# Ion-acoustic solitary waves in multi-ion dusty plasmas

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*Introduction.* Many authors often model dust ion acoustic (DIA) waves [1] via the Korteweg-de Vries (KdV) equation. The validity of this description is restricted to some range of amplitudes (which does not always exist) [2]. Based on the Sagdeev potential technique [3] for arbitrary amplitude solitary waves, new features of the existence domain of the produced waves are found [4]. Sayed *et al.* [5] have studied DIA wave in a cold plasma system containing negative and positive ions, yet neglecting thermal effects, which may change drastically the existence domain of solitary waves [4].

*Model equations and analysis.* We consider a collisionless unmagnetized dusty plasma, consisting of electrons, two ion species of opposite charge sign $q_j = \gamma_j Z_j e$ (here $j = 1$ or 2; $\gamma_1 = -\gamma_2 = +1$) and dust particles $q_d = \alpha Z_d e$, where $\alpha = -1(+1)$ refers to negative (positive) dust. Employing the pseudo-potential technique, the model is reduced to the dynamics of a point mass moving in a potential $V(\phi) = V_0(\phi) - V_0(0)$, where

$$V_0(\phi) = \alpha Z_d n_d \phi - \exp(s\phi) - \sum_{j=1}^{2} \frac{Z_j \left[b_j - (b_j^2 - a_j)^{1/2}\right]^{1/2} \left[2b_j + (b_j^2 - a_j)^{1/2}\right]}{3\psi_j \sqrt{6\sigma_j}}. \quad (1)$$

We define the quantities $a_j = 12\sigma_j n_{j0}^2 M^2$, $b_j = M^2 - 2\gamma_j \psi_j \phi + 3\sigma_j n_{j0}^2$, $\sigma_j = \beta_j/\mu_j$, $\beta_j = T_j/T_e$ and $\mu_j = m_j/m_1$, $\psi_j = Z_j/\mu_j$ and $s = 1/Z_1$. Here, we restrict ourselves to the DIA wave with velocity $M \geq \max\{n_{10}\sqrt{3\sigma_1}, n_{20}\sqrt{3\sigma_2}\} \equiv M_c$, where $n_{j0}$ is the equilibrium number density of the $j-$th ion fluid. To ensure a real number density of ions, the following condition should be verified: $(|M| - n_{j0}\sqrt{3\sigma_j})^2 - 2\gamma_j \psi_j \phi \geq 0$. (cf. Fig 1). Moreover, to get DIA solitary wave solutions, three restrictions have to be

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imposed on (1), yielding

$$f(M) = \frac{n_{10}}{M^2 - 3\sigma_1 n_{10}^2} + \frac{Z_2 \psi_2 n_{20}}{M^2 - 3\sigma_2 n_{20}^2} < s^2 . \tag{2}$$

(which is solved graphically as shown in Fig 2a).

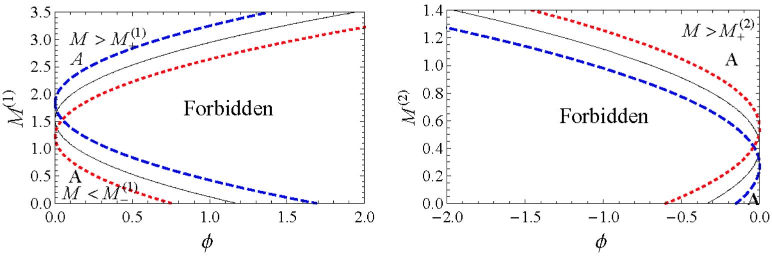


**FIGURE 1.** The allowed (A) and forbidden regimes of $M$ against $\phi$. The solid curve is plotted without dust grains and $n_{10} = 2.5$, $n_{20} = 1.5$, $Z_1 = Z_2 = 1$, $\beta_1 = 0.125$, $\beta_2 = 0.1$ and $\mu_2 = 4$. The dashed blue (dotted red) curve corresponds to including negative (positive) dust grains, respectively, with $Z_d n_{d0} = 0.5$.

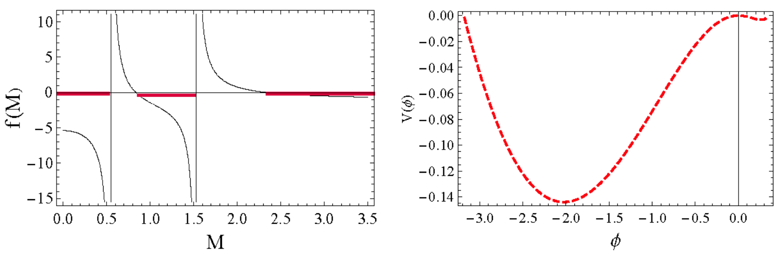


**FIGURE 2.** (a) The solutions of (2), with negative dust. (b) The asymmetric Sagdeev potential is plotted for positive dust, with $M = 2.06$. Other parameter values as in the solid curve in Fig. 1.

*Conclusions.* Including dust temperature effects, we have extended the study by Sayed *el al.* [5]. Different regimes are found (cf. Fig 2a). The Sagdeev potential function is asymmetric, and predicts either positive or negative potential pulses ($\phi > 0$ or $\phi < 0$). The latter possess a higher amplitude and a wider width than those of the positive ones.

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