Propagation of large amplitude ion acoustic waves in an electron beam plasma consisting of two temperature electrons and warm ions

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Abstract. The propagation of arbitrary amplitude nonlinear ion-acoustic waves in an electron-beam-plasma system consisting of two temperature electrons (hot/cold) and warm ions is investigated by using a pseudopotential method, applied in a two-fluid model. The effects of hot-to-cold electron temperature and density ratio (μ and ν, respectively) and beam-to-ion density ratio (β) are studied numerically. The conditions for the existence of large amplitude ion-acoustic waves in terms of these parameters are investigated. It is remarked that the maximum Mach number M increases (decreases) as β (ν) increases, for fixed σ and μ. Also, the maximum Mach number M increases (decreases) as β (μ) increases for fixed σ and μ. In addition, it is found that the amplitude of compressive solitons increases as μ rises to a given limit, after which the compressive solitons do not occur, provided other parameters remain fixed. On the other hand, increasing ν up to a given limit leads to an enhancement in the amplitude of compressive solitons. However, if ν rises above this limit, the amplitude of solitons decreases again (very low and very high values of ν have the same physical meaning, i.e. a single-electron species limit).

Keywords: Large Amplitude Ion Acoustic Waves, Pseudopotential Method, Electron Beam and Two Fluid Plasma.
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INTRODUCTION

This study focuses on a situation of particular interest, when an electron beam is present in a two-electron-temperature plasma. Such a situation is typically encountered in the upper layers of the magnetosphere, where a co-existence of two different electron populations (say, cold inertial and warm energetic ones) has been reported by satellite missions [1, 2]. Recently, a lot of research work has focused on plasmas in the presence of an electron beam e.g. [3, 4, 5, 6] or two-temperature electrons e.g. [8, 7]. It is therefore tempting to investigate the existence of large amplitude ion-acoustic solitary waves in a plasma consisting of warm ions, two distinct temperature electrons and an electron beam. In the following, we shall adopt a pseudopotential (Sagdeev) method.

BASIC EQUATION AND FORMULATION

We consider a plasma consisting of warm ions, two temperature electrons and a non-relativistic electron beam. Assuming a one-dimensional (1D) geometry, the basic set of normalized fluid equations for this system is as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0\,, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\partial \phi}{\partial x}+\frac{\sigma}{n}\frac{\partial p}{\partial x}=0\,, \tag{2}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+3p\frac{\partial u}{\partial x}=0\,, \tag{3}$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial (n_b u_b)}{\partial x}=0\,, \tag{4}$$

$$\frac{\partial u_b}{\partial t}+u_b\frac{\partial u_b}{\partial x}-\mu'\frac{\partial \phi}{\partial x}=0\,. \tag{5}$$

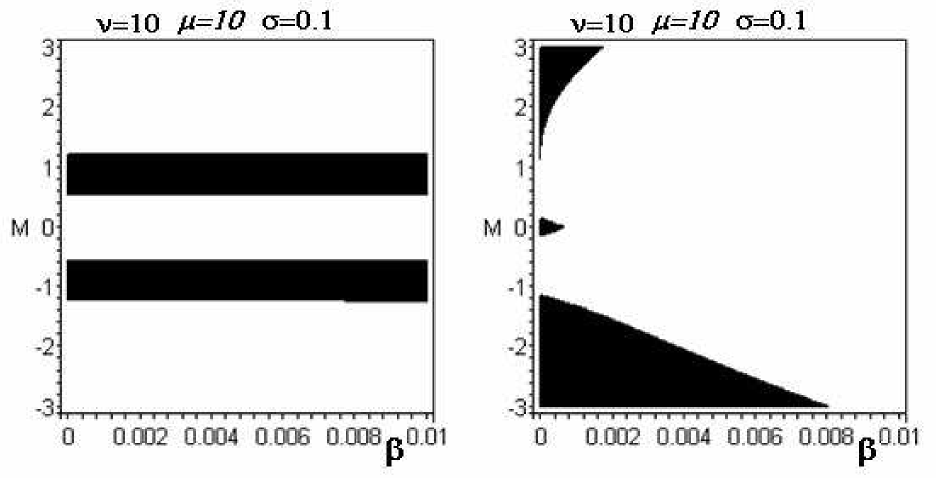

FIGURE 1. The zero pseudopotential value $V(\phi_{max}) = 0$ (left) and curvature $d^2V(\phi)/d\phi^2|_{\phi=0} = 0$ (right) contours are depicted versus the electron -beam-to-background-ion density ratio β and the Mach number M, for $\nu = 10$, $\mu = 10$, $\sigma = 0.1$, $u_{b_0} = 1.1$ and $\mu' = 1836$. The black (white) region corresponds to negative (positive) values.

