Electrostatic mode envelope excitations in warm pair ion plasmas with a small fraction of stationary positive ions - application in e-p-i and doped fullerene plasmas

A. Esfandyari-Kalejahi*, I. Kourakis† and P. K. Shukla **

*Azarbaijan University of Tarbiat Moallem, Faculty of Science, Department of physics, 51745-406, Tabriz, Iran.
†Ruhr-Universität Bochum, Institut für Theoretische Physik IV, D-44780 Bochum, Germany.
**Ruhr-Universität Bochum, Institut fürTheoretische Physik IV, D-44780 Bochum, Germany.

Abstract. The nonlinear propagation of electrostatic wave packets in electron-positron-ion (e-p-i) plasmas, or pair- (e.g. fullerene) ion plasmas in the presence of a small fraction of uniform and stationary positive ions is studied. A two-fluid plasma model is employed. Two distinct electrostatic modes are obtained, namely a quasi-ion-thermal lower mode and a Langmuir-like optic-type upper one, as in pure pair plasmas, in agreement with previous experimental observations and theoretical studies of equal-temperature pair plasmas. The basic set of model equations is reduced to a nonlinear Schrödinger equation for the slowly varying electric field perturbation amplitude. The analysis reveals that the stability range of lower (acoustic) mode increases as the positive-to- negative-ion (or positron-to-electron) density ratio increases, so this quasi-thermal mode may propagate in the form of a dark-type envelope soliton (i.e. a potential dip, or a void) modulating a carrier wave packet for small wave-numbers, for a fixed value of the positive-to-negative-ion (or positron-to-electron) temperature ratio. On the other hand, the upper mode is modulationally unstable, and may thus favor the formation of bright- type envelope soliton (pulse) modulated wave-packets in the same wave-number region.

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THE MODEL EQUATIONS

The present study is devoted to an investigation of the nonlinear amplitude modulation of electrostatic modes [1] propagating parallel to the external magnetic field, in e-p-i plasmas, which is an extension to our previous work on pure pair plasma [2]. Recently, the production of pair fullerene-ion plasmas in laboratory [3, 4] has enabled experimental studies of pair plasmas rid of intrinsic problems involved in electron-positron plasmas, namely pair recombination processes and strong Landau damping. Here, we consider the nonlinear propagation of electrostatic wave packets in e-p-i plasmas or pair- (e.g. fullerene) ion plasmas in the presence of a small fraction of uniform and stationary positive ions, by employing a two-fluid plasma model. The two-fluid plasma-dynamical (moment) equations for our three-component plasma include the two density (continuity) equations

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial (n_\alpha \vec{U}_\alpha)}{\partial x} = 0, \tag{1}$$

and the two momentum equations

$$\frac{\partial \vec{U}_\alpha}{\partial t} + (\vec{U}_\alpha . \vec{\nabla})\vec{U}_\alpha = -\alpha \vec{\nabla}\phi - \frac{\gamma T_\alpha}{T_-}(n_\alpha)^{\gamma-2}\vec{\nabla} n_\alpha, \tag{2}$$

where the subscript α denotes either species 1 (i.e. the positive ions, or positrons) for $\alpha = +$, or species 2 (i.e. the negative ions, or electrons)for $\alpha = -$. The moment variables n_α, $\vec{U}_\alpha$ denote the density and fluid velocity of species α, respectively. The electric field is provided by the electric potential ϕ, which obeys Poisson's equation

$$\vec{\nabla}^2\phi = (n_- - n_+ - \frac{Z_3}{Z} n_3). \tag{3}$$

where Z (Z_3) denote the charge states of positrons and electrons (background ions, respectively). In equations (1)-(3), all quantities are normalized: the time and space variables as $t' \equiv \omega_p t$ and $x' \equiv x/\lambda_{D,-}$, respectively, where the characteristic scales are defined by the plasma frequency $\omega_{p,\alpha} = (4\pi n_0 q_\alpha^2/m_\alpha)$ and the Debye frequency $\lambda_{D,\alpha} = (K_B T_\alpha / m\omega_{p,\alpha})^{1/2}$. The density, velocity and electric potential state variables are scaled as $n'_\alpha = n_\alpha/n_{-,0}$, $u'_\alpha = u_\alpha/c_s$ and $\phi' = \phi/\phi_0$ respectively, where we have defined the characteristic (sound) speed $c_s = (K_B T_-/m)^{(1/2)}$ (for negative ions) and the characteristic potential scale $\phi_0 = (K_B T_-/Ze)$; the primes will be dropped for simplicity. It is assumed that the neutrality condition holds in equilibrium and the background ion density n_3 is constant.

