quad (4)$$

$$\frac{\partial^2 u_0^{(1)}}{\partial X^2} = -\frac{p_0 k^2}{v_{g,L}^2 - \omega_{L,0}^2 r_0^2} \frac{\partial}{\partial X} |u_1^{(1)}|^2 . \quad (5)$$

Here $v_{g,L} = \omega_L'(k)$, and $\{X, T\}$ are slow variables (as above). We have defined: $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20} r_0^3$ and $q_0 = U''''(r_0) r_0^4/(2M) \equiv 3a_{30} r_0^4$ (both positive, and similar in magnitude for Debye interactions [4, 10]). Eqs. (4), (5) can be combined into an NLSE in the form of Eq. (2), for $A = u_1^{(1)}$ here, with $P = P_L = \omega_L''(k)/2 < 0$. The exact form of $Q > 0$ (< 0) [9] prescribes stability (instability) at low (high) k. Envelope excitations are now *asymmetric*, i.e. rarefactive bright or compressive dark envelope structures.

4. Longitudinal solitons & Intrinsic Localized Modes. Equation (3) is essentially identical to the equation of atomic motion in a chain with anharmonic springs, i.e. in the celebrated FPU (*Fermi-Pasta-Ulam*) problem. At a first step, one may adopt a continuum description, viz. $\delta x_n(t) \to u(x,t)$. This leads to different nonlinear evolution equations (depending on the simplifying hypotheses adopted), some of which are critically discussed in [10]. What follows is a summary of the lengthy analysis therein.

Keeping lowest order nonlinear and dispersive terms, $u(x,t)$ obeys[1]:

$$\ddot{u} + \nu\,\dot{u} - c_L^2\, u_{xx} - \frac{c_L^2}{12}\, r_0^2\, u_{xxxx} = -\,p_0\, u_x\, u_{xx} + q_0\, (u_x)^2\, u_{xx}\,, \quad (6)$$

where $(\cdot)_x \equiv \partial(\cdot)/\partial x$; $c_L = \omega_{L,0}\, r_0$; p_0 and q_0 were defined above. Assuming *near-sonic propagation* (i.e. $v \approx c_L$), and defining the relative displacement $w = u_x$, one has

$$w_\tau - a w w_\zeta + \hat{a} w^2 w_\zeta + b w_{\zeta\zeta\zeta} = 0 \quad (7)$$

[6] Here, $\omega_{0,L} = [U''(r_0)/M)]^{1/2}$, e.g. $\omega_{L,0}^2 = 2\,\omega_{DL}^2 \exp(-\kappa)\,(1+\kappa+\kappa^2/2)/\kappa^3$ in the Debye case.

(for $\nu = 0$), where $a = p_0/(2c_L) > 0$, $\hat{a} = q_0/(2c_L) > 0$, and $b = c_L r_0^2/24 > 0$. Following Melandsø [11], various studies have relied on the *Korteweg - deVries* (KdV) equation, i.e. Eq. (7) for $\hat{a} = 0$, to gain analytical insight in the *compressive* structures observed in experiments [1]. Indeed, the KdV Eq. possesses *negative* (*only*, here, since $a > 0$) supersonic pulse soliton solutions for w, implying a compressive (anti-kink) excitation for u; the KdV soliton is thus interpreted as a density variation in the crystal, viz. $n(x,t)/n_0 \sim -\partial u/\partial x \equiv -w$. Also, the pulse width L_0 and height u_0 satisfy $u_0 L_0^2 = cst.$, a feature which is confirmed by experiments [1]. However, $\hat{a} \approx 2a$ in real Debye crystals (for $\kappa \approx 1$), which invalidates the KdV approximation $\hat{a} \approx 0$ [10]). Instead, one may employ the *extended KdV* Eq. (eKdV) (7), which accounts for *both* compressive *and* rarefactive lattice excitations (exact expressions in [10]). Alternatively, Eq. (6) can be reduced to a *Generalized Boussinesq* (GBq) Equation [10]; again, for $q_0 \sim \hat{a} \approx 0$, one recovers a *Boussinesq* (Bq) equation, widely studied in solid chains. The GBq (Bq) equation yields, like its eKdV (KdV) counterpart, both compressive and rarefactive (only compressive, respectively) solutions; however, the (supersonic) propagation speed v now does *not* have to be close to c_L. The lengthy analysis [10] is not reproduced here.

Following existing studies on *Discrete Breathers* (ILMs) *in FPU chains* [cf. (3) above], it is straightforward to show the existence of such localized excitations in the longitudinal direction. A detailed investigation, in terms of real experimental parameters, is on the way and will be reported soon.

Concluding, we have reviewed recent results on nonlinear excitations (solitary waves and discrete breathers) occurring in a (1d) dust mono-layer, due to sheath and coupling nonlinearity. One encounters modulated envelope TDL and LDL structures. *Both* compressive and rarefactive longitudinal excitations are predicted and may be observed by appropriate experiments. Finally, highly localized discrete excitations may also occur, and should be sought for by appropriate experiments.

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REFERENCES

1. D. Samsonov *et al.*, *PRL* **88** 095004 (2002); K. Avinash *et al.*, *PRE* **68**, 046402 (2003).
2. T. Misawa *et al.*, *PRL* **86**, 1219 (2001); B. Liu *et al.*, *PRL* **91**, 255003 (2003).
3. C. Zafiu *et al.*, *Phys. Rev. E* **63** 066403 (2001); A. V. Ivlev *et al.*, Phys. Rev. Lett. **85**, 4060 (2000).
4. I. Kourakis and P. K. Shukla, *Physica Scripta* T**113**, 97 (2004).
5. I. Kourakis and P. K. Shukla, *Phys. Plasma*s, **11**, 2322 (2004); *idem*, **11**, 3665 (2004).
6. I. Kourakis and P. K. Shukla, *Phys. Plasmas*, **12**, 014502 (2005).
7. I. Kourakis, P. K. Shukla and V. Basios, *Proc. 2004 ICPP* (Nice, France): `hal.ccsd.cnrs.fr/ccsd-00001892/en/`; V. Basios, I. Kourakis and P.K. Shukla, *Detection and controlability aspects of intrinsic localized modes in dusty plasma crystals*, in preparation; V. Koukouloyannis and I. Kourakis, *Existence of multibreathers in systems with an inverse dispersion law.*, in preparation.
8. See in: D. K. Campbell, S. Flach and Yu. S. Kivshar, *Physics Today*, **57** (1) (2004), for a brief review.
9. I. Kourakis and P. K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).
10. I. Kourakis and P. K. Shukla, *Eur. Phys. J. D*, **29**, 247 (2004).
11. F. Melandsø, Phys. Plasmas **3**, 3890 (1996).