Charge polarization (dressed electrostatic interaction) effects in dusty (complex) crystals

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Abstract. The influence of dust charge polarization (dressing) on lattice vibrations is investigated. Both one-dimensional (1D) and hexagonal (2D) monolayer configurations are considered. It is shown that dressed interactions lead to a reduction (increase) in the frequency of lattice vibrations, as regards longitudinal (transverse) degrees of freedom. The possibility of a new crystal instability (melting) entirely due to the dressing effect is pointed out. On the other hand, the occurrence of crystals consisting of opposite (...+-+-+-+...) charge dust grains may be anticipated via this mechanism.

Keywords: Dusty plasma crystals, complex plasmas, dust-lattice waves, electrostatic interactions, phonons, solitons.
PACS: 52.27.Lw, 52.35.Fp, 52.25.Vy

It is now established that the presence of massive mesoscopic (micron-sized, typically) particulates ("dust grains") may modify plasma properties substantially [1]. Of particular importance is the occurrence of strongly-coupled crystalline-like dust configurations, due to strong inter-grain interactions [2]. These dust quasi-lattices are now known to support a variety of linear and nonlinear excitations, which may be of potential use in future applications.

At a first approach, *ab initio* studies show that inter-grain electrostatic interactions may be considered to be of the screened Coulomb (Debye - Hückel) type [3]. More refined theoretical studies have later shown that taking into account plasma polarization due to the sheath region (near the grain surface) associated with the grains [4, 5] results in a strong modification of the (oppositely charged) charge cloud surrounding the particles. This "dressing" effect leads to a change in the very nature of the inter-particle interactions, which may even become attractive for equal-sign charged particles (inversely, repulsive interactions may appear in the case of opposite neighboring grain charges).

The influence of dust charge polarization (dressing) on lattice vibrations is investigated in this brief report. Both one-dimensional (1D) and hexagonal (2D) monolayer configurations are considered. It is shown that dressed interactions lead to a reduction in the frequency of lattice vibrations [6, 7], as regards both longitudinal and transverse degrees of freedom. The possibility of a new crystal instability (melting) entirely due to the dressing effect is pointed out. On the other hand, the occurrence of crystals consisting of opposite (...+-+-+-+...) charge dust grains may be anticipated [8].

DUST-LATTICE WAVES IN ONE-DIMENSIONAL (1D) DUST CRYSTALS

The potential (energy) of interaction between two particles (charges Q_1 and Q_2) located at a distance r reads [1, 4, 5]

$$U_{drD}(r) = Q_1 Q_2 \frac{e^{-r/\lambda_D}}{r}\left(1 - \delta \frac{r}{2\lambda_D}\right) \equiv \left(\frac{|Q_1 Q_2|}{\lambda_D}\right) s \frac{e^{-x}}{x}\left(1 - \delta \frac{x}{2}\right), \tag{1}$$

where $x = r/\lambda_D \equiv \kappa r'$, and λ_D denotes the effective Debye radius [1, 3]; here, we have defined the lattice parameter $\kappa = r_0/\lambda_D$ and the reduced space variable $r' = r/r_0$. The parameter $s = \mathrm{sgn}(Q_1 Q_2) = \pm 1$ is equal to 1 (-1) for equal- (opposite-)-charge-sign particles, respectively. The parameter δ simply takes the values 1 (for "dressed" Debye interactions) and 0 (recovering the familiar unperturbed Debye form); unless otherwise stated, $\delta = 1$ in the following.

The potential form (1), studied in Refs. [4, 5], is depicted in Fig. 1. For $s = 1$ (equal charge-sign grains), it changes sign at $x = 2$, shifting from repulsive to attractive interactions (among equal charge signs, here). Furthermore, it bears a minimum at $x = 1 + \sqrt{3} \approx 2.732$, which may play the role of a potential well for neighboring particles located at an appropriate distance; naturally, this potential form was suggested as a simple model for dust molecule formation in earlier works [1, 5].

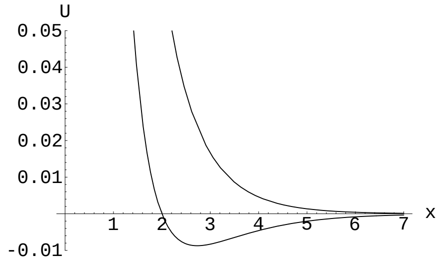

FIGURE 1. The interaction potential (energy) U, as given by Eq. (1) [scaled by $|Q_1Q_2|/\lambda_D$] vs. space $x = r/\lambda_D$. Here, $s = +1$ (equal-sign grain charges) and δ equals, respectively, 0 (1), for simple (dressed) Debye interactions, in the upper (lower) curve.

