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Effect of strong electrostatic interactions of microparticles on the modulational stability of dust acoustic waves

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Abstract. The modulational instability of dust-acoustic waves is investigated, relying on a recently proposed model for strong electrostatic interactions between the highly charged dust particles. The resulting effect on the occurrence (threshold, growth rate) of modulational instability is investigated. Our results can in principle be tested experimentally.

Keywords: Dust-acoustic wave, modulational instability
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Strong electrostatic interactions of highly charged microparticles, a common feature of complex plasma dynamics, have been shown to modify the behavior of dust acoustic waves, which behave like thermal waves at large wavenumbers [1]. Comparisons with recent experiments seem to confirm this approach [2]. This work investigates the amplitude modulation of dust-acoustic wavepackets via a modified fluid approach presented earlier [2], relying on an anomalous pressure term which involves an effective dust temperature related to the dusty plasma characteristics (namely, the lattice parameter and the plasma coupling parameter).

We consider an unmagnetized collisionless three-component plasma consisting of Maxwellian electrons and ions and negatively charged mobile dust. The dynamics of dust-acoustic waves, whose phase speed lies in the region between the dust thermal speed and the ion thermal speed, is described by

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0 \,, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{eZ_d}{m_d}\frac{\partial \Phi}{\partial x} - \frac{T_d^{(eff)}}{m_d n_d}\,\frac{\partial n_d}{\partial x} \,, \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = 4\pi e\big(n_e - n_i + Z_d n_d\big) \,. \tag{3}$$

The effective dust temperature $T_d^{(eff)}$ is [2, 3]

$$T_d^{(eff)} = \frac{N_{nn}}{3}\Gamma T_d (1+\kappa) e^{-\kappa}, \tag{4}$$

where N_{nn}, Γ and T_d is the number of nearest neighbors, coupling parameter and particle kinetic temperature, respectively. It is stressed that $T_d^{(eff)}$ is here a function of the plasma density and of the electric potential [1], a fact missing in earlier works [4]. For our analysis, we have scaled the dust number density n_d, velocity u_d and electrostatic potential Φ by the equilibrium dust number density n_{d0}, the effective dust sound speed $c_d^{(eff)} = (k_B T_0/m_d)^{1/2}$ and $\Phi_0 = k_B T_0/Z_d e$, respectively. The space and time variables are normalized by the effective dust Debye length $\lambda_{Deff} = (k_B T_0)/(4\pi e^2 Z_d^2 n_{d0})^{1/2}$ and the inverse dust plasma frequency $\omega_{pd}^{-1} = (4\pi e^2 Z_d^2 n_{d0}/m_d)^{-1/2}$, respectively. We have defined $T_0 = (Z_d^2 n_{d0} T_i T_e)/(T_i n_{e0} + T_e n_{i0})$ and the temperature ratio $d = T_d^{(eff)}/T_0$.

We anticipate modelling localized wavepackets (modulated envelope multi-harmonic modes) as *envelope solitons* of the bright/dark type [4]. A multiscale approach is employed [4], to separate the fast carrier from the slow envelope wave. We consider small ($\varepsilon \ll 1$) deviations of all state variables, say S $(= n, u, \phi)$ from equilibrium as

$$S = S^{(0)} + \Sigma_{n=1}^{\infty} \varepsilon^n \sum_{l=-n}^{n} S^{(nl)} e^{il(kx-\omega t)} \,, \; d = d_0 + \sum_{n=1}^{\infty} \varepsilon^n d_n \,,$$

where

$$d_0 = \frac{T_{d0}^{(eff)}}{T_0} \,,\; T_{d0}^{(eff)} = \frac{N_{nn}}{3} Z_d^2 e^2 \times \sqrt[3]{n_{d0}}\,(1+\kappa_0)\, e^{-\kappa_0} \,,$$

$$\kappa_0 = \frac{\sqrt{A}}{\sqrt[3]{n_{d0}}\lambda_{Di}} \,, \quad A = \frac{1-p}{\tau} + 1 \,, \quad \tau = \frac{T_e}{T_i} \,,$$

and p denotes the Havnes parameter $p = \frac{Z_d n_{d0}}{n_{i0}}$.

