Localized excitations in dusty plasma crystals: on the interface among plasma physics and nonlinear lattice theories

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Introduction. Dusty plasma crystals (DPCs) are strongly-coupled charged particle configurations, which occur in dysty plasmas (DP) [1] when the average electrostatic potential energy substantially exceeds the mean kinetic energy. In laboratory, DPCs are formed in low-temperature plasma discharges, wherein the charged dust particles are suspended under the combined action of gravity and electric forces [2]. DPC configurations typically consist of two-dimensional (2D) – hexagonal in general – layers, but also one- (1D) chains, when appropriate trapping potentials are employed for lateral confinement [3]. Our aim here is to revisit the nonlinear aspects of dust grain motion in 1D and 2D DPCs, from first principles, by reviewing earlier analytical results [4] and presenting more recent ones [5, 6].

Origin of nonlinearity. Plasma discharges provide a nonlinear environment, *par excellence*. Let us briefly review what nonlinearity in DPCs originates from.

Coupling nonlinearity. Electrostatic inter-grain interactions are generally associated with a Debye-type interaction potential. Assuming an infinite chain of oscillators (confined at the boundaries by an appropriate trapping potential to ensure static stability), the interaction force acting on the $n-$th grain is $F_n = -\nabla U_{int}(|\mathbf{r}_n - \mathbf{r}_{n-1}|)$. For small displacements, the interaction potential U_{int} can be expanded near the equilibrium grain position $\{x_n, y_n, z_n\} = \{nr_0, mr_0, 0\}$ ($n, m = 0, \pm 1, \pm 2, ...$; assuming gravity along $\hat{z}$), thus yielding a polynomial in the (small) displacements δx_n, δy_n and δz_n [4, 7, 8].

Sheath "substrate" potential nonlinearity. The plasma sheath environment provides an on-site substrate potential which is intrinsically nonlinear, and may (for low density and pressure) be strongly *anharmonic* (see Figure 1):

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2}M\omega_g^2(\delta z_n)^2 + \frac{1}{3}M\alpha\,(\delta z_n)^3 + \frac{1}{4}M\beta\,(\delta z_n)^4 + \mathscr{O}[(\delta z_n)]^5\,. \quad (1)$$

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The anharmonicity coefficients α and β may be obtained from experiments [9, 10, 11] or from *ab initio* calculations.

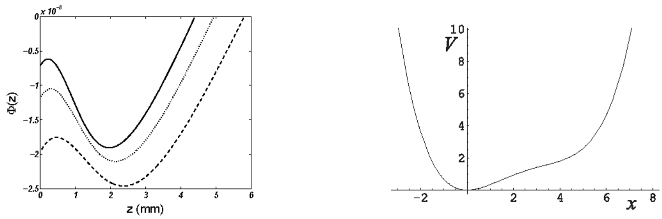

FIGURE 1. The (anharmonic) sheath potential $\Phi(z)$ is depicted vs. the vertical distance z from the negative electrode: (left) as results from *ab initio* numerical simulations (increasing from bottom to top) (data courtesy of G. Sorasio); (Right) Based on the experimental data in [10].

Geometric effects. Dust-grain motion combines 2 or 3 degrees of freedom, and thus introduces a nonlinear transverse-to-longitudinal mode-coupling effect [8].

Solitary waves. The intrinsic crystal characteristics provide the necessary ingredients for the formation of localized excitations, sustained via a mutual balance among nonlinearity and dispersion. The nonlinear horizontal (longitudinal, acoustic) as well as vertical (transverse, inverse-optic) dust grain motion in a 1D dust monolayer has been studied thoroughly [4, 7], so results need only be summarized here.

Longitudinal solitons. Dust crystals are known to support supersonic longitudinal solitary excitations (density solitons), related to longitudinal (in-plane) dust grain displacement. In theory, these structures are associated with Korteweg - de Vries and/or Boussinesq Equation soliton solutions [7, 12]. Experimentally they are observed as localized density perturbations, which sustain a stationary shape [13] and survive collisions [14], as predicted by theory [4, 7]. Importantly, although only compressional pulses have been sought for (and observed) experimentally [13], *rarefactive* solitons are also predicted by the theory; see separate presentation [7].

Off-plane (transverse) envelope solitons. Modulated envelope wavepackets associated with *backward*-propagating (negative group velocity) transverse (off-plane) oscillations are predicted by the nonlinear Schrödinger (NLS) theory in a quasi-continuum lattice approximation [15]. Such wavepackets are also observed in experiments [16].

