AIP Conf. Proc. 1397, 86-91 (2011)
doi:http://dx.doi.org/10.1063/1.3659745

Electrostatic waves in superthermal dusty plasmas: review of recent advancement[1]

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Abstract. Real plasmas are often caracterized by the presence of excess energetic particle populations, resulting in a long-tailed non-Maxwellian distribution. In Space plasma physics, this phenomenon is usually modelled via a kappa-type distribution. This presentation is dedicated to an investigation, from first principles, of the effect of superthermality on the characteristics of dusty plasma modes. We employ a kappa distribution function to model the superthermality of the background components (electrons and/or ions). Background superthermality is shown to modify the charge screening mechanism in dusty plasmas, thus affecting the linear dispersion laws of both low- and higher frequency DP modes substantially. Various experimentally observed effects may thus be interpreted as manifestations of superthermality. Focusing on the features of nonlinear excitations (solitons) as they occur in different dusty plasma modes, we investigate the role of superthermality in their propagation dynamics (existence laws, stability profile) and characteristics (geometry).

Keywords: Dusty plasmas, complex plasmas, electrostatic waves, solitons, modulational instability
PACS: PACS 52.27.Lw, 52.35.Fp, 52.35.-g

Plasma environments in Space and in the laboratory are often characterized by the presence of energetic particles in the background, due to various acceleration mechanisms [1]. Following the ubiquitous observation of a long-tailed behavior in Space plasmas, a kappa- (κ) parametrized distribution function was proposed, and later widely employed to fit observed data with higher accuracy than the widespread Maxwellian approach [2]. It has been shown from first principles that the omnipresent superthermal feature of plasmas may alter the propagation characteristics of plasma modes, and modify the screening properties rather dramatically [3-6]. Recently, superthermality was employed to fit the observed data in experiments on electron-holes [7], while non-Maxwellian electron distribution was also observed in high-power laser plasma interactions, where electron acceleration was induced by plasma expansion into tenuous plasma [8]. Superthermality was thus shown to be an ubiquitous feature of real plasmas. Dusty plasmas (DP), where the dust component evolves against a background of light particles, often in a nonthermal (off-Maxwellian) state, should be no exception.

We have recently carried out a series of investigations, from first principles, of the

[1] Invited talk at: *International Conference on the Physics of Dusty Plasmas* (ICPDP 2011), 16-20 May 2011, Garmisch-Partenkirchen, Germany. The project is carried out in the framework of an active collaboration with N.S. Saini (Guru Nanak Dev University, India), T. Baluku (Mbarara University of Science and Technology, Uganda) and M.A. Hellberg (University of KwaZulu-Natal, South Africa). Joint work with S.M. Hosseini Jenab (Amirkabir University of Technology, Tehran, Iran), A. Danekhar (Macquarie University, Sydney, Australia) and O. Adeyemi (University of Hamburg, Germany) is also acknowledged.

effect of superthermality on the characteristics of dusty plasma modes [3-6, 9-10]. We have employed the kappa-distribution paradigm to model the influence of superthermality of the plasma components (electrons and/or ions) on nonlinear waves, e.g., soliton pulses and shocks. In a generic manner, for all DP modes considered, superthermality is shown to modify the charge screening mechanism in dusty plasmas, thus affecting the linear dispersion laws of both low- and higher frequency DP modes substantially. Various experimentally observed effects, in particular deviating from the expected theoretical prediction, may thus be interpreted as manifestations of superthermality. Focusing on the features of nonlinear excitations as they occur in different dusty plasma modes, we have investigated the role of superthermality in their propagation dynamics (soliton existence laws, stability profile) and characteristics.

In this article, we shall limit ourselves to pin-pointing the combined effect of superthermality and dust on *dust-ion acoustic (DIA) waves*, as a fundamental paradigm for our purpose. Superthermality affects the wave characteristics via the dynamical charge balance, which takes into account a non-Maxwellian distribution for the electrons (in the former case). The results presented below are generic, in that the same qualitative features are encountered in other modes as well.

