Effect of superthermality on nonlinear electrostatic modes in plasmas

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Abstract

The nonlinear propagation of electron-acoustic solitary structures is investigated in a plasma containing κ distributed (superthermal) electrons. Different types of localized structures are shown to exist. The occurence of modulational instability is investigated.

1. Introduction. Superthermal particles in laboratory, space and astrophysical plasmas are often modelled by a κ-type distribution function (df) [1-2]. The superthermality parameter κ measures the deviation from a Maxwellian distribution (the latter is recovered for infinite κ). Our twofold aim here is to investigate the effect of superthermality on electrostatic solitary waves, and also on self-modulated wavepackets.

EA waves (EAW) occur in plasmas containing two distinct temperature electron populations (here referred to as "cold" and "hot" electrons) [3-4]. These are high frequency electrostatic electron oscillations, where the restoring force comes from the hot electrons pressure and the cold electrons provide the inertia [3,5], while ions plainly provide a neutralizing background. The phase speed v_{ph} of the EAW is much larger than the thermal speeds of both cold electrons and ions but much smaller than the cold electrons (i.e., $v_{ph,c}, v_{ph,i} \ll v_{ph} \ll v_{ph,h}$). EAWs survive Landau damping in the region $T_h/T_c \geq 10$ and $0.25 \leq n_{c0}/n_{h0} \leq 4$ [3, 4], where we have defined the temperature (T_c, T_h) and density (n_c, n_h) of the electron constituents ('c' for cold, 'h' for hot).

2. Electron fluid model. We consider a three component plasma consisting of inertial ("cool") electrons, $\kappa-$distributed ("hot") electrons and stationary background ions. In a 1D geometry, the dynamics of the cold electrons is governed by the following normalized equations:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0\,, \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n}\frac{\partial P}{\partial x}\,,$$

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + 3P\frac{\partial u}{\partial x} = 0\,, \qquad \frac{\partial^2 \phi}{\partial x^2} = -(\beta+1) + n + \beta\left(1 - \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa+1/2}.$$

We have scaled all relevant physical quantities in (1) below as: cold electron density $n = n_c/n_{c0}$; fluid speed $u = u_c/v_0$; electric potential $\phi = \Phi/\Phi_0$; time $t = t\omega_{pc}$; space $x = x/\lambda_0$; pressure $P = P/n_{c0}k_BT_c$; we have defined: $v_0 \equiv (k_BT_h/m_e)^{1/2}$ $\lambda_0 = \left(k_BT_h/4\pi n_{c0}e^2\right)^{1/2}$ and $\omega_{pc}^{-1} = (4\pi n_{c0}e^2/m_e)^{-1/2}$, and also the density- and temperature- ratio(s): $\beta = n_{h,0}/n_{c,0}$ and $\sigma = T_c/T_h$.

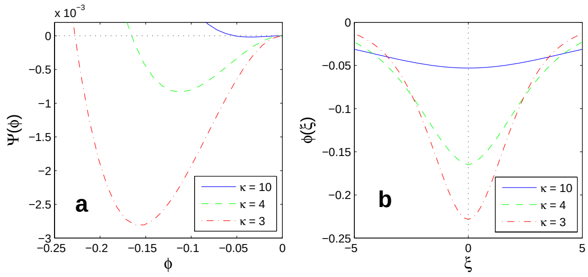

Figure 1: Variation of the pseudopotential $\Psi(\phi)$ with ϕ (left); the electric potential ϕ vs. ξ (right). We have considered various values of κ, and $\sigma = 0.01$, $\beta = 1$, and $M = 1$.