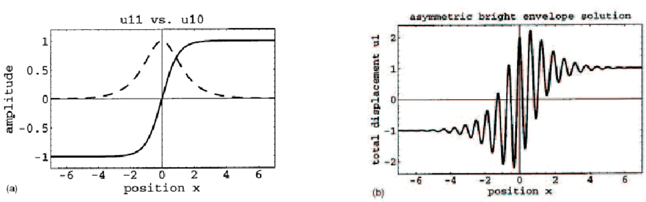

FIGURE 2. Bright (asymmetric) longitudinal envelope solitons: (a) the zeroth-harmonic (pulse) and first harmonic (kink) amplitudes; (b) the resulting asymmetric wavepacket.

In-plane modulated envelope wavepackets. *Asymmetric* localized envelope solitons, involving a non-zero zeroth harmonic contribution, occur in the longitudinal (in-plane, acoustic mode) degree of freedom [17].

In 2D, hexagonal dust lattices sustain modulated envelope bell-shaped structures, which may be formed as a result of modulational instability of in-plane vibrations [18].

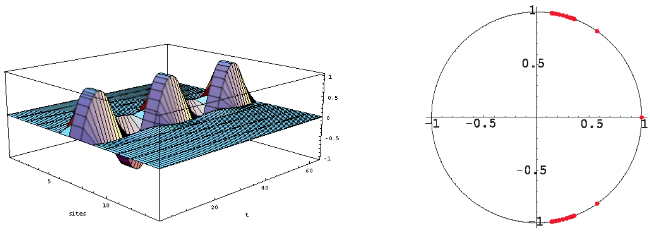

FIGURE 3. (From left to right) (a) Time evolution of a stable (1D, 1:1 here) 2-breather for $\varepsilon = -0.016$; (b) The corresponding linear stability profile: all eigenvalues lie on the imaginary circle.

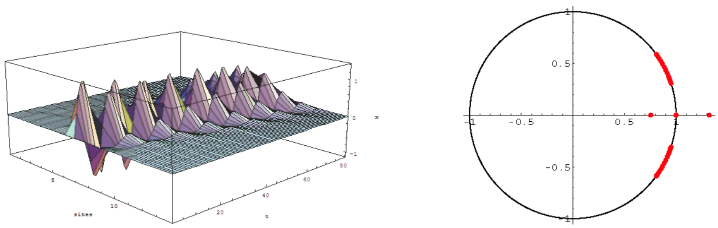

FIGURE 4. (From left to right) (a) Time evolution of an unstable 1D breather: see that the excitation is not localized anymore, after some time; (b) The corresponding linear stability profile: two eigenvalues have departed from the imaginary circle.

Discrete dust-lattice modes. *Intrinsic localized modes* (*Discrete Breathers*) form a topic of frontier research in present-time nonlinear science. They consist of highly localized (only few sites moving) periodic oscillatory lattice eigenmodes, which owe their stability to the crystal discreteness, in combination with nonlinearity [19].

Discrete Breathers (DBs) may occur, related with transverse dust lattice vibrations, as can be shown from first principles, both in 1D [20, 21] and in 2D [22] crystals. A discrete analysis of hexagonal crystals [6] from first principles suggests, apart from 1D discrete modes, the occurrence of ultra-localized multipole modes (discrete vortices; see Fig. 5) [22]. These may also be modelled as 2D discrete DNLS soliton modes [23]. The stability profile of DBs depends on the discreteness parameter $\varepsilon = \omega_{T,0}^2/\omega_g^2$, which is the (square) ratio of the transverse mode eigenfrequency by the transverse gap frequency (related to the sheath potential well as $\sim \Phi''(0)$). In general, the smaller the value of ε, the "more dicrete" a lattice system is. Detailed results will be reported soon [22].

Discrete dust lattice excitations, so far essentially unexplored, open new directions for Debye crystal applications, once experimentally confirmed and eventually manipulated.

Conclusions. DPCs provide an excellent test-bed for continuum and discrete nonlinear theories. With the exception of compressional (only) density solitons [13] and trans-

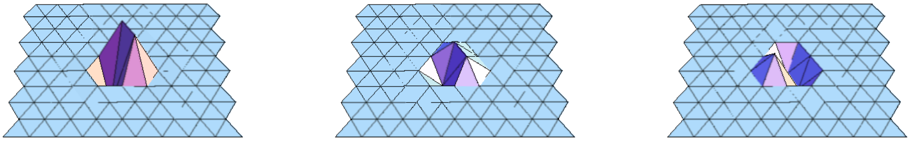

FIGURE 5. Time snapshots of a *multipole breather* (a *discrete vortex*): 3 sites oscillating at $\Delta\phi = 2\pi/3$; preliminary results [5]; further results, based on a discrete NLS theory [6, 23], confirm these findings.

verse wavepackets [16] (and not overseeing observations of dust vortices [24], though not in a crystalline phase), these theoretical predictions have still not rigorously been tested in the laboratory. This provides a challenging direction for experimental investigations, which will hopefully confirm these results.

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