Nonlinear modelling of a rotating multi-component dusty plasma

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The Coriolis force may have a dominant role in rotating laboratory plasmas, as well as in space plasma environments, where dusty plasmas (DP) abide [1, 2, 3]. In this work, we investigate the propagation of nonlinear electrostatic excitations in a four-component DP, consisting of two dust species of opposite polarity (d^+ and d^-, of mass m and charge $\pm Z_d e$, respectively), electrons (mass m_e, charge e) and ions (mass m_i, charge $+Ze$).

We consider a rotating DP embedded in an external magnetic field $\mathbf{B} = B_0\hat{x}$. A two-fluid model is employed, for the two dust species (distinguished by indices $+$ and $-$). Both Lorentz and Coriolis forces are considered, associated with gyroscopic (Larmor) and mechanical plasma rotation, respectively via the dust Larmor $\omega_c\hat{x} = \pm\frac{Z_d e B_0}{mc}\hat{x}$ and rotation $\Omega_0\hat{x}$ frequencies. The density $n_\pm$, velocity $\mathbf{u}_\pm$ and potential ϕ perturbations vanish at equilibrium. The system of fluid equations

$$\frac{\partial n_{-,+}}{\partial t} + \nabla.(n_{-,+}u_{-,+}) = 0, \tag{1}$$

$$m\left(\frac{\partial}{\partial t} + \mathbf{u}_- \cdot \nabla\right)\mathbf{u}_- = Z_d e\nabla\phi - \frac{1}{n_-}\nabla p_- - \frac{Z_d e}{c}(\mathbf{u}_- \times B_0\hat{x}) + 2m(\mathbf{u}_- \times \Omega_0\hat{x}), \tag{2}$$

$$m\left(\frac{\partial}{\partial t} + \mathbf{u}_+ \cdot \nabla\right)\mathbf{u}_+ = -Z_d e\nabla\phi - \frac{1}{n_+}\nabla p_+ + \frac{Z_d e}{c}(\mathbf{u}_+ \times B_0\hat{x}) + 2m(\mathbf{u}_+ \times \Omega_0\hat{x}). \tag{3}$$

is closed by the equation(s) of state $p_\alpha \sim n_\pm^{5/3}$, and by Poisson's equation

$$\nabla^2\phi = 4\pi e\left[Z_d(n_- - n_+) - Zn_i + n_e\right]. \tag{4}$$

We assume that $n_{+,0}/n_{-,0} \equiv \delta = 1-\gamma$, where $\delta = n_{+,0}/n_{-,0}$, $\gamma = (Z_i n_i - n_e)/n_{-,0} \equiv \hat{n}_0/n_{-,0}$), i.e. the *e*-*i* background maintains charge neutrality while remaining uniform.

Perturbation theory, introducing slow scales $X = \varepsilon^{1/2}(x-\lambda t)$, $Y = \varepsilon^{1/2}y$ and $\tau = \varepsilon^{3/2}$ ($\varepsilon \ll 1$) reduces the system (1)-(4) to a ZAKHAROV-KUZNETSOV (ZK) equation

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X} + B\frac{\partial^3\phi^{(1)}}{\partial X^3} + D\frac{\partial^3\phi^{(1)}}{\partial X\partial Y^2} = 0. \tag{5}$$

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where

$$A = B\left[\frac{3\delta\lambda^2}{(\lambda^2-2\delta\sigma_+)^3} - \frac{3\lambda^2}{(\lambda^2-2)^3}\right], \tag{6}$$

$$B = \left[\frac{2\lambda}{(\lambda^2-2)^2} + \frac{2\delta\lambda}{(\lambda^2-2\delta\sigma_+)^2}\right]^{-1}, \tag{7}$$

$$D = B\left[1 + \frac{1}{\Omega_-^2}\frac{\lambda^4}{(\lambda^2-2)^2} + \frac{\delta\lambda^4}{\Omega_+^2(\lambda^2-2\delta\sigma_+)^2}\right]. \tag{8}$$

We have defined the effective frequencies $\Omega_\pm = (2\Omega_0 \pm \omega_c)/\omega_{p,-}$ [where $\omega_{p,-} = (4\pi Z_d^2 e^2 n_0/m)^{1/2}$] and the positive-to-negative dust temperature ratio $\sigma_+ = T_+/T_-$, as well as the (reduced) soliton velocity λ. Taking $\chi = \ell X + mY - U\tau$, the solution for the potential $\phi \simeq \varepsilon^{3/2}\phi^{(1)} + \mathscr{O}(\varepsilon^2)$ reads

$$\phi^{(1)} = \phi_0 \mathrm{sech}^2\left(\frac{\chi}{W}\right), \tag{9}$$

where the amplitude $\phi_0 = 3U/A\ell$ and the width $W = \sqrt{4\ell(B\ell^2 + Dm^2)/U}$.

The behavior of these nonlinear solutions is under investigation. Our preliminary results suggest a significant dependence of the width W on the rotation frequency Ω_0 and on the dust cyclotron frequency ω_c. In particular, an interesting effect is witnessed when $\Omega_\pm \sim 2\Omega_0 \pm \omega_c \to 0$, where W diverges, leading to $\phi^{(1)} \to \phi_0$ everywhere; notice the white region in Fig. 1b. Extensions of this work currently underway include the

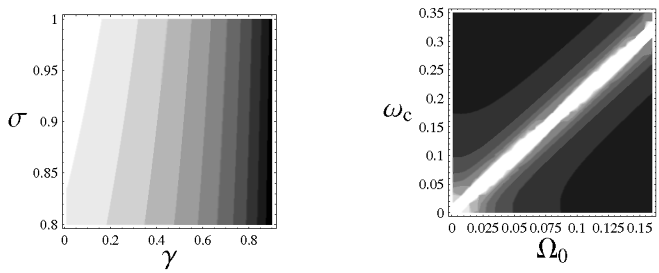

FIGURE 1. Contour plots of: (left frame) the phase velocity λ vs. σ and γ and (right frame) the soliton width W vs. Ω_0 and ω_c, for $\lambda = 0.6$, $\ell = 0.7$ and $\gamma = 0.4$. In both plots, higher values are shown as lighter colored areas.

derivation of an extended ZK equation for critical plasma compositions where $A \simeq 0$, as well as a prediction for the existence of electrostatic double layers in rotating plasmas.

Details on this investigation will be presented in an extensive report, to appear soon.

REFERENCES

1. F. Verheest, *Waves in Dusty Space Plasmas*, (Kluwer, Dordrecht, 2000).
2. P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics*, (Institute of Physics, Bristol, 2002).
3. G. C. Das and A. Nag, Phys. Plasmas **13**, 082303 (2006).
4. I. Kourakis, U. M. Abdelsalam, W. M. Moslem, P. K. Shukla, in preparation.