Transverse dust-lattice waves. The dispersion relation for *transverse 1D dust-lattice (TDLW) oscillations* is [1, 9]

$$\omega_T^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(kr_0/2), \tag{2}$$

where r_0 is the lattice spacing and k is the wavenumber. The gap frequency $\omega_g = \lim_{k\to 0}\omega_T(k)$ is related to the plasma sheath environment (assumed to form a parabolic potential in the transverse direction, centered at the crystal levitation height), and need not be discussed here; we retain the condition $\omega_g^2/\omega_{T,0}^2 > 4$, which should be imposed for stability (so that $\omega_T^2 > 0$ in the entire Brillouin zone $[0, \pi/r_0]$). The characteristic constant $\omega_{T,0}^2$ is related to $U(r)$ as

$$\omega_{0,T}^2 = -U'(r_0)/(Mr_0), \tag{3}$$

where M denotes the dust grain mass. Combining with Eq. (1), one obtains

$$\omega_{0,T\,(drD)}^2 = s\left(\frac{|Q_1Q_2|}{M\lambda_D^3}\right)\frac{e^{-\kappa}}{\kappa^3}\left(1+\kappa-\delta\frac{\kappa^2}{2}\right). \tag{4}$$

The right-hand side changes sign at the potential extremum, viz. $U'(r_0) = 0$. Specifically, for $s = 1$ (equal-sign charges), $\omega_{0,T\,(drD)}^2$ will be a positive (negative) quantity for values of κ below (above) $\kappa_1 = 1+\sqrt{3} \approx 2.732$ (and the inverse qualitative picture holds for $s = -1$). This behaviour is depicted in Fig. 2a. This change in sign is not possible for $\delta = 0$ (simple Debye case), where the well-known (positive) form $\omega_{0,T}^2 = [|Q_1Q_2|/(M\lambda_D^3)]\, e^{-\kappa}(1+\kappa)/\kappa^3$ is recovered [1, 9].

It is clear from (2) and (4) that ω_T increases for $\delta = 1$, as compared to $\delta = 0$ (ordinary screening). The change in the sign of $\omega_{0,T\,(drD)}^2$ at $\kappa = \kappa_1$ results in a structural change in the dispersion curve, which obtains a normal (inverse) optic-like form for negative (positive) values of $\omega_{0,T\,(drD)}^2 \sim -U'(r_0)$. This behavior is depicted in Fig. 2b. Transverse dust-lattice waves may thus lose their long-discussed (and experimentally confirmed) *backward-wave* property (viz. group and phase velocities of opposite signs) if the lattice parameter κ attains values higher than κ_1.

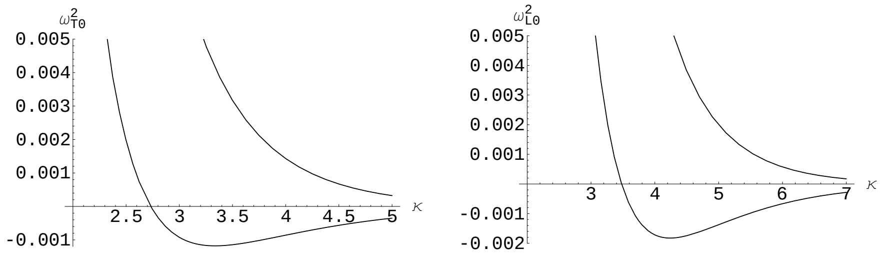

FIGURE 2. The TDLW characteristic constants $\omega_{T,0}^2$ and $\omega_{L,0}^2$, as given by Eqs. (4) and (7) respectively [scaled by $|Q_1Q_2|/(M\lambda_D^3)$] vs. $\kappa = r_0/\lambda_D$. Here, $s = +1$ and δ is 0 (1), for simple (dressed) Debye interactions, in the upper (lower) curve.

