Propagation of large amplitude ion acoustic waves in an electron beam plasma consisting of two temperature electrons and warm ions

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Abstract. The propagation of arbitrary amplitude nonlinear ion-acoustic waves in an electron-beam-plasma system consisting of two temperature electrons (hot/cold) and warm ions is investigated by using a pseudopotential method, applied in a two-fluid model. The effects of hot-to-cold electron temperature and density ratio (µ and ν, respectively) and beam-to-ion density ratio (β) are studied numerically. The conditions for the existence of large amplitude ion-acoustic waves in terms of these parameters are investigated. It is remarked that the maximum Mach number $M$ increases (decreases) as $β$ ($ν$) increases, for fixed $σ$ and $µ$. Also, the maximum Mach number $M$ increases (decreases) as $β$ ($µ$) increases for fixed $σ$ and $µ$. In addition, it is found that the amplitude of compressive solitons increases as $µ$ rises to a given limit, after which the compressive solitons do not occur, provided other parameters remain fixed. On the other hand, increasing $ν$ up to a given limit leads to an enhancement in the amplitude of compressive solitons. However, if $ν$ rises above this limit, the amplitude of solitons decreases again (very low and very high values of $ν$ have the same physical meaning, i.e. a single-electron species limit).

Keywords: Large Amplitude Ion Acoustic Waves, Pseudopotential Method, Electron Beam and Two Fluid Plasma.

PACS: 52.35.Mw,52.40.Mj,52.30.Ex

INTRODUCTION

This study focuses on a situation of particular interest, when an electron beam is present in a two-electron-temperature plasma. Such a situation is typically encountered in the upper layers of the magnetosphere, where a coexistence of two different electron populations (say, cold inertial and warm energetic ones) has been reported by satellite missions [1, 2]. Recently, a lot of research work has focused on plasmas in the presence of an electron beam e.g. [3, 4, 5, 6] or two-temperature electrons e.g. [8, 7]. It is therefore tempting to investigate the existence of large amplitude ion-acoustic solitary waves in a plasma consisting of warm ions, two distinct temperature electrons and an electron beam. In the following, we shall adopt a pseudopotential (Sagdeev) method.

BASIC EQUATION AND FORMULATION

We consider a plasma consisting of warm ions, two temperature electrons and a non-relativistic electron beam. Assuming a one-dimensional (1D) geometry, the basic set of normalized fluid equations for this system is as follows:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{\sigma p}{n} \frac{\partial p}{\partial x} = 0,$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + 3p \frac{\partial u}{\partial x} = 0,$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b u_b)}{\partial x} = 0,$$

$$\frac{\partial u_b}{\partial t} + u_b \frac{\partial u_b}{\partial x} - \mu' \frac{\partial \phi}{\partial x} = 0.$$
The Lorentz force term is neglected, since wave propagation parallel to the external magnetic field is assumed. The electric field derives from an electric potential $\phi$, which obeys Poisson’s equation

$$\frac{\partial^2 \phi}{\partial \xi^2} = 1 - \frac{\beta}{Z} \frac{n_b}{Z} + (1 - \frac{\beta}{Z})(\phi + \alpha \phi^2 + \alpha' \phi^3) - n.$$  \hspace{1cm} (6)

where $n, n_b, n_e$ and $n_h$ are the ion, electron beam, cold electron and hot electron density respectively, normalized to the unperturbed ion density $n_0$; the ion (also, electron beam) velocity $u_b$, the ion pressure $p$ and the electrostatic potential $\phi$ are normalized to the ion acoustic speed $C_{s,i} = (Zk_BT_{i,\text{eff}}/m)^{1/2}$, $n_0k_BT_i$ and $k_BT_{i,\text{eff}}/e$, respectively ($k_B$ is Boltzmann’s constant); the space and time variables have been scaled by the effective Debye length $\lambda_{D,\text{eff}} = (k_BT_{i,\text{eff}}/4\pi Z e^2 n_0)^{1/2}$ and the ion plasma frequency $\omega_{pi} = (4\pi Z e^2 n_0/m)^{1/2}$, respectively. The parameters $\alpha, \alpha', \beta, \beta', \nu$ and $\sigma$ are given by

