

# On the existence of rarefactive longitudinal solitons in dusty plasma lattices

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The existence and stability of solitary excitations associated with longitudinal dust grain motion in a dusty plasma crystal [1, 2] is considered in this investigation. The theoretical modelling of this problem has originally relied on a KORTEWEG – DE VRIES (*KdV*) type description [3], which predicted compressive solitary density excitations (*only*). Not surprisingly, such excitations were experimentally found to exist [4]. A more rigorous recent investigation of dust lattice dynamics, from first principles [5], has shown that the *KdV* picture, despite its analytical simplicity, appears to be rather incomplete: in specific, it neglects higher- (than cubic) order interaction nonlinearity [5]. Our aim here is to revisit the problem of nonlinear longitudinal dynamics in DP crystals and, in fact, to show that *rarefactive* solitons may also occur in dusty plasma crystals.

The *discrete* equation of longitudinal motion for the  $n$ -th dust-grain in a crystal reads:

$$\frac{d^2(\delta x_n)}{dt^2} + \nu \frac{d(\delta x_n)}{dt} = \omega_{0,L}^2 (\delta x_{n+1} + \delta x_{n-1} - 2\delta x_n) - a_{20} [(\delta x_{n+1} - \delta x_n)^2 - (\delta x_n - \delta x_{n-1})^2] + a_{30} [(\delta x_{n+1} - \delta x_n)^3 - (\delta x_n - \delta x_{n-1})^3], \quad (1)$$

where  $\delta x_n = x_n - nr_0$  is the longitudinal dust grain displacement; we have defined the longitudinal “sound” velocity  $c_L = \omega_{0,L}r_0$ ;  $M$  is the grain mass;  $r_0$  is the lattice spacing. We shall henceforth neglect the damping coefficient  $\nu$ . Eq. (1) essentially defines a *Fermi-Pasta-Ulam (FPU)* type dynamical system, long-known from anharmonic spring-chain models [6]. The coefficients  $\omega_{0,L}^2$ ,  $a_{20}$  and  $a_{30}$  are prescribed by the electrostatic inter-grain interaction law [2]; for Debye-type interactions, these are positive quantities.

A standard procedure in lattice dynamics is the *quasi-continuum approximation*; anticipating excitations whose length, say  $L$ , is much larger than  $r_0$ , this hypothesis consists in expanding  $\delta x_n \ll r_0$  near zero; (1) is thus reduced to a PDE in the form

$$\frac{d^2u}{dt^2} - c_L^2 u_{xx} - \frac{c_L^2}{12} r_0^2 u_{xxxx} = -2a_{20}r_0^3 u_x u_{xx} + 3a_{30}r_0^4 (u_x)^2 u_{xx}, \quad (2)$$

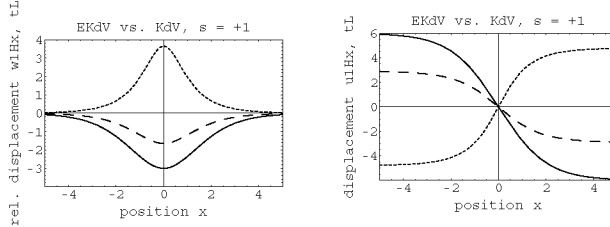
where subscripts denote differentiation. Although it would be straightforward to work out a BOUSSINESQ-type theory from (2) (which smoothly leads to the anticipated solitary wave excitations in an extended velocity range, as shown in [5] in full rigor), we shall follow Melandso [3] and seek for quasi-stationary near-sonic localized forms, by setting  $u = u(x - vt, t) \equiv u(\zeta, \tau)$ , where  $v \simeq c_L$ . This leads to an EXTENDED KORTEWEG

- DE VRIES (EKdV) equation in the form

$$w_\tau - saww_\zeta + \hat{a}w^2w_\zeta + bw_\zeta\zeta\zeta = 0, \quad (3)$$

for  $w = du/d\zeta$ . Here,  $a = |p_0|/(2c_L) > 0$ ,  $\hat{a} = q_0/(2c_L) > 0$  and  $b = c_L r_0^2/24 > 0$ , while  $s$  is the sign of  $p_0$ , i.e.  $s = p_0/|p_0| = \pm 1$ . The characteristic quantities  $p_0 = -r_0^3 U'''(r_0)/M \equiv 2a_{20}r_0^2$  and  $q_0 = U''''(r_0)r_0^4/(2M) \equiv 3a_{30}r_0^4$  are related to cubic and quartic potential nonlinearity; for Debye-type interactions, both are positive quantities of similar order of magnitude, while  $s = +1$  (see in [5] for details).

Adopting the setting above, we may revisit the original nonlinear theory furnished by Melandso [3], which consisted in considering the cubic interaction nonlinearity (yet neglecting the quartic-order one) in (1) and, subsequently, in (2). Melandso's recipe essentially amounts to setting  $a_{30} = 0$  in (1) and (2), and thus  $\hat{a} = 0$  in (3). This leads exactly to a KdV equation, bearing supersonic pulse soliton solutions in the form  $w(\zeta, \tau) = -s w_m \text{sech}^2[(\zeta - M\tau - \zeta_0)/L_0]$ , where  $\zeta_0$  and  $M$  are arbitrary real constants. Obviously, in our case of interest (and for Debye interactions, viz.  $s = +1$ ), the KdV description provides only *negative* pulses, i.e. the positive density excitations  $\delta n/n_0 \sim -u_x$  which were detected in [4].



**FIGURE 1.** (a) The two localized pulse solutions of the EKdV Eq. (3) for the relative displacement  $w(x,t) \sim \partial u(x,t)/\partial x$  are depicted for arbitrary (positive) values of the  $p_0$  and  $q_0$  coefficients (i.e.  $s = +1$ ): the first, dashed (second, short-dashed) curve, represents the smaller negative (larger positive) pulses. The larger negative pulse (solid curve) denotes the solution of the KdV equation for the same parameter set. (b) The corresponding solutions for the particle displacement  $u(x,t)$ .

Advancing to a fourth-order theory, i.e. retaining  $\hat{a} \neq 0$  in (3), it is straightforward to show that *both* rarefactive and compressive excitations [5] may occur in a dust crystal. An experimental investigation would hopefully confirm these challenging results.

The description of DP crystals is, in fact, significantly modified if one takes into account interaction polarization ("dressing") effects [7]. A change in sign is thus possible, which leads to a structural change in the excitations supported by the crystal.

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