THE PERTURBATIVE ANALYSIS.

In order to obtain an explicit evolution equation describing the propagation of modulated EA envelopes, from the model Eqs (1)-(3), we shall employ the standard reductive perturbation (multiple scales) technique [5]. The independent variables x and t are stretched as $\xi = \varepsilon(x - \lambda t)$ and $\tau = \varepsilon^2 t$, where ε is a small (real) parameter; here, λ is a free (real) parameter, which is to be later determined as the wave's group velocity by compatibility requirements. The dependent variable vector $\mathbf{S}_\alpha$ is expanded as

$$\mathbf{S}_\alpha = \mathbf{S}_{\alpha,0} + \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \varepsilon^n \mathbf{S}_{\alpha,l}^{(n)}(\xi,\tau)\, e^{il(kx-\omega t)} \tag{4}$$

where $\mathbf{S}_{\alpha,0}$ denotes the equilibrium case. Substituting the expansion ansatz (4) and the stretched variables ξ,τ into Eqs. (1)- (3), and then isolating distinct orders in ε, we obtain, in the lowest-order, $n = 1$ and $l = 1$

$$n_{-,1}^{(1)} = \frac{k^2}{-\omega^2 + 3k^2}\phi_1^{(1)}, \;\; n_{+,1}^{(1)} = \frac{\beta k^2}{\omega^2 - 3\sigma\beta^2 k^2}\phi_1^{(1)}, \;\; U_{-,1}^{(1)} = \frac{\omega k}{-\omega^2 + 3k^2}\phi_1^{(1)}, \;\; U_{+,1}^{(1)} = \frac{\beta k^2}{\omega^2 - 3\sigma\beta^2 k^2}\phi_1^{(1)}. \tag{5}$$

The following dispersion relation is deduced

$$\frac{\beta}{\omega^2 - 3\sigma\beta^2 k^2} + \frac{1}{\omega^2 - 3k^2} = 1 \tag{6}$$

as a compatibility requirement, where $\beta = n_+/n_-$ and $\sigma = T_+/T_-$. Two real solutions are thus obtained for the frequency square ω^2, defined by

$$\omega_1^2 = \frac{1+\beta}{2} + \frac{\gamma}{2}(1+\sigma\beta^2)k^2 - \frac{1}{2}\sqrt{\gamma^2 k^4 (1-\sigma\beta)^2 + 2\gamma(\beta-1)(\sigma\beta^2-1)k^2 + (1+\beta)^2}, \tag{7}$$

$$\omega_2^2 = \frac{1+\beta}{2} + \frac{\gamma}{2}(1+\sigma\beta^2)k^2 + \frac{1}{2}\sqrt{\gamma^2 k^4 (1-\sigma\beta)^2 + 2\gamma(\beta-1)(\sigma\beta^2-1)k^2 + (1+\beta)^2} \tag{8}$$

which respectively denote a an acoustic mode (lower branch), and a Langmuir-like optical mode (higher branch). These two dispersion curves are depicted in Figure 1. For the second-order ($n = 2$) equatins with $l = 1$ (1^{st} harmonics), we deduce the following compatibility condition

$$\lambda = \frac{\omega}{k} - \frac{1}{k\omega\left[\frac{1}{(\omega^2-3k^2)^2} + \frac{\beta}{(\omega^2-3\sigma\beta^2k^2)^2}\right]}. \tag{9}$$

It is easy to show that $\lambda = \frac{\partial\omega}{\partial k}$.

