Electrostatic mode envelope excitations in warm pair ion plasmas with a small fraction of stationary positive ions - application in e-p-i and doped fullerene plasmas

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Abstract. The nonlinear propagation of electrostatic wave packets in electron-positron-ion (e-p-i) plasmas, or pair- (e.g. fullerene) ion plasmas in the presence of a small fraction of uniform and stationary positive ions is studied. A two-fluid plasma model is employed. Two distinct electrostatic modes are obtained, namely a quasi-ion-thermal lower mode and a Langmuir-like optic-type upper one, as in pure pair plasmas, in agreement with previous experimental observations and theoretical studies of equal-temperature pair plasmas. The basic set of model equations is reduced to a nonlinear Schrödinger equation for the slowly varying electric field perturbation amplitude. The analysis reveals that the stability range of lower (acoustic) mode increases as the positive-to-negative-ion (or positron-to-electron) density ratio increases, so this quasi-thermal mode may propagate in the form of a dark-type envelope soliton (i.e. a potential dip, or a void) modulating a carrier wave packet for small wave-numbers, for a fixed value of the positive-to-negative-ion (or positron-to-electron) temperature ratio. On the other hand, the upper mode is modulationally unstable, and may thus favor the formation of bright-type envelope soliton (pulse) modulated wave-packets in the same wave-number region.

Keywords: Pair plasma, Electron-Positron-Ion Plasma, Modulational Instability, Envelope soliton

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THE MODEL EQUATIONS

The present study is devoted to an investigation of the nonlinear amplitude modulation of electrostatic modes [1] propagating parallel to the external magnetic field, in e-p-i plasmas, which is an extension to our previous work on pure pair plasma [2]. Recently, the production of pair fullerene-ion plasmas in laboratory [3, 4] has enabled experimental studies of pair plasmas rid of intrinsic problems involved in electron-positron plasmas, namely pair recombination processes and strong Landau damping. Here, we consider the nonlinear propagation of electrostatic wave packets in e-p-i plasmas or pair- (e.g. fullerene) ion plasmas in the presence of a small fraction of uniform and stationary positive ions, by employing a two-fluid plasma model. The two-fluid plasma-dynamical (moment) equations for our three-component plasma include the two density (continuity) equations

$$\frac{\partial n_{\alpha}}{\partial t} + \frac{\partial (n_{\alpha}U_{\alpha})}{\partial x} = 0,$$

(1)

and the two momentum equations

$$\frac{\partial U_{\alpha}}{\partial t} + (U_{\alpha} \cdot \nabla)U_{\alpha} = -\alpha \nabla \phi - \frac{\gamma T_{\alpha}}{T_{-}} (n_{\alpha})^{\gamma - 2} \nabla n_{\alpha},$$

(2)

where the subscript $\alpha$ denotes either species 1 (i.e. the positive ions, or positrons) for $\alpha = +$, or species 2 (i.e. the negative ions, or electrons) for $\alpha = -$. The moment variables $n_{\alpha}$, $U_{\alpha}$ denote the density and fluid velocity of species $\alpha$, respectively. The electric field is provided by the electric potential $\phi$, which obeys Poisson’s equation

$$\nabla^2 \phi = (n_{-} - n_{+} - \frac{Z_{1}}{Z} n_{3}).$$

(3)

where $Z$ ($Z_{1}$) denote the charge states of positrons and electrons (background ions, respectively). In equations (1)-(3), all quantities are normalized: the time and space variables as $t' \equiv \omega_{ph} t$ and $x' \equiv x/\lambda_{D_{-}}$, respectively, where the
characteristic scales are defined by the plasma frequency \( \omega_{p,a} = (4\pi n_0 q_a^2 / m_a) \) and the Debye frequency \( \lambda_0 = (k_B T_a / m_0) \). The density, velocity and electric potential state variables are scaled as \( n'_a = n_a / n_{-0}, u'_a = u_a / c_s \) and \( \phi' = \phi / \phi_0 \) respectively, where we have defined the characteristic (sound) speed \( c_s = (k_B T_a / m)^{1/2} \) (for negative ions) and the characteristic potential scale \( \phi_0 = (k_B T_a / Z e) \); the primes will be dropped for simplicity. It is assumed that the neutrality condition holds in equilibrium and the background ion density \( n_0 \) is constant.