The first order expressions provide the (known [2]) dust-acoustic wave *dispersion relation*:

$$\omega^2 = \frac{k^2}{k^2+1} + k^2 d_0 \,,$$

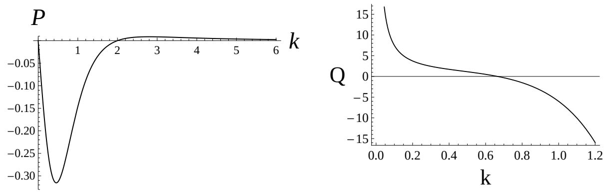

FIGURE 1. Variation of dispersion P (left) and dissipation Q (right) coefficient versus wavenumber k for $\kappa_0 = 2.5,\ p = 0.9,\ d_0 = 0.6,\ \sigma = 0.009,\ \tau = 100$.

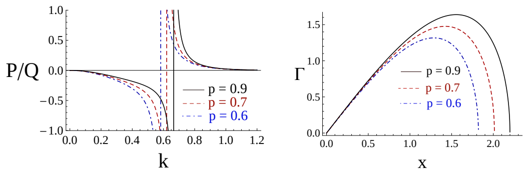

FIGURE 2. Variation of the P/Q ratio (left) and of the modulational instability growth rate Γ (right) for different p. Here $\kappa_0 = 2.5,\ d_0 = 0.6,\ \sigma = 0.009,\ \tau = 100$ for both cases.

along with $n_1^{(1)} = -(k^2+1)\phi_1^{(1)}$, $u_1^{(1)} = \frac{\omega}{k} n_1^{(1)}$.

Combining the algebra upto third order in ε, we obtain a compatibility condition in the form of a *nonlinear Schrödinger* equation (NLSE)

$$i\frac{\partial \psi}{\partial \tau} + P\frac{\partial^2 \psi}{\partial \zeta^2} + Q\mid \psi \mid^2 \psi = 0\,, \qquad (5)$$

where ψ denotes the electric potential correction $\phi_1^{(1)}$ and the slow variables are $\zeta = \varepsilon(x - v_g t)$ and $\tau = \varepsilon^2 t$. The group velocity v_g is given by $v_g = \frac{k}{\omega(k^2+1)^2} + \frac{k}{\omega}\, d_0$. The dispersion coefficient $P = \frac{1}{2}\frac{d^2\omega}{dk^2}$ reads

$$P(k;d_0) = \frac{k^2}{2\omega(k^2+1)^3}\left(-3 + \frac{k^4 d_0}{\omega^2}\right), \qquad (6)$$

which may change sign depending on the relevant plasma parameters. In Fig.1 we give an example of the dispersion and the nonlinear coefficient for a typical set of plasma parameters. The nonlinearity coefficient Q is omitted here (lengthy expression, to be reported elsewhere [5]). Considering a small harmonic *amplitude perturbation* $\tilde{\psi}_1 = \psi_{1,0}\, e^{i(\tilde{k}\zeta - \tilde{\omega}\tau)}$

$$\tilde{\omega}^2 = P^2\,\tilde{k}^2\,(\tilde{k}^2 - \frac{2Q}{P}|\psi_0|^2)\,, \qquad (7)$$

where we have considered $\psi = \psi_m e^{i\theta}$ with $\psi_m = \psi_0 + \varepsilon\tilde{\psi}_1$ in (5). Eq. (7) implies that the carrier wave will be modulationally stable for $Q/P < 0$, while modulational instability occurs (i.e., $\tilde{\omega}^2$ becomes negative and a purely growing mode develops) for $Q/P > 0$, if $\tilde{k}$ lies below $\tilde{k}_{cr} = \sqrt{(2Q/P)}|\psi_0|$. The instability growth rate $\sigma(= |Im\tilde{\omega}(\tilde{k})|)$ will attain a maximum $\sigma_{max}(= |