Proceeding to n=2, l=2 in combination with n=3, l=0, 1, we obtain a compatibility condition in the form of the nonlinear Schrödinger equation (NLSE) [6]

$$i\frac{\partial\phi}{\partial\tau} + P\frac{\partial^2\phi}{\partial\xi^2} + Q|\phi|^2\phi = 0, \tag{10}$$

which describes the slow evolution of the first-order amplitude of the plasma potential perturbation $\phi_1^{(1)}$. The dispersion coefficient P, which is related to the dispersion curve as $P = \frac{\partial^2\omega}{2\partial k^2}$ and the nonlinearity coefficient Q which is due to the carrier wave self-interaction, are given in the Appendix. The localized solutions of the NLSE (10) describe (arbitrary amplitude) nonlinear excitations, in the form of bright (for $PQ > 0$) or dark (i.e. black/gray, for $PQ < 0$) envelope

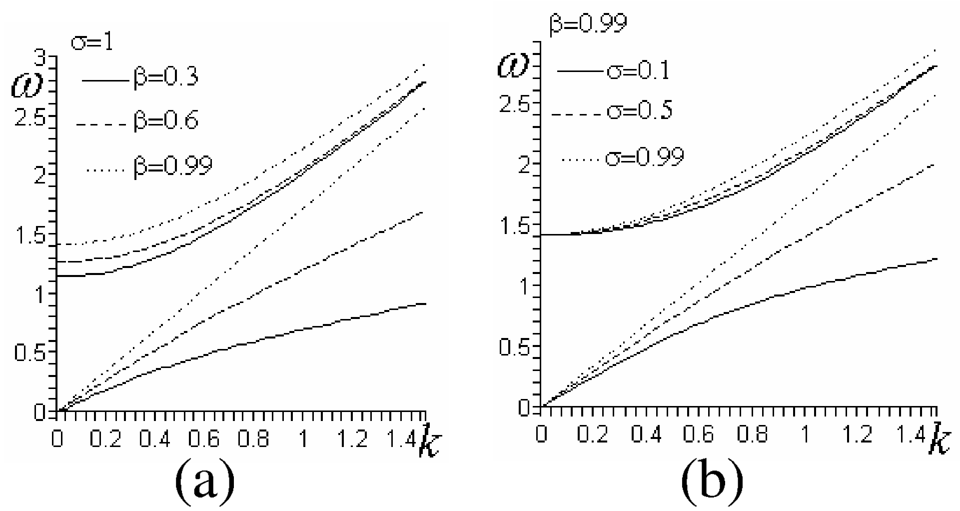

FIGURE 1. The two dispersion curves defined by Eq. (6) are depicted, as a frequency ω variation vs. the reduced wavenumber k.

solitons. Exact expressions for these envelope structures can be found by substituting with $\phi = \sqrt{\rho}\, e^{i\theta}$ into Eq.(10), and then separating real and imaginary parts. The final formulae are exposed e.g. in Refs. [7, 8]. It is remarked that the ratio P/Q determines the spatial extension of the localized envelope structures for a given maximum amplitude (and vice versa), in an inverse-proportional manner. The stability of the NLS equation (10) consists in linearizing around the monochromatic wave solution $\psi = \tilde{\psi} e^{iQ|\tilde{\psi}|^2\tau}$, i.e. by setting $\tilde{\psi} = \tilde{\psi}_0 + \varepsilon \tilde{\psi}_1$, and then taking the perturbation $\tilde{\psi}_1$ to be of the form $\tilde{\psi}_1 = \tilde{\psi_1}_{,0} e^{i(\hat{k}\xi - \hat{\omega}\tau)}$(the perturbation wave number $\hat{k}$ and frequency $\hat{\omega}$ should be distinguished from the carrier wave quantities k and ω). One thus obtains the dispersion relation $\hat{\omega}^2 = P\hat{k}^2(P\hat{k}^2 - 2\frac{Q}{P}|\tilde{\psi}^0|^2)$. In order for the wave to be stable, the product PQ must be negative.

NUMERICAL ANALYSIS

We have seen that two distinct electrostatic modes, namely a quasi-thermal lower mode and a Langmuir-like optic-type upper one, may propagate in our plasma system in the linear approximation; see Eqs. (7) and (8). Now, We may investigate the numerical value of the quantities PQ and P/Q in terms of the relevant physical parameters, namely the positron-to-electron (or positive-to-negative ion) density and temperature ratio(s), β and σ, respectively, for these modes. The results of the calculations for the lower and higher modes are shown in Figs. 1 and 2 respectively. We conclude that the lower (acoustic) mode is generally stable, for realistic large wavelength situations (cf. Fig. 2) and may propagate in the form of a dark-type envelope soliton (i.e. a potential dip, a void). On the other hand, the upper (Langmuir-like) mode is modulationally unstable (cf. Fig. 3), and may favor the formation of bright-type envelope soliton (pulse) modulated wave packets at low wave-numbers. Fig.1 reveals that the stability range of the lower (acoustic) mode increases as the positive ion (or positron) to negative ion (or electron) ion density ratio β increases. Furthermore, careful inspection of Figs. 1 and 2 shows that the temperature ratio σ is an important factor, from the point of view of stability, for both modes. In specific, one may anticipate that a local coexistence of positrons with a colder (warmer), say, population of negative electrons, viz. $\sigma < 1$ ($\sigma > 1$), may critically affect the stability profile of electrostatic modes, for instance by stabilizing the lower mode, or by destabilizing the upper mode.