3. Arbitrary amplitude solitary excitations. Anticipating stationary profile localized excitations, we shift from variables $\{x,t\}$ to $\xi = x - Mt$, where M is the solitary wave speed, scaled by v_0 (defined above). We obtain $u = M\left(1 - \dfrac{1}{n}\right)$, $u = M - (M^2 + 2\phi - 3n^2\sigma + 3\sigma)^{1/2}$, and $P = n^3$. Eqs. (1) are thus combined into a pseudo-energy balance equation

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \Psi(\phi) = 0\,, \tag{1}$$

where the “Sagdeev" pseudopotential function $\Psi(\phi)$ reads

$$\Psi(\phi) = (1+\beta)\phi + \beta\left[1 - \left(1 + \frac{\phi}{-\kappa + \frac{3}{2}}\right)^{-\kappa+3/2}\right] + \frac{1}{6\sqrt{3\sigma}}\left[\left(M + \sqrt{3\sigma}\right)^3 - \left(M - \sqrt{3\sigma}\right)^3 - \left(2\phi + \left[M + \sqrt{3\sigma}\right]^2\right)^{3/2} + \left(2\phi + \left[M - \sqrt{3\sigma}\right]^2\right)^{3/2}\right] \tag{2}$$

Soliton existence region. In order for solitary waves to exist, we need to impose [5]: $\Psi'(\phi = 0) = 0$ and $\Psi''(\phi = 0) < 0$ (where the prime denotes differentiation wrt ϕ), leading to the (true sound speed) threshold $M_1 = \left[\frac{\kappa - \frac{3}{2}}{\beta(\kappa - \frac{1}{2})} + 3\sigma\right]^{1/2}$. An upper limit for M is obtained by imposing the reality requirement [6] $F_2(M) = \Psi(\phi)|_{\phi = \phi_{\max}} > 0$ (where $\phi_{\max}$ is the root of $\Psi(\phi)$; see Fig. 1a). The region thus obtained is depicted in Figure 2.

4. Modulated EA wavepackets. We consider small ($\varepsilon \ll 1$) deviations of the state variables, say S $(= n, u, \phi)$, from the equilibrium state, viz. $S = S^{(0)} + \Sigma_{n=1}^{\infty} \varepsilon^n \sum_{l=-n}^{n} S^{(nl)} e^{il(kx - \omega t)}$, and allow for a weak space-/time-dependence of the $l-$th harmonic amplitudes $S^{(nl)}$. In what follows, we ignore the pressure term (*cold electron model*) for simplicity, and set $\alpha = n_{c,0}/n_{h,0}$ $(= \beta^{-1})$.

The 1st order ($\sim \varepsilon^1$) expressions provide the EAW *dispersion relation* $\omega^2 = \frac{k^2\alpha}{k^2 + c_1}$, along with the amplitudes of the first harmonics. The 2nd and 0th harmonics are obtained at order ε^2. Annihilation of secular terms at 3rd order yields a nonlinear Schrödinger (NLS) type equation:

$$i\frac{\partial \psi}{\partial \tau} + P\frac{\partial^2 \psi}{\partial \zeta^2} + Q|\psi|^2\psi = 0\,, \tag{3}$$

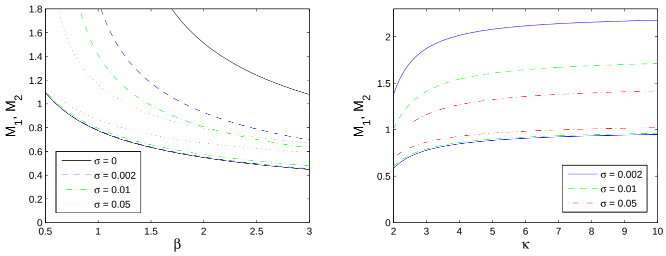

Figure 2: Soliton existence region ($M_1 < M < M_2$) for different temperature ratio σ values, versus β for $\kappa = 3$ (left panel); also, versus κ for $\beta = 1$ (right panel).

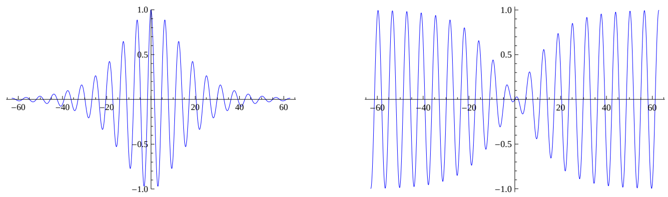

Figure 3: Envelope type solitary excitations: bright type (left panel) and dark type (right panel).

