Nonlinear electromagnetic waves in pair plasmas

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Nonlinear electromagnetic waves in magnetized plasmas that are complex, with a number of ion species, or charged dust grains, or pair plasmas, are of increasing interest for laboratory, space and astrophysical applications. Imbalanced electron and positron number densities occur (1) in the plasma of a rotating pulsar magnetosphere, or (2) due to background additional ions or dust grains. In each case the background charge may be considered to be due to effectively infinitely massive particles. Imbalance of the electron and positron number densities leads to circularly polarized, rather than linearly polarized, modes propagating along the magnetic field. Here we analyse the unique features of nonlinear waves in the example of pair plasmas, and find general conditions for modulational instabilities, firstly for propagation parallel to the magnetic field, and then for perpendicular propagation of ordinary (O) modes.

1. Introduction

Pair-plasmas, i.e., plasmas consisting of negatively and positively charged particles bearing the same mass and (absolute) charge, have been gathering increasing interest among plasma researchers in the last years. Magnetized electron-positron plasmas exist in pulsar magnetospheres [1], in bipolar outflows in active galactic nuclei, at the center of the Galaxy, in the early universe, and in inertial confinement fusion schemes using ultraintense lasers [2]. Nonrelativistic pair plasmas have been created in experiments [3]. There is the possibility of pair production in large tokamaks due to collisions between multi-MeV runaway electrons and thermal particles [4]. Plasmas composed of two populations of fully ionized particles with the same mass and absolute charges of opposite charge polarity, have recently been created in the laboratory [5] by creating a large ensemble of fullerene ions, in equal numbers, thus allowing for a study of pair plasma properties with no concern for mutual annihilation which limits electron-positron plasma lifetime.

The physics of pair plasmas is substantially different from that of electron-ion plasmas, since the large time and space scale separation among constituents due to the large ion-to-electron mass ratio, in an electron-ion plasma, is absent in a pair plasma. In magnetized pair plasma, besides the electrostatic upper-hybrid waves, we have the perpendicularly propagating ordinary and extraordinary modes as well as electromagnetic waves propagating parallel to the magnetic field, featuring a linear polarization. However, when the number densities of the equal mass, oppositely charged particles are unequal, due to a background charge such as due to very heavy particles, the natural small amplitude modes are circularly polarized. Imbalanced electron and positron number densities can occur either due to the Goldreich-Julian charge in the plasma of a rotating pulsar magnetosphere [1], or due to background additional (relatively massive) ions or dust grains [6]. In each case the background charge may be considered to be due to effectively infinitely massive particles. Here perturbation theory is employed for a fluid model of such a plasma and shown to lead to a Nonlinear Schrödinger-type equation for the wave amplitude. The linear stability of the wave envelope and the occurrence of envelope solitons are discussed.

2. The Model and Solution

A non-relativistic two-fluid plasma model is used, with the momentum equations for each of the two oppositely charged fluids with equal mass particles, but allowing for unequal number densities:

$$\frac{\partial \mathbf{u}_j}{\partial t} + \mathbf{u}_j \cdot \nabla \mathbf{u}_j = \frac{q_j}{m_j}\left(\mathbf{E} + \mathbf{u}_j \times \mathbf{B}\right), \tag{1}$$

The total equilibrium charge is ensured to be zero by including a background of neutralising infinitely massive particles. Maxwell's equations complete the set. Full details of the model equations are found in Ref [7].

A reductive perturbation technique is used, with stretched time and space variables. The first order solution is the well known linear mode dispersion relation. Ensuring that secular terms in the second order solution vanish produces a compatibility condition, taking the form of the nonlinear Schrödinger-type equation (NLSE) for the magnetic field component perpendicular to the background field:

$$i\frac{\partial B_y'}{\partial \tau} + P\frac{\partial^2 B_y'}{\partial \xi^2} + Q|B_y'|^2 B_y' = 0. \tag{2}$$

Here $\tau=\varepsilon^2 t$ is the slow time scale, and $\xi=\varepsilon(x-v_g t)$ is the moving envelope space coordinate, with v_g the linear wave group velocity. The dispersion coefficient P is related to the curvature of the dispersion relation $\omega(k)$:

$$P = \frac{1}{2}\omega''(k) \tag{3}$$

which becomes for perpendicular propagation

$$P = \frac{c^2\omega_{p,\mathrm{eff}}^2}{2\omega^3} \tag{4}$$

The nonlinearity coefficient Q is a complicated function of the plasma parameters (see [7] for details), but simplifies for the balanced case for perpendicular propagation:

$$Q = \frac{3\Omega^2\omega_p^2}{2\omega(\Omega^2 - 3\omega^2)}. \tag{5}$$

Here Ω is the electron cyclotron frequency, $\omega_{p,eff}$ is the plasma frequency based on the sum of the electron (n_-) and positron (n_+) number densities, and ω_p is the electron or positron plasma frequency for the balanced case.