The Lorentz force term is neglected, since wave propagation parallel to the external magnetic field is assumed. The electric field derives from an electric potential ϕ, which obeys Poisson's equation

$$\frac{\partial^2\phi}{\partial x^2} = 1 - \frac{\beta}{Z} + \frac{n_b}{Z} + (1 - \frac{\beta}{Z})(\phi + \alpha\phi^2 + \alpha'\phi^3) - n. \tag{6}$$

where n, n_b, n_c and n_h are the ion, electron beam, cold electron and hot electron density respectively, normalized to the unperturbed ion density n_0; the ion (also, electron beam) velocity u (u_b), the ion pressure p and the electrostatic potential ϕ are normalized to the ion acoustic speed $C_{s,eff} = (Zk_BT_{eff}/m)^{1/2}$, $n_0k_BT_i$ and k_BT_{eff}/e, respectively (k_B is Boltzmann's constant); the space and time variables have been scaled by the effective Debye length $\lambda_{D,eff} = (K_BT_{eff}/4\pi Ze^2n_0)^{1/2}$ and the ion plasma frequency $\omega_{pi} = (4\pi Z^2e^2n_0/m)^{1/2}$, respectively. The parameters σ, μ', β, α and α' are given by

$$\sigma = \frac{T_i}{ZT_{eff}}, \quad \mu' = \frac{m}{Zm_e}, \quad \beta = \frac{n_{b_0}}{n_0}, \quad \alpha = \frac{1}{2}(1+\nu)\frac{(\nu+\mu^2)}{(\nu+\mu)^2}, \quad \alpha' = \frac{1}{6}(1+\nu)^2\frac{(\nu+\mu^3)}{(\nu+\mu)^3}$$

in which T_i is the ion temperature, $T_{eff} = (n_{c_0} + n_{h_0})/(n_{h_0}/T_h + n_{c_0}/T_c)$ is an effective temperature, m (m_e) is the ion (electron) mass, Z is the ion charge state, μ' is the mass ratio m_i/m_e, $\nu = n_{h_0}/n_{c_0}$ (here n_{h_0/c_0} is the hot/cold unperturbed electron density) and $\mu = T_h/T_c$ (where $T_{h/c}$ is the hot/cold electron temperature).

Derivation of pseudopotential

Anticipating a solitary travelling-wave solution, we assume all the dependent variables to depend on a single independent variable $\xi = x - Mt$, where M is the Mach number (the velocity of the solitary wave v normalized to ion acoustic speed $C_{s,eff}$). Under appropriate boundary conditions for localized waves: $\phi \to 0, n \to 1, u \to 0, p \to 1, n_b \to \beta$ and $u_b \to u_{b0}$at $\xi \to \pm\infty$, upon integrating Eqs. (1)-(3) we obtain

$$n = \frac{\sigma_1}{\sqrt{2}\sigma_0}[1 - \frac{2\phi}{M^2\sigma_1^2} - \sqrt{(1 - \frac{2\phi}{M^2\sigma_1^2})^2 - \frac{4\sigma_0^2}{\sigma_1^4}}]^{1/2}, \tag{7}$$

where $\sigma_0 = \sqrt{3\sigma/M^2}$ and $\sigma_1 = \sqrt{1+\sigma_0^2}$. Integrating Eqs. (4) and (5), we get