Longitudinal dust-lattice waves. The dispersion relation for *longitudinal 1D dust-lattice (LDL) oscillations* is

$$\omega_L^2 = 4\omega_{L,0}^2 \sin^2(kr_0/2), \tag{5}$$

where r_0 and k have been defined above. Contrary to TDLWs, here $\omega_L(k)$ goes to zero as $\omega_L(k) \approx \omega_{L,0} r_0 k \equiv c_s k$, where c_s is the LDL sound speed. The characteristic constant $\omega_{T,0}^2$ is related to the interaction potential as [1, 9]

$$\omega_{0,L}^2 = U''(r_0)/M. \tag{6}$$

Combining with Eq. (1), one obtains

$$\omega_{0,L(drD)}^2 = 2s\left(\frac{|Q_1 Q_2|}{M\lambda_D^3}\right)\frac{e^{-\kappa}}{\kappa^3}\left(1+\kappa+\frac{\kappa^2}{2}-\delta\frac{\kappa^3}{4}\right). \tag{7}$$

Clearly, dressing (i.e. $\delta = 1$) leads to a decrease in the LDL vibration frequency. The RHS in (7) changes sign at the potential deflection point, viz. $U''(r_0) = 0$. In specific, for $s = 1$ (equal-sign charges), $\omega_{0,L(drD)}^2$ will be a positive (negative) quantity for values of κ below (above) $\kappa_2 \approx 3.48$ (and the inverse qualitative picture holds for $s = -1$). This behaviour is depicted in Fig. 2b. Again, this change in sign is not possible for $\delta = 0$ (simple Debye case), where the known (positive) expression $\omega_{0,L}^2 = 2[|Q_1 Q_2|/(M\lambda_D^3)]\, e^{-\kappa}\,(1+\kappa+\kappa^2/2)/\kappa^3$ is recovered [1, 9].

We see that, for κ values above κ_2, i.e. resulting in negative values of $\omega_{0,L(drD)}^2 \sim U''(r_0)$, LDL oscillations will be *unstable*. For lower κ values, LDLWs will be stable (recall that $\kappa \approx 1$ or slightly higher in today's experiments).

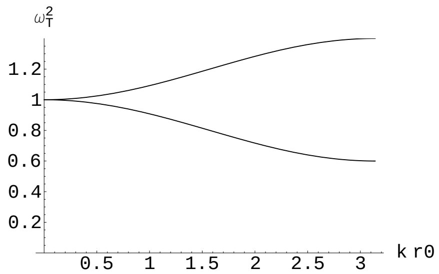

FIGURE 3. The TDLW dispersion curve: the square frequency ω_T^2, as given by Eq. (2) (scaled by ω_g^2) vs. the reduced wavenumber kr_0), for arbitrary values of all parameters except s (here $s = +1$) and δ. $\delta = 0$ (1) in the lower (upper) curves.

Stabilization of LDL waves in crystals of alternating charge-sign grains. An interesting consequence of the electrostatic potential "dressing" effect is the following. Let us consider the 1D alternating charge sign pattern:

$$..., +, -, +, -, +, -, +, -, +...$$

Coulomb-like interactions are attractive here, giving rise to *unstable* longitudinal displacements [8].

Taking into account the dressed Debye potential given by Eq. (1), for opposite grain charge-signs, i.e. for $s = -1$, one essentially obtains an reversed, qualitatively speaking, picture, as compared with the case $s = +1$ treated above; cf. Fig. 1, upon setting $U \to -U$, which yields the mirror-symmetric plot, with respect to the horizontal axis; the corresponding figure is omitted here, for brevity. Most interestingly, considering this type of interaction among one (any) grain and its first order neighbors, we see that the total force $F_n = F_{n-1,n} + F_{n+1,n}$ felt by the $n-$th grain, viz. $F_n = -\left[\frac{\partial U_{n-1,n}}{\partial z_n} + \frac{\partial U_{n+1,n}}{\partial z_n}\right] = -\frac{\partial}{\partial z_n}[U_{drD}(r_0+z_n) + U_{drD}(r_0-z_n)] \equiv -\left[\frac{\partial}{\partial x}U_{total}(x)\right|_{x=z_n}$, derives from a total potential, say $U_{total}(x)$, which may here, for $\delta = 1$, present a local minimum (hence a stable equilibrium position for the $n-$th grain). It turns out that the extremum at $x = 0$, viz. $U'_{total}(0) = 0$, is a local minimum (maximum), i.e. $U''_{total}(0)$ is positive (negative) for κ values above (below) a critical value $\kappa_3 \approx 3.4798$. This qualitative behavior is depicted in Fig. 4a. Therefore, the electrostatic dressing effect may result in stabilization of longitudinal grain displacements in a bi-lattice, consisting of oppositely charged neighboring grains. Remarkably, this possibility is inexistent in the absence of the dressing effect. Indeed, analyzing the form of $U_{total}(x)$ in the case $\delta = 0$ (i.e. for simple, unperturbed Debye interactions), one sees that no stable equilibrium point occurs in this case; cf. Fig. 4b.