The NLSE supports plane wave solutions, and it is well known that the solutions are stable to perturbations if $PQ<0$. If $PQ>0$ the solution is unstable for perturbation wavelengths greater than a critical value proportional to $\sqrt{(Q/P)}$, the *modulational instability*. If the carrier wave is modulationally unstable, it can still lead to *bright* soliton solutions. If the wave is modulationally stable, it can give rise to *dark* soliton solutions, i.e. a propagating localized hole in a uniform wave energy region.

3. Results

3.1. Parallel propagating waves

The dispersion relation for small amplitude waves of frequency ω and wave number k is

$$(\omega^2 - \Omega^2)(\omega^2 - c^2k_z^2) - \omega^2\omega_{p,\mathrm{eff}}^2 \pm \eta\,\omega_{p,\mathrm{eff}}^2\Omega\omega = 0. \tag{6}$$

The parameter $\eta = (n_+-n_-)/(n_++n_-)$ measures the imbalance of the two species. For $\eta\neq 0$ there are two oppositely circularly polarized modes, separated by a stop-band.

Plots of the coefficients P and Q (normalized) against carrier wave frequency, normalized to the electron cyclotron frequency, are shown in Figure 1. The case of balanced electron and positron number densities ($\eta=0$) is shown, when the wave is linearly polarized. In addition, the cases of unbalanced electron and positron number densities are shown ($\eta=0.5$), when the waves are circularly polarized. The shaded regions indicate the stop-band range of frequency, with lower and upper cutoffs. For $\eta=0$ a modulational instability occurs for $f<0.95$. For $\eta>0$, at low frequencies both the LHP and RHP waves become stable.

3.2 Perpendicular propagating waves

The small amplitude O-mode has a dispersion relation

$$\omega^2 = \omega^2_{p,eff} + c^2k^2 \tag{7}$$

Thus the dispersion relation is relatively simple, giving a single mode but the nonlinear behaviour shows some complexities. In this case the coefficient P is always positive, from equation (4). Some results are shown in Figure 2, where Q/P is plotted against normalized wavenumber for two fixed values of Ω/ω_p. Balanced and unbalanced electron and positron numbers are considered: it is found that an imbalance tends to destabilize the wave envelope, and also affects the envelope width and form.

4. Conclusions

We have considered the propagation of nonlinear amplitude-modulated EM wave packets in a pair plasma, allowing for an imbalance of electron and positron numbers (assuming the background balancing charge is due to immobile particles). The carrier wave was assumed to be (a) a parallel propagating wave, or (b) a perpendicularly propagating ordinary-mode wave. The waves can be modulationally unstable, depending on the parameter range, and can lead to bright or dark-type solitons.