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REFERENCES

1. T. STIX, *"Waves in plasmas" (New York: American Institute of Physics, 1992).*

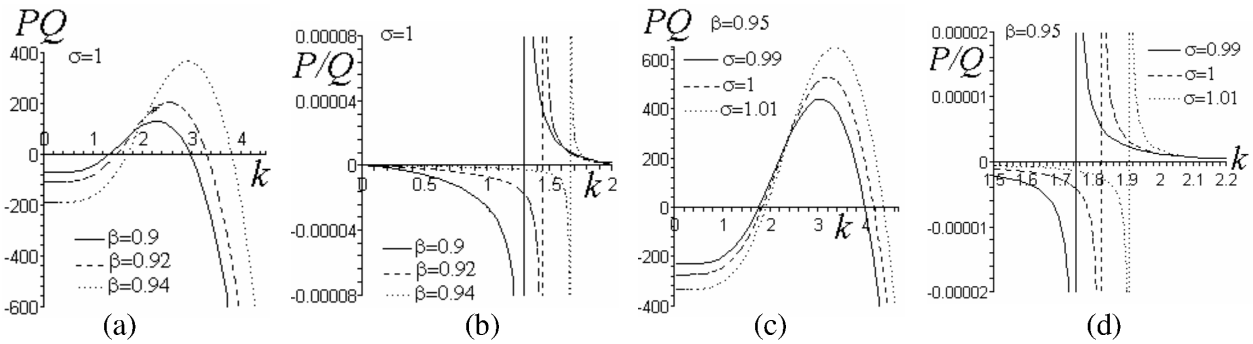

FIGURE 2. The NLSE coefficient product PQ (a and c) and ratio P/Q (b and d) corresponding to the lower dispersion branch , are depicted against the reduced wavenumber *k* (in abscissa everywhere).

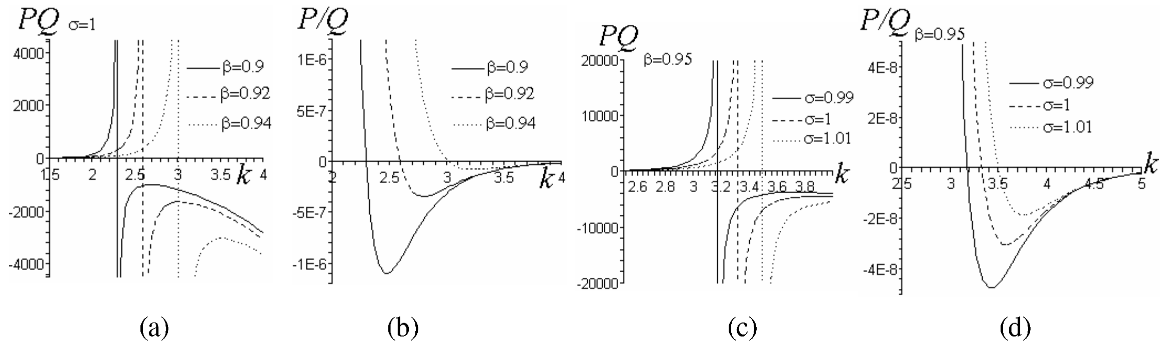

FIGURE 3. The NLSE coefficient product PQ (a and c) and ratio P/Q (b and d) corresponding to the higher dispersion branch , are depicted against the reduced wavenumber *k* (in abscissa everywhere)