DIA solitary waves in superthermal dusty plasmas.. We consider an unmagnetized collisionless three-component dusty plasma consisting of electrons, ions and negatively charged mobile dust. The dynamics of dust-ion acoustic waves is described by a fluid model modelling the ionic inertia, i.e., adopting exactly Eqs. (1)-(5) in [11]. Given the (ionic) scale of interest, the dust is assumed to be stationary (fixed). The electron density, modeled via a superthermal $\kappa-$ type electron distribution, has the form [2]: $n_e = n_{e0}\left\{1 - e\Phi/[(\kappa - \frac{3}{2})k_BT_e]\right\}^{-\kappa+1/2}$, where the index "0" denotes the unperturbed (equilibrium) number density values and the spectral index κ measures the strength of the distribution function. Small values of κ correspond to a strong deviation from the Maxwellian distribution, which is recovered in the limit $\kappa \to \infty$ (and practically, in fact, for finite values of κ above $\simeq 10$).

In the following, we scale the state variables by the equilibrium ion number density n_{i0}, the ion sound speed $c_s = (Z_ik_BT_e/m_i)^{1/2}$ and $\Phi_0 = k_BT_e/e$, respectively. Space and time will be normalized by the ion plasma screening length $\lambda_{D,eff} = [k_BT_e/(4\pi Z_in_{i0}e^2)]^{1/2}$ and by the inverse ion plasma frequency $\omega_{pi}^{-1} = (4\pi n_{i0}Z_i^2e^2/m_i)^{-1/2}$, respectively. We also define the dimensionless (Havnes) dust parameter $\mu = Z_dn_{d0}/(Z_in_{i0}) = 1 - \frac{n_{e0}}{Z_in_{i0}}$. Note that μ takes values between zero (dust-free limit) and unity (for complete electron depletion, i.e. in ion-dust plasma).

Linear dispersion relation.. The fluid system introduced above leads to the dispersion relation: $\omega^2 = k^2/(k^2 + c_1)$, i.e., restoring dimensions for a minute, $\omega^2 = k^2\omega_{pi}^2/[k^2 + (\sqrt{c_1}/\lambda_{D,eff})^2]$, where the constant parameter c_1 is defined by $c_1 = (1-\mu)\frac{\kappa-1/2}{\kappa-3/2}$. We see that the charge (Debye) screening mechanism is strongly affected by both the dust concentration (via μ) and by superthermality. Defining the (κ-dependent) *modified screening length* as $\lambda_{Di}^{(\kappa)} = \lambda_{D,eff}/\sqrt{c_1} = \left\{(\kappa - 3/2)/[(\kappa-1/2)(1-\mu)]\right\}^{\frac{1}{2}}\lambda_{D,eff}$, we notice that a stronger deviation from the Maxwellian (i.e., for lower κ) leads to a reduced charge screening length, viz., $\lambda_{Di}^{(\kappa)} < \lambda_{Di}^{(Max)}$, which even reaches zero at $\kappa \to 3/2$. On the other hand, a stronger dust concentration (i.e., lower μ) leads to a larger Debye sphere, thus somehow competing with superthermality, for negative dust (it is straightforward to see that this effect should be reversed for positive dust). We stress the fact that superthermality results in a *modified sound speed*, since the *real* sound speed in the plasma becomes $c_s^{(\kappa)} = \omega_{pi}\lambda_{Di}^{(\kappa)} = c_s^{(\kappa\to\infty)}/\sqrt{c_1}$, i.e. it is reduced by a factor $\sqrt{c_1}$. It is straightforward to obtain $\omega^2 \simeq c_s k/\sqrt{c_1}$ in the large wavelength limit, thus $v_{ph} = c_s^{(\kappa)}$ defined above is the real phase speed $v_{ph} = \omega/k$ of ion-acoustic waves (in the cold ion model).