$$n_b = \beta/(\sqrt{1 + 2\mu'\phi/(u_{b_0} - M)^2}). \tag{8}$$

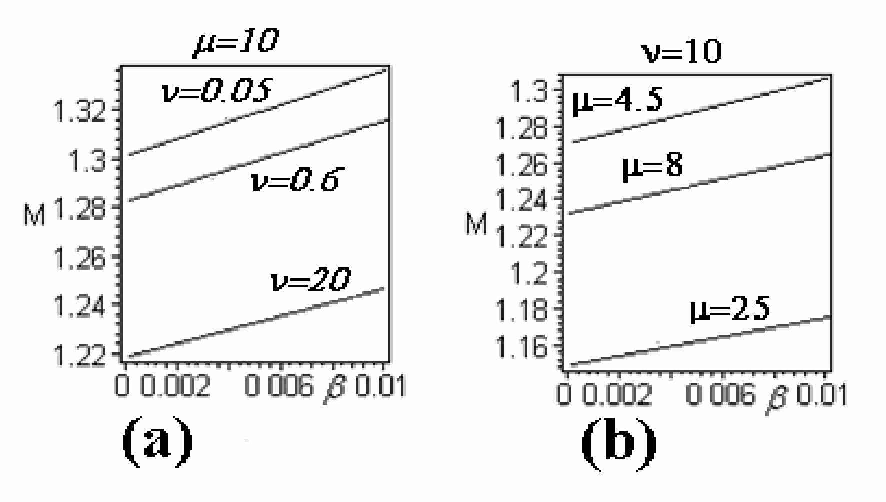

FIGURE 2. The maximum Mach number M as a function of electron beam density β $\sigma = 0.1$, $u_{b_0} = 1.1$ and $\mu' = 1836$.

The densities n and n_b are real if one of the following conditions is satisfied

$$-\frac{(u_{b_0}-M)^2}{2\mu'} \leq \phi \leq \frac{M^2\sigma_1^2}{2}\left(1-\frac{2\sigma_0}{\sigma_1^2}\right), \tag{9}$$

or if

$$\phi \geq \max\left\{-\frac{(u_{b_0}-M)^2}{2\mu'}, \frac{M^2\sigma_1^2}{2}\left(1+\frac{2\sigma_0}{\sigma_1^2}\right)\right\}. \tag{10}$$

The region of the existence of ϕ is determined by these conditions. Now, integration of Poisson equation , Eq (6), gives rise to

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{11}$$

where$V(\phi)$ is the pseudopotential and given by

$$\begin{aligned} V(\phi) = &-[(1-\tfrac{\beta}{Z})\phi + (1-\tfrac{\beta}{Z})(\tfrac{\phi^2}{2} + \alpha\tfrac{\phi^3}{3} + \alpha'\tfrac{\phi^4}{4}) + \tfrac{\beta(u_{b_0}-M)^2}{Z\mu'}(\sqrt{1+\tfrac{2\mu'\phi}{(u_{b_0}-M)^2}} - 1)] \\ &-M^2\sqrt{\sigma_0}[(e^{\theta/2} - e^{\theta_0/2}) + \tfrac{1}{3}(e^{-3\theta/2} - e^{-3\theta_0/2})] \end{aligned} \tag{12}$$

where $\theta = \cosh^{-1}[\frac{\sigma_1^2}{2\sigma_0}(1-\frac{2\phi}{M^2\sigma_1^2})]$ and $\theta_0 = \theta(\phi = 0)$ is the integration constant which $V(\phi) = 0$ at $\phi = 0$. The solitary solution for nonlinear ion acoustic waves exists when the following two conditions are satisfied:

i) The pseudopotential $V(\phi)$ has a maximum at $\phi = 0$, i.e. if $(d^2V(\phi)/d\phi^2|_{\phi=0} < 0$, so that the fixed point at the origin is unstable, and

ii) $V(\phi) < 0$ for $0 < \phi < \phi_{\max}$, for positive solitary waves, or for $0 > \phi > \phi_{\min}$, for negative solitary waves, where $\phi_{\max(\min)}$ is the positive (negative) value of ϕ for which $V(\phi) = 0$; these values are here determined by

$$\phi_{max} = M^2\sigma_1^2(1-2\sigma_0/\sigma_1^2)/2, \quad \phi_{\min} = -\frac{(u_{b_0}-M)^2}{2\mu'} \tag{13}$$

It must be noticed that the first critical value does not depend on ν and μ, so the effects of physical parameters on the existence of large amplitude ion-acoustic solitary waves will bear its origin in the second condition.