$$\sigma = \frac{T_i}{Z T_{i,\text{eff}}}, \hspace{1cm} \beta = \frac{n_h}{n_b}, \hspace{2cm} \alpha = \frac{1}{2} \frac{(1 + \nu)(\nu + \mu)}{v + \mu}, \hspace{2cm} \alpha' = \frac{1}{6} \frac{(1 + \nu)^2(\nu + \mu)}{v + \mu},$$

in which $T_i$ is the ion temperature, $T_{i,\text{eff}} = (n_{e,0} + n_{h,0})/(n_{b,0}/T_b + n_{c,0}/T_e)$ is an effective temperature, $m$ ($m_e$) is the ion (electron) mass, $Z$ is the ion charge state, $\beta'$ is the mass ratio $m_b/m_e$, $\nu = n_b/n_c$ (here $n_{b,0}/c_0$ is the hot/cold unperturbed electron density) and $\mu = T_b/T_e$ (where $T_b/T_e$ is the hot/cold electron temperature).

**Derivation of pseudopotential**

Anticipating a solitary travelling-wave solution, we assume all the dependent variables to depend on a single independent variable $\xi = x - Mt$, where $M$ is the Mach number (the velocity of the solitary wave $v$ normalized to ion acoustic speed $C_{s,i}$). Under appropriate boundary conditions for localized waves: $\phi \rightarrow 0, n \rightarrow 1, u \rightarrow 0, p \rightarrow 1, n_b \rightarrow \beta$ and $u_b \rightarrow u_{b,0}$ at $\xi \rightarrow \pm \infty$, upon integrating Eqs. (1)-(3) we obtain

$$n = \frac{\sigma_1}{\sqrt{2} \sigma_0} \left[ 1 - \frac{2 \phi}{M^2 \sigma_1^2} \right] = \sqrt{\left( 1 - \frac{2 \phi}{M^2 \sigma_1^2} \right)^2 - 4 \sigma_0^2} \left[ 1 + \sigma_0^2 \right]^{1/2},$$ \hspace{1cm} (7)

where $\sigma_0 = \sqrt{3\sigma / M^2}$ and $\sigma_1 = \sqrt{1 + \sigma_0^2}$. Integrating Eqs. (4) and (5), we get

$$n_b = \beta / \left( \sqrt{1 + 2 \mu' \phi/(u_{b,0} - M)^2} \right).$$ \hspace{1cm} (8)
FIGURE 2. The maximum Mach number $M$ as a function of electron beam density $\beta \sigma = 0.1$, $u_{b0} = 1.1$ and $\mu' = 1836$.

The densities $n$ and $n_b$ are real if one of the following conditions is satisfied

$$-\frac{(u_{b0} - M)^2}{2\mu'} \leq \phi \leq \frac{M^2\sigma^2_1}{2} \left(1 - \frac{2\sigma_0}{\sigma_1^2}\right),$$

or if

$$\phi \geq \max \left\{ -\frac{(u_{b0} - M)^2}{2\mu'}, \frac{M^2\sigma^2_1}{2} \left(1 + \frac{2\sigma_0}{\sigma_1^2}\right) \right\}.$$

The region of the existence of $\phi$ is determined by these conditions. Now, integration of Poisson equation, Eq (6), gives rise to

$$\frac{1}{2} \left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0,$$

where $V(\phi)$ is the pseudopotential and given by

$$V(\phi) = -\left[\left(1 - \frac{\beta}{2}\right)\phi + \left(1 - \frac{\beta}{2}\right)\left(\frac{\phi^2}{2} + \alpha \frac{\phi^3}{3} + \alpha' \frac{\phi^4}{4}\right) + \frac{\beta_0 (u_{b0} - M)^2}{2\mu'} (\sqrt{1 + \frac{2\mu'\phi}{(u_{b0} - M)^2}} - 1) \right]$$

$$-M^2 \sqrt{\theta_0} \left[\left(e^{\theta/2} - e^{\theta_0/2}\right) + \frac{1}{2} \left(e^{-3\theta/2} - e^{-3\theta_0/2}\right)\right]$$