**THE PERTURBATIVE ANALYSIS.**

In order to obtain an explicit evolution equation describing the propagation of modulated EA envelopes, from the model Eqs. (1)-(3), we shall employ the standard reductive perturbation (multiple scales) technique [5]. The independent variables \( x \) and \( t \) are stretched as \( \xi = \varepsilon (x - \lambda t) \) and \( \tau = \varepsilon^2 t \), where \( \varepsilon \) is a small (real) parameter; here, \( \lambda \) is a free (real) parameter, which is to be later determined as the wave’s group velocity by compatibility requirements. The dependent variable vector \( \mathbf{S}_a \) is expanded as

\[
\mathbf{S}_a = \mathbf{S}_{a,0} + \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} e^{i n \phi} \mathbf{S}_{a,n,l}(\xi, \tau) e^{i (k_l x - \omega_l t)}
\]

(4)

where \( \mathbf{S}_{a,0} \) denotes the equilibrium case. Substituting the expansion ansatz (4) and the stretched variables \( \xi, \tau \) into Eqs. (1)-(3), and then isolating distinct orders in \( \varepsilon \), we obtain, in the lowest-order, \( n = 1 \) and \( l = 1 \)

\[
\begin{align*}
n_{-1}^{(1)} &= \frac{k^2}{\omega^2 + 3 \alpha^2} \phi_1^{(1)}, & n_{-1}^{(1)} &= \frac{\beta k^2}{\omega^2 + 3 \alpha^2} \phi_1^{(1)}, & U_{-1}^{(1)} &= \frac{\omega k}{\omega^2 + 3 \alpha^2} \phi_1^{(1)}, & U_{-1}^{(1)} &= \frac{\beta k^2}{\omega^2 + 3 \alpha^2} \phi_1^{(1)}.
\end{align*}
\]

(5)

The following dispersion relation is deduced

\[
\frac{\omega}{\omega^2 - 3 \alpha^2} + \frac{1}{\omega^2 - 3 \alpha^2} = 1
\]

(6)

as a compatibility requirement, where \( \beta = n_+ / n_- \) and \( \sigma = T_+ / T_- \). Two real solutions are thus obtained for the frequency square \( \omega^2 \), defined by

\[
\begin{align*}
\omega_1^2 &= \frac{1 + \beta}{2} + \frac{\gamma}{2} (1 + \sigma \beta^2) k^2 - \frac{1}{2} \sqrt{\gamma^2 k^4 (1 - \sigma \beta^2)^2 + 2 \gamma (\beta - 1) (\sigma \beta^2 - 1) k^2 + (1 + \beta)^2}, \\
\omega_2^2 &= \frac{1 + \beta}{2} + \frac{\gamma}{2} (1 + \sigma \beta^2) k^2 + \frac{1}{2} \sqrt{\gamma^2 k^4 (1 - \sigma \beta^2)^2 + 2 \gamma (\beta - 1) (\sigma \beta^2 - 1) k^2 + (1 + \beta)^2}
\end{align*}
\]

(7)

(8)

which respectively denote an acoustic mode (lower branch), and a Langmuir-like optical mode (higher branch). These two dispersion curves are depicted in Figure 1. For the second-order \( n = 2 \) equatinos with \( l = 1 \) (1st harmonics), we deduce the following compatibility condition

\[
\lambda = \frac{\omega}{k} = \frac{1}{k\omega(\omega^2 - 3 \alpha^2)} \frac{1}{\sqrt{\lambda^2 (\omega^2 - 3 \alpha^2) + 2 \beta}}.
\]

(9)

It is easy to show that \( \lambda = \frac{\partial \phi}{\partial \tau} \).