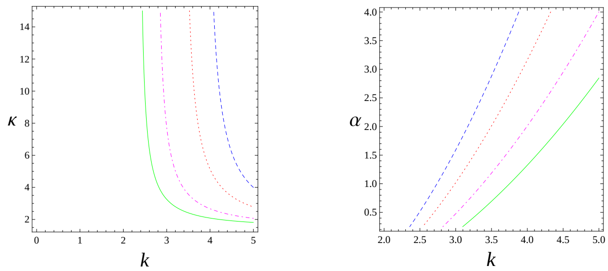

Figure 4: $PQ = 0$ (or $k = k_{cr}$) contours *vs* carrier wavenumber k and superthermality parameter κ, or density ratio α. Left panel: α=0.25 for the green curve; 1 for magenta; 2.5 for red; and 4 for blue. Right panel: κ=3 for green; 4 for magenta; 8 for red; and 100 for blue.

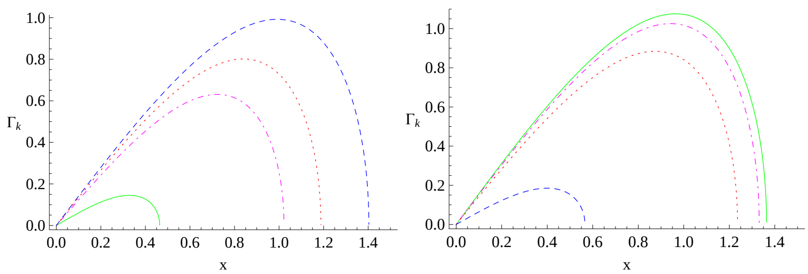

Figure 5: Modulational instability growth rate (normalised by its value for infinite κ) versus perturbation wavenumber. Left panel: $\kappa = 100$, 7, 5, 3.5 (top to bottom) for $\alpha = 0.5$, $k = 3.2$. Right panel: $\alpha = 0.5$, 1, 2, 4 (top to bottom) for $k = 4.5$, $\kappa = 7$.

where the amplitude $\psi \equiv \phi_1^{(1)}(\zeta, \tau)$ depends on $\zeta = \varepsilon(x - v_g t)$, $\tau = \varepsilon^2 t$, while $v_g = \frac{d\omega}{dk} = \frac{\omega^3 c_1}{k^3 \alpha}$ and P and Q are dispersion and nonlinear coefficients respectively.

Modulational instability. Adopting standard procedure [2], we investigate the occurrence of modulational instability by considering a harmonic solution of (3) and then a harmonic amplitude perturbation with (wavenumber, frequency)=($\tilde{k}, \tilde{\omega}$). A nonlinear dispersion relation is thus obtained: $\tilde{\omega}^2 = P^2 \tilde{k}^2 (\tilde{k}^2 - 2\frac{Q}{P}|\tilde{\psi}_{1,0}|^2)$. Provided that $PQ > 0$, wavenumbers above $\tilde{k}_{cr} = (\frac{2Q}{P})^{1/2}|\tilde{\psi}_{1,0}|$ lead to (amplitude) modulational instability (MI). For $PQ < 0$, wavepackets are stable.

Envelope solitons. In the modulationally stable region ($PQ < 0$, in fact essentially for large wavelengths) *"dark"* solitons may occur, i.e. exact solutions in the form [7]: $\psi = \psi_0\{1 - d^2 \mathrm{sech}^2[(\zeta - V\tau)/L]\}$. On the other hand, for $PQ > 0$, *"bright"* envelope solitons occur in the form [7]: $\psi = \psi_0 \,\mathrm{sech}[(\zeta - V\tau)/L]$. In the above, ψ_0 is the asymptotic electric potential amplitude value, V is the propagation speed and L is the soliton width, while the positive constant d regulates the depth of the void ($d = 1$ for black solitons or $d < 1$ for grey ones).

5. Summary. Stronger superthermality leads to higher amplitude solitary excitations (as suggested by Fig. 1). Both the cold electron temperature and concentration significantly effect on the soliton existence domain, as the upper Mach number limit M_2 increases for higher "cold" electron temperature, while the sonic threshold M_1 is decreased for higher n_{c0} (see Fig. 4). The modulational instability growth rate may be reduced due to stronger superthermality (see Fig. 5a), while (somewhat counter-intuitively) the presence of more excess hot (superthermal) electrons increases the instability growth rate (see Fig. 5b).

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