Nonlinear Korteweg - de Vries description for small-amplitude solitary waves.. Adopting the so-called reductive-perturbation technique, one may consider the stretched (slow) coordinates $\xi = \varepsilon^{1/2}(x - v_{ph}t)$, $\tau = \varepsilon^{3/2}t$, where $\varepsilon \ll 1$ and v_{ph} is the phase speed defined above, and expand the dependent variables (n, u and ϕ) near equilibrium as $\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \cdots$ (along with analogous expressions for n and u near 1 and 0, respectively), in order to obtain the nonlinear Korteweg - de Vries (KdV) equation

$$\frac{d\phi_1}{d\tau} + A\phi_1\frac{d\phi_1}{d\xi} + B\frac{d^3\phi_1}{d\xi^3} = 0\,, \tag{1}$$

for the potential disturbance ϕ_1. The nonlinearity (A) and dispersion (B) coefficients read

$$A = \frac{3\mu + \kappa(4-6\mu) - 4}{2\sqrt{(2\kappa-3)(2\kappa-1)(1-\mu)}}\,, \qquad B = \frac{1}{2}\left[\frac{(2\kappa-1)(1-\mu)}{2\kappa-3}\right]^{-3/2}\,, \tag{2}$$

or, in the Maxwellian electron limit ($\kappa \to \infty$), $A = \frac{2-3\mu}{2\sqrt{1-\mu}}$, $B = \frac{1}{2}\left(\frac{1}{1-\mu}\right)^{3/2}$. In the dust-free limit (i.e. for $\mu = 0$), one recovers for ordinary ion-acoustic waves $A = \frac{2(\kappa-1)}{\sqrt{(2\kappa-3)(2\kappa-1)}}$, $B = \frac{1}{2}\left(\frac{2\kappa-1}{2\kappa-3}\right)^{-3/2}$, which yields $A = 1\,, B = 1/2$ as expected [12] in the (dust-free) Maxwellian limit. The KdV equation (1) bears the soliton solution

$$\phi_1(\xi,\tau) = \phi_0\,\mathrm{sech}^2\left[(\xi - V\tau)/L_0\right]\,, \tag{3}$$

where the pulse amplitude ϕ_0 and the pulse width L_0, defined as $\phi_0 = 3V/A$ and $L_0 = \sqrt{4B/V}$ respectively satisfy the relation $\phi_0 L_0^2 = 12B/A$.

Numerical stability investigation. We note in Fig. 1 that nonlinearity is stronger while dispersion is weaker for lower κ. The delicate balance between these two mechanisms – for given, say, κ– is thus lost if the value of κ is altered. We have considered a numerical investigation of the following scenario: an electrostatic pulse which propagates in a Maxwellian plasma in a stable manner (see Fig. 2) enters a region which characterized by a lower value of κ (here, $\kappa = 3$): see Fig. 3. We have observed that the energy stored in the pulse allows for the pulse to decompose exactly into a faster (and thus taller and thinner, as expected for KdV solitons) pulse, in addition to a weaker (and wider) pulse following it at a slower speed: see Fig. 3. Of course, the soliton energy balance transfer between one and two pulses which occurred here was rather accidental: in general,

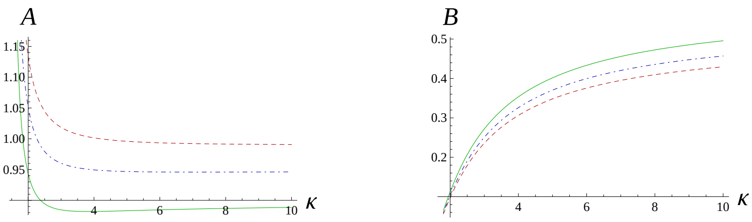

FIGURE 1. The nonlinearity coefficient A (left) and the dispersion coefficient B (right) defined in (2) are depicted versus κ, for different values of the dust parameter μ: $\mu = 0.01$ (dashed curve), $\mu = 0.05$ (dot-dashed curve), and $\mu = 0.1$ (solid curve).