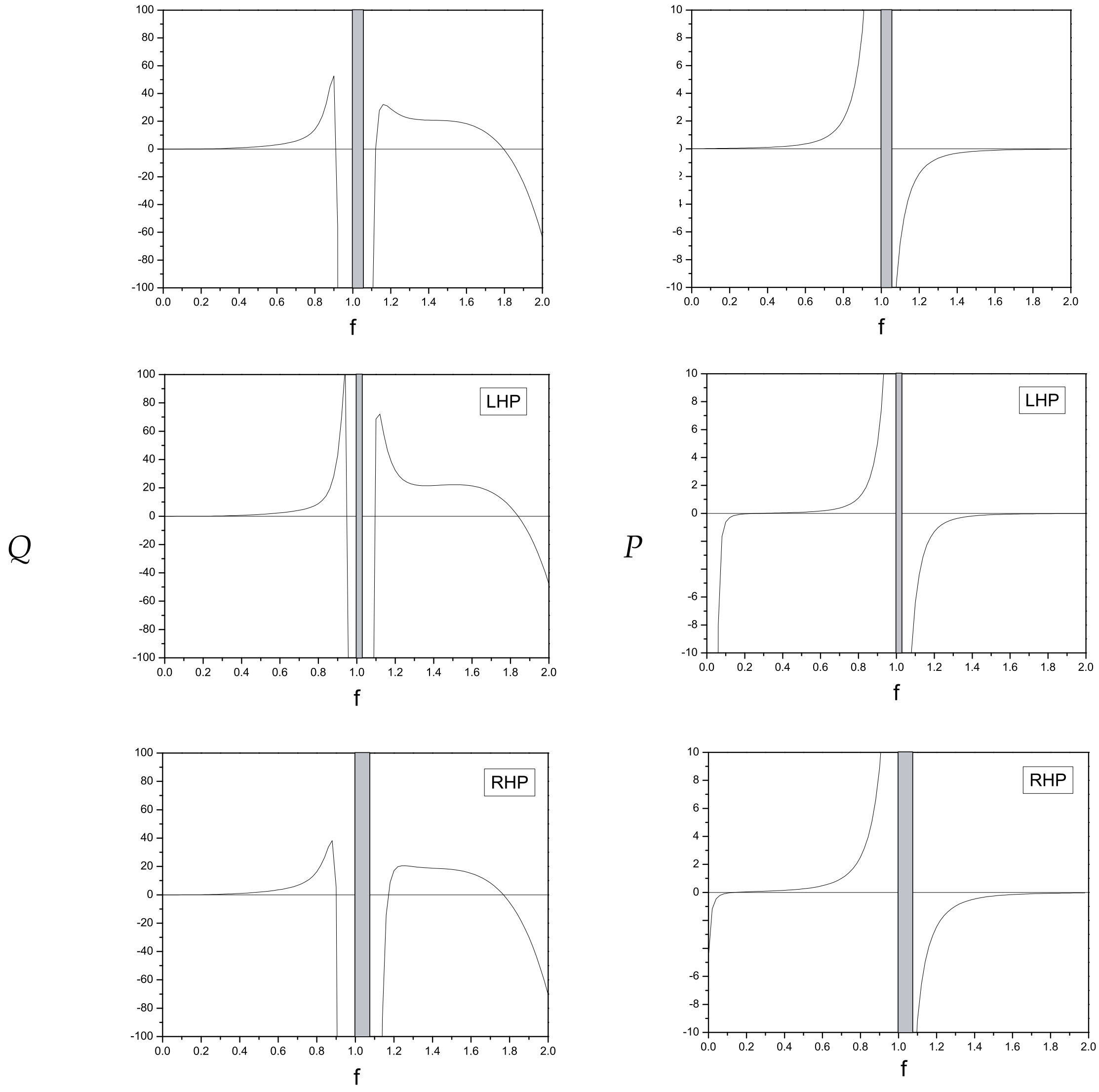

Figure 1: The nonlinear (Q) and dispersive (P) coefficients of the NLSE equation for waves propagating parallel to the magnetic field, as a function of normalized frequency, for balanced electron and positron numbers (η=0) (linearly polarized waves) (top two), and unbalanced numbers (η=0.5), for left hand circularly polarized (LHP) (middle two) and right hand circularly polarized (RHP) waves (bottom two).

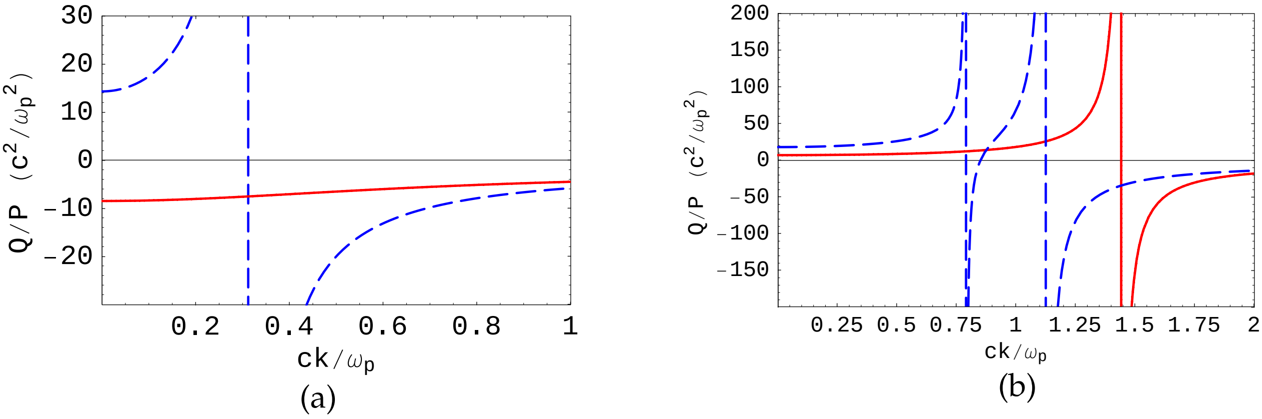

Figure 2: The coefficient ratio *Q/P* versus wavenumber for perpendicular propagation. (a) Ω/ω_p =2.1, for balanced electron and positron numbers (η=0) (solid curve) and unbalanced numbers (η=1/3) (dashed curves). (b) Ω/ω_p =3.5, for η=0 (solid curve) and η=-1/3 (dashed curves).

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6. References

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