Proceeding to \( n = 2, l = 2 \) in combination with \( n = 3, l = 0, 1 \), we obtain a compatibility condition in the form of the nonlinear Schrödinger equation (NLSE) [6]

\[
i \frac{\partial \phi}{\partial \tau} + P \frac{\partial^2 \phi}{\partial \xi^2} + Q |\phi|^2 \phi = 0,
\]

(10)

which describes the slow evolution of the first-order amplitude of the plasma potential perturbation \( \phi_1^{(1)} \). The dispersion coefficient \( P \) which is related to the dispersion curve as \( P = \frac{\partial \omega}{\partial k} \) and the nonlinearity coefficient \( Q \) which is due to the carrier wave self-interaction, are given in the Appendix. The localized solutions of the NLSE (10) describe (arbitrary amplitude) nonlinear excitations, in the form of bright (for \( P Q > 0 \)) or dark (i.e. black/gray, for \( P Q < 0 \)) envelope
The two dispersion curves defined by Eq. (6) are depicted, as a frequency $\omega$ variation vs. the reduced wavenumber $k$.

solitons. Exact expressions for these envelope structures can be found by substituting with $\phi = \sqrt{\rho} e^{i\theta}$ into Eq.(10), and then separating real and imaginary parts. The final formulae are exposed e.g. in Refs. [7, 8]. It is remarked that the ratio $P/Q$ determines the spatial extension of the localized envelope structures for a given maximum amplitude (and vice versa), in an inverse-proportional manner. The stability of the NLS equation (10) consists in linearizing around the monochromatic wave solution $\psi = \psi_0 e^{iQ|\psi_0|^2\tau}$, i.e. by setting $\psi = \tilde{\psi}_0 + \varepsilon \tilde{\psi}_1$, and then taking the perturbation $\tilde{\psi}_1$ to be of the form $\tilde{\psi}_1 = \tilde{\psi}_1, e^{i(\tilde{k} \xi - \tilde{\omega} \tau)}$ (the perturbation wave number $\tilde{k}$ and frequency $\tilde{\omega}$ should be distinguished from the carrier wave quantities $k$ and $\omega$). One thus obtains the dispersion relation $\tilde{\omega}^2 = P\tilde{k}^2(\tilde{k}^2 - 2\frac{P}{Q}|\psi_0|^2)$. In order for the wave to be stable, the product $PQ$ must be negative.

NUMERICAL ANALYSIS

We have seen that two distinct electrostatic modes, namely a quasi-thermal lower mode and a Langmuir-like optictype upper one, may propagate in our plasma system in the linear approximation; see Eqs. (7) and (8). Now, We may investigate the numerical value of the quantities $PQ$ and $P/Q$ in terms of the relevant physical parameters, namely the positron-to-electron (or positive-to-negative ion) density and temperature ratio(s), $\beta$ and $\sigma$, respectively, for these modes. The results of the calculations for the lower and higher modes are shown in Figs. 1 and 2 respectively. We conclude that the lower (acoustic) mode is generally stable, for realistic large wavelength situations (cf. Fig. 2) and may propagate in the form of a dark-type envelope soliton (i.e. a potential dip, a void). On the other hand, the upper (Langmuir-like) mode is modulationally unstable (cf. Fig. 3), and may favor the formation of bright-type envelope soliton (pulse) modulated wave packets at low wave-numbers. Fig.1 reveals that the stability range of the lower (acoustic) mode increases as the positive ion (or positron) to negative ion (or electron) ion density ratio $\beta$ increases.

Furthermore, careful inspection of Figs. 1 and 2 shows that the temperature ratio $\sigma$ is an important factor, from the point of view of stability, for both modes. In specific, one may anticipate that a local coexistence of positrons with a colder (warmer), say, population of negative electrons, viz. $\sigma < 1$ ($\sigma > 1$), may critically affect the stability profile of electrostatic modes, for instance by stabilizing the lower mode, or by destabilizing the upper mode.

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REFERENCES

3. The NLSE coefficient product $PQ$ (a and c) and ratio $P/Q$ (b and d) corresponding to the lower dispersion branch, are depicted against the reduced wavenumber $k$ (in abscissa everywhere).

4. The NLSE coefficient product $PQ$ (a and c) and ratio $P/Q$ (b and d) corresponding to the higher dispersion branch, are depicted against the reduced wavenumber $k$ (in abscissa everywhere).


Appendix