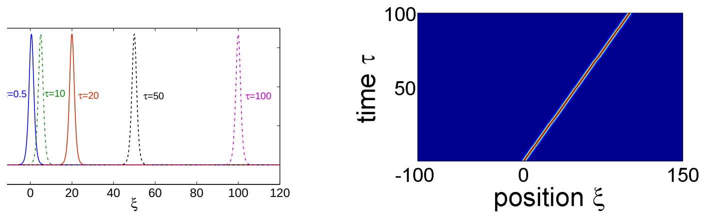

FIGURE 2. Stable propagation of an electrostatic pulse in a quasi-Maxwellian plasma (here $\kappa = 100$, $\mu = 0.1$): different time snapshots (left) and 3D plot vs space and time (right). The exact pulse solution (3) was used as initial condition, for $\kappa = 100$, $\mu = 0.1$ and $V = 1$, into the KdV (1) for $\kappa = 100$, $\mu = 0.1$.

a pulse-shaped initial condition would be expected to evolve into a pulse soliton, if energetically permitted, in addition to a background of oscillations lagging behind the pulse. This was indeed the case when we considered the inverse scenario, i.e. assuming that an exact soliton solution for $\kappa = 3$ enters a Maxwellian (high κ) region. The result is depicted in Figs. 4 and 5, where we observe a(n) (initially stable) pulse propagating in a low-κ plasma (see Fig. 4) enter into a Maxwellian region (see Fig. 5): the pulse slows down slightly and extends itself laterally, while a sea of linear waves clearly appears to follow the pulse, due to the energetic lack of balance between the exact soliton solutions for high and for low κ. We note in the above paradigms an acceleration of pulses crossing into a lower κ region, and a slowing down in the reverse case (in qualitative agreement with earlier results [13, 14]), i.e., lower κ values support faster solitons.

Large-amplitude nonlinear theory. The occurrence and stability of large-amplitude

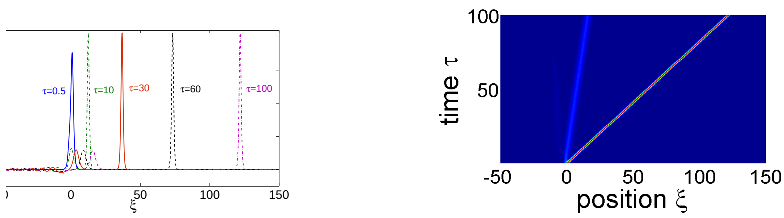

FIGURE 3. High-to-low κ shift: a pulse initially moving in a quasi-Maxwellian plasma (at $t = 0$) *crosses into a superthermal dusty plasma region* (here $\kappa = 3$, $\mu = 0.1$) at $t = 0$: different time snapshots (left) and 3D plot vs space and time (right). The same initial condition as in Fig. 2 was considered here.

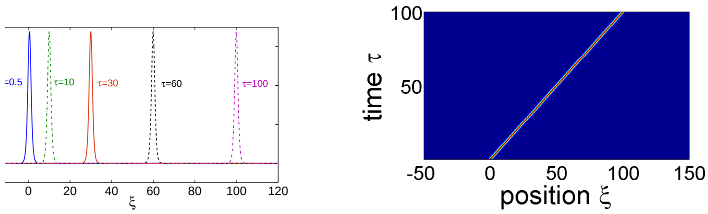

FIGURE 4. Stable propagation of an electrostatic pulse in a strongly *superthermal* plasma (here $\kappa = 3$, $\mu = 0.1$): different time snapshots (left) and 3D plot vs space and time (right). The exact pulse solution (3) was used as initial condition, for $\kappa = 3$, $\mu = 0.1$ and $V = 1$, into the KdV (1) for $\kappa = 3$, $\mu = 0.1$.