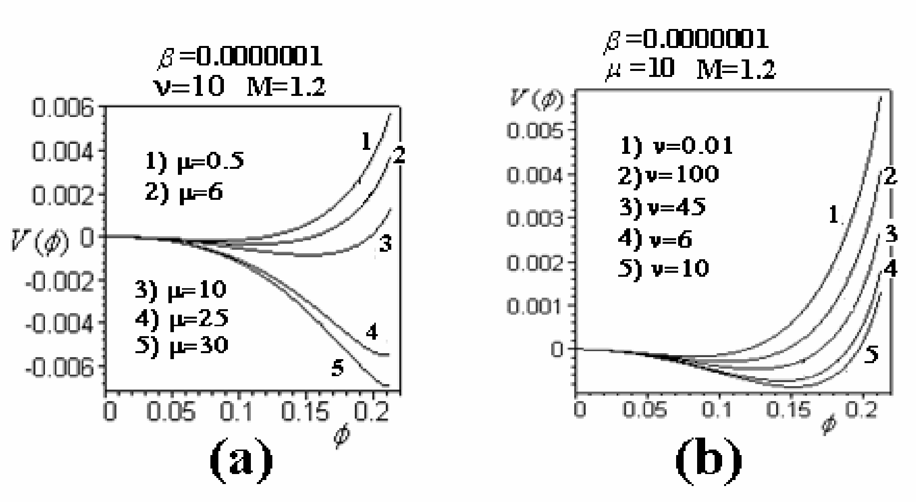

FIGURE 3. The pseudopotential $V(\phi)$ versus ϕ for $\beta = 0.0000001$, $M = 1.2$, $\sigma = 0.1$, $u_{b_0} = 1.1$ and $\acute{\mu} = 1836$.

THE REGION OF THE EXISTENCE OF LARGE AMPLITUDE ION-ACOUSTIC WAVES

We depict the zero-value contour plots for $V(\phi_{max})$ and $d^2V(\phi)/d\phi^2$ at $\phi = 0$ against the Mach number M and electron beam to background ion density ratio β in Figs. 1a and 1b respectively, in the case $\nu = 10$, $\mu = 10$, $\sigma = 0.1$, $u_{b_0} = 1.1$ and $\mu' = 1836$. In Fig. 1a, the area in black/white represents the regions in the $(M-\beta)$ plane where $V(\phi_{max})$ is positive/negative. We remark that there is a maximum and minimum limit for M. On the other hand, the area in black/white in Fig. 1b represents the regions in the $(M-\beta)$ plane where $V(\phi_{max}) < 0/ > 0$. As it is mentioned in previous section, fig. 1a does not modified as μ and ν change. Careful inspection Fig. 1a and 1b shows that large amplitude ion-acoustic solitary waves occur when β is very very small, for instance $\beta = 0.0000001$, and $|M| > 1$ namely supersonic case.

We show the maximum Mach number M as a function of β in figure 2a(2b) in the case: $\mu = 10$; $\nu = 0.05$, $\nu = 0.6$, and $\nu = 20$ ($\nu = 10$; $\mu = 4.5$, $\mu = 8$, and $\mu = 25$). It is remarked that the maximum Mach number increases (decreases) as $\beta(\nu)$ increases for fixed μ. Also the maximum Mach number M increases (decreases) as $\beta(\mu)$ increases for fixed ν. We illustrate dependence of $V(\phi)$ on the electrostatic potential ϕ when $\beta = 0.0000001$, $M = 1.2$, σ=0.1 for two case: $\nu = 10$ but different values μ, and μ=10 but different values ν in Figs. 3a-b. It is seen the amplitude of compressive solitons increases as μ rises to a given limit, after which the compressive solitons do not occur (Fig .3a). On the other hand, enhancing ν up to a given limit leads to an increase in the amplitude of compressive solitons. However, if ν rises to more than this limit, amplitude of solitons will decrease (Fig.3b), since very low and very high valuesν have the same physical meaning.

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