DUST-LATTICE WAVES IN TWO-DIMENSIONAL (2D) HEXAGONAL CRYSTALS

Let us now consider a 2D hexagonal dust monolayer; see in Fig. 5. Linear vibrations along the $x-$ or the $y-$axis, propagating in an arbitrary direction ($0 < \theta < \pi/2$), have been studied for dressed interactions in [7]. Various

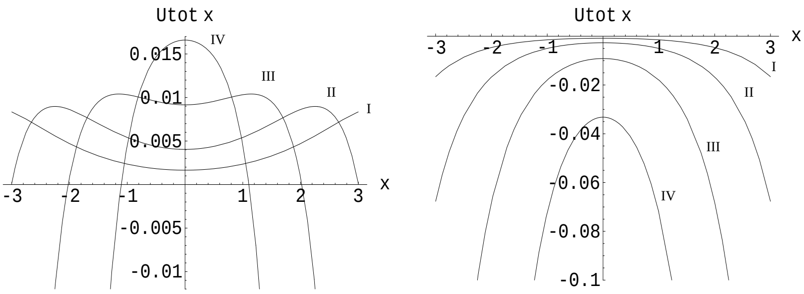

FIGURE 4. The effective potential $U_{tot}(x) = U_{n-1,n}(r_0 + x) + U_{n+1,n}(r_0 - x)$ [here scaled by $|Q_1 Q_2|/\lambda_D$], which is felt by a dust grain in a alternating charge-sign bi-lattice, is depicted vs. the reduced position (displacement) variable x/λ_D. Here, $s = -1$ (opposite-sign grain charges) and $\delta = 1$ ($\delta = 0$) in the left (right) plot. The lattice κ is equal to 6, 5, 4, 3, in curves I, II, III, IV.

combinations exist; for instance, for longitudinal excitations [$\delta x_n \sim \exp i(kx - \omega t)$, $\delta y_n = 0$] propagating along the principal axis x (i.e. $\theta = 0$), we have [7]: $\omega^2 \sim \omega^2_{0,L(drD)} \sin^2(kr_0/4)[1 + 4\cos^2(kr_0/4)]$ [cf. (7) above]. As in the 1D case treated above, one finds out that taking the polarization effect into account results in a decrease in the vibration frequency ω, in addition to a consequent slowing down in the phase speed. The same qualitative effect is witnessed for transverse DL waves, for various values of θ. Finally, "dressing" yields a considerable effect on the characteristics of *nonlinear* modulated envelope DL structures, as shown in [10].

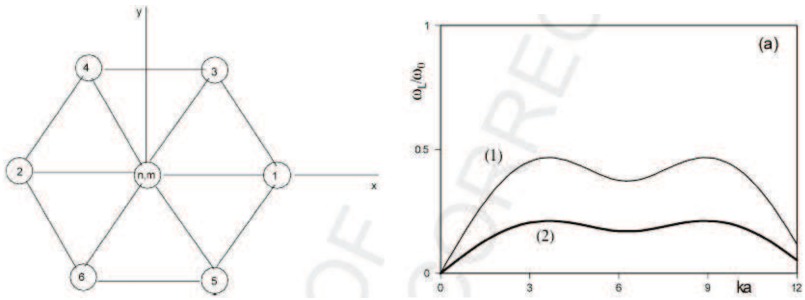

FIGURE 5. (a) Elementary cell in a 2D hexagonal crystalline configuration. (b) The normalized LDL frequency ω_L^2 vs. kr_0, for wave propagation in the x direction. Here κ =2.5, and $\delta = 0$ (1) in curve 1 (2).

In conclusion, charge polarization (electrostatic "dressing") results in a significant modification of the propagation characteristics of dust-lattice waves, which may even be destabilized (for high values of the lattice parameter κ, essentially). Furthermore, transverse off-plane 1D vibrations may shift from a backward- to a forward-propagating wave, due to the polarization effect. There results may be investigated by appropriate experiments.

Acknowledgements. I.K. acknowledges partial support by the Deutsche Forschungsgemeinschaft (Bonn, Germany) through the Sonderforschungsbereich (SFB) 591 Programme.

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