where $\theta = \cosh^{-1}\left(\frac{\sigma_1^2}{2\sigma_0} (1 - \frac{2\phi}{M^2\sigma_1^2})\right)$ and $\theta_0 = \theta(\phi = 0)$ is the integration constant which $V(\phi) = 0$ at $\phi = 0$. The solitary solution for nonlinear ion acoustic waves exists when the following two conditions are satisfied:

i) The pseudopotential $V(\phi)$ has a maximum at $\phi = 0$, i.e. if $(d^2V(\phi)/d\phi^2)|_{\phi=0} < 0$, so that the fixed point at the origin is unstable, and

ii) $V(\phi) < 0$ for $0 < \phi < \phi_{\text{max}}$, for positive solitary waves, or for $0 > \phi > \phi_{\text{min}}$, for negative solitary waves, where $\phi_{\text{max(min)}}$ is the positive (negative) value of $\phi$ for which $V(\phi) = 0$; these values are here determined by

$$\phi_{\text{max}} = \frac{M^2\sigma^2_1}{2} \left(1 - 2\sigma_0/\sigma_1^2\right), \quad \phi_{\text{min}} = -\frac{(u_{b0} - M)^2}{2\mu'}$$

It must be noticed that the first critical value does not depend on $\nu$ and $\mu$, so the effects of physical parameters on the existence of large amplitude ion-acoustic solitary waves will bear its origin in the second condition.
FIGURE 3. The pseudopotential $V(\phi)$ versus $\phi$ for $\beta = 0.0000001$, $M = 1.2$, $\sigma = 0.1$, $u_{b0} = 1.1$ and $\dot{\mu} = 1836$.

THE REGION OF THE EXISTENCE OF LARGE AMPLITUDE ION-ACOUSTIC WAVES

We depict the zero-value contour plots for $V(\phi_{\text{max}})$ and $\frac{d^2V(\phi)}{d\phi^2}$ at $\phi = 0$ against the Mach number $M$ and electron beam to background ion density ratio $\beta$ in Figs. 1a and 1b respectively, in the case $\nu = 10$, $\mu = 10$, $\sigma = 0.1$, $u_{b0} = 1.1$ and $\mu' = 1836$. In Fig. 1a, the area in black/white represents the regions in the $(M - \beta)$ plane where $V(\phi_{\text{max}})$ is positive/negative. We remark that there is a maximum and minimum limit for $M$. On the other hand, the area in black/white in Fig. 1b represents the regions in the $(M - \beta)$ plane where $V(\phi_{\text{max}}) < 0/ > 0$. As it is mentioned in previous section, fig. 1a does not modified as $\mu$ and $\nu$ change. Careful inspection Fig. 1a and 1b shows that large amplitude ion-acoustic solitary waves occur when $\beta$ is very very small, for instance $\beta = 0.0000001$ and $|M| > 1$ namely supersonic case.

We show the maximum Mach number $M$ as a function of $\beta$ in figure 2a(2b) in the case: $\mu = 10$; $\nu = 0.05$, $\nu = 0.6$, and $\nu = 20$ ($\nu = 10$; $\mu = 4.5$, $\mu = 8$, and $\mu = 25$). It is remarked that the maximum Mach number increases (decreases) as $\beta(\nu)$ increases for fixed $\mu$. Also the maximum Mach number $M$ increases (decreases) as $\beta(\mu)$ increases for fixed $\nu$. We illustrate dependence of $V(\phi)$ on the electrostatic potential $\phi$ when $\beta = 0.0000001$, $M = 1.2$, $\sigma = 0.1$ for two case: $\nu = 10$ but different values $\mu$, and $\mu = 10$ but different values $\nu$ in Figs. 3a-b. It is seen the amplitude of compressive solitons increases as $\mu$ rises to a given limit, after which the compressive solitons do not occur (Fig. 3a). On the other hand, enhancing $\nu$ up to a given limit leads to an increase in the amplitude of compressive solitons. However, if $\nu$ rises to more than this limit, amplitude of solitons will decrease (Fig.3b), since very low and very high values $\nu$ have the same physical meaning.

REFERENCES