2. I. KOURAKIS, A. ESFANDYARI-KALEJAHI, M. MEHDIPOOR AND P.K. SHUKLA, Modulated electrostatic modes in pair plasmas, submitted to *Physics of Plasmas*, (2006).
3. W. OOHARA AND R. HATAKEYAMA, *Phys. Rev. Lett,* **91**, 205005 (2003).
4. W. OOHARA, D. DATE AND R. HATAKEYAMA, *Phys. Rev. Lett,* **95**, 175003 (2005).
5. T. TANIUTI AND N. YAJIMA, *J. Math. Phys,* **10**, 1369 (1969).
6. P. SULEM AND C. SULEM , *"Nonlinear Schrödinger Equation" (Speringer, Berlin, 1999).*
7. R. FEDELE, *Phys. Scr,* **65**, 502 (2002).
8. I. KOURAKIS AND P.K. SHUKLA, *Nonlinear Proc. Geophys,* **12**, 407 (2005).

Appendix

$$P=\frac{(\omega^2-k\lambda\omega)^2(\omega-k\omega)}{2\omega^2k^2}\Big[\frac{\omega^2+3k^2}{(\omega^2-3k^2)^3}+\frac{\beta(\omega^2+3\sigma k^2\beta^2)}{(\omega^2-3\sigma k^2\beta^2)^3}\Big]+\frac{3(\omega^2-k\lambda\omega)^2}{\omega}\Big[\frac{1}{(\omega^2-3k^2)^3}+\frac{\sigma\beta^3}{(\omega^2-3\sigma k^2\beta^2)^3}\Big]$$
$$-\frac{(\omega^2-k\lambda\omega)}{2\omega k^2}-\frac{(\omega^2-k\lambda\omega)^2\lambda}{k}\Big[\frac{1}{(\omega^2-3k^2)^3}+\frac{\beta}{(\omega^2-3\sigma k^2\beta^2)^3}\Big],$$

$$Q=-\frac{k^3(2\omega+k\lambda)(\omega^2-k\lambda\omega)}{2\lambda\omega}\Big[\frac{(\omega^2+3k^2)}{(\omega^2-3k^2)^4}+\frac{\beta(\omega^2+3\sigma k^2\beta^2)}{(\omega^2-3\sigma k^2\beta^2)^4}\Big]-\frac{3k^4(\omega^2-k\lambda\omega)}{4\omega}\Big[\frac{(\omega^2+3k^2)(\omega^2+k^2)}{(\omega^2-3k^2)^5}+\frac{\beta(\omega^2+3\sigma k^2\beta^2)(\omega^2+\sigma k^2\beta^2)}{(\omega^2-3\sigma k^2\beta^2)^5}\Big]$$
$$-\frac{3k^4(\omega^2-k\lambda\omega)}{4\omega}\Big[\frac{(\omega^2+k^2)[\omega^2+k^2+6k^2(\omega^2-3k^2)]}{(\omega^2-3k^2)^6}+\frac{\beta^2(\omega^2+\sigma k^2\beta^2)[\omega^2+\sigma k^2\beta^2+6\sigma k^2\beta(\omega^2-3\sigma k^2\beta^2)]}{(\omega^2-3\sigma k^2\beta^2)^6}\Big]$$
$$+\frac{3\beta k^4(\omega^2+k^2)(\omega^2+\sigma k^2\beta^2)(\omega^2-k\lambda\omega)}{2\omega(\omega^2-3k^2)^3(\omega^2-3\sigma k^2\beta^2)^3}+\frac{(2k\lambda\omega+\omega^2+3k^2)(\omega^2-k\lambda\omega)}{2\omega[\lambda^2-3\sigma\beta^2+(\lambda^2-3)\beta]}\Big\{\frac{2\omega k^3(\lambda^2-3\sigma\beta^2-3\beta)-k^2\beta\lambda(\omega^2+3k^2)}{\lambda(\omega^2-3k^2)^4}$$
$$-\frac{4\omega k^3\lambda\beta}{(\omega^2-3k^2)^2(\omega^2-3\sigma k^2\beta^2)^2}-\frac{k^2\beta(2\omega^2+3k^2+3\sigma k^2\beta^2)}{(\omega^2-3k^2)^2(\omega^2-3\sigma k^2\beta^2)^2}+\frac{2\omega k^3\beta^2(\lambda^2-3\sigma\beta-3)-k^2\beta\lambda(\omega^2+3\sigma k^2\beta^2)}{\lambda(\omega^2-3\sigma k^2\beta^2)^4}\Big\}.$$