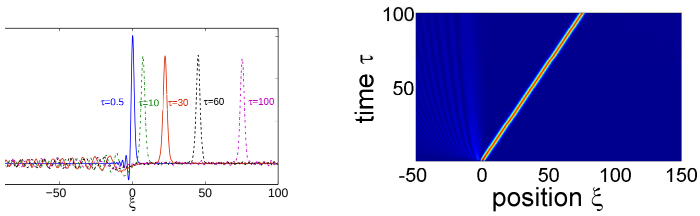

FIGURE 5. Low-to-high κ interface: evolution of a pulse initially moving in a superthermal (for $\kappa = 3$, here) dusty plasma (at $t = 0$) *crossing into a Maxwellian plasma region* (here $\kappa = 100$, $\mu = 0.1$) at $t = 0$: different time snapshots (left) and 3D plot vs space and time (right). The exact pulse solution (3) was considered as initial condition, for $\kappa = 3$, $\mu = 0.1$ and $V = 1$, into the KdV (1) for $\kappa = 100$, $\mu = 0.1$.

ion acoustic solitary waves was investigated in the past via a pseudopotential phenomenology (known as the "Sagdeev" approach), both in dusty plasmas [13] and, earlier, in the absence of dust [14]. These studies have shown that *smaller κ values support faster solitons*, in agreement with our findings above. A positive-to-negative soliton polarity shift was predicted for high (negative) dust concentration, while the associated dust density threshold was shown to be lower for smaller values of κ: superthermality was thus shown to favour the existence of negative-potential ion-acoustic pulses. No negative pulses may occur for positive dust though. As a matter of fact, a co-existence of weak negative and large positive pulses may occur in the presence of negative dust, as predicted by both Sagdeev and modified-KdV (mKdV) theories [13]. The existence of solitons is generally permitted in specific regions in configuration space, which may be conveniently expressed in terms of the Mach number M, i.e. the soliton speed scaled by the (reference) sound speed. Generally, solitary waves occur in the region between two boundary values, viz., for $M_1 < M < M_2$ (e.g., for $1 < M < 1.58$ in the dust-free ion-acoustic model), where M_1 is essentially the sonic point (which depends on both κ and dust via μ, as discussed above) –imposing supersonic soliton propagation– and M_2 is the infinite compression limit, imposed by reality requirements for the state variables. It was earlier shown [14] that both M_1 and M_2 decrease for lower κ (see Fig. 1 in [14]), hence the soliton existence region is significantly affected by superthermality, and in fact shrinks to nil in the limit $\kappa \to 3/2$. Furthermore, it was shown in [13] that a stronger negative dust concentration results in higher (region of permitted) soliton speed values, in apparent competition with superthermality. These considerations agree with our numerical observations reported above.

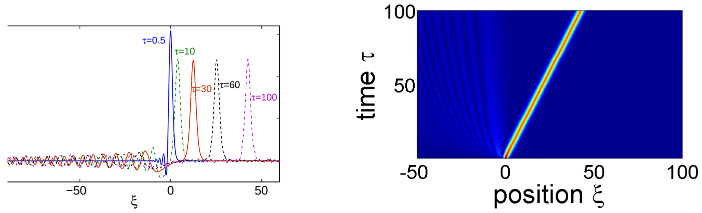

FIGURE 6. Low-to-high *dust* shift: a pulse initially moving in a superthermal (for $\kappa = 3$ and $\mu = 0.1$, here) dusty plasma (at $t = 0$) crosses into a *higher-dust concentration* region (here $\mu = 0.3$) at $t = 0$: different time snapshots (left) and 3D plot vs space and time (right). The exact pulse solution (3) was considered as initial condition, for $\kappa = 3$, $\mu = 0.1$ and $V = 1$, into the KdV (1) for $\kappa = 3$, $\mu = 0.3$.

Summarizing, we have reported a series of extensive analytical and numerical investigations of the effect of superthermal particle on the propagation of electrostatic waves in dusty plasmas. In particular, in the limited space available, we have briefly analyzed the combined effect of superthermality and negative dust on (dust-)ion acoustic solitary waves and envelope solitons. The two mechanisms were shown to cooperate, or sometimes compete, towards a strong modification of the stability characteristics of electrostatic excitations. Further investigations are currently underway and will be reported elsewhere [16] soon.

ACKNOWLEDGMENTS

Support from UK EPSRC under S&I grant No. EP/D06337X/1 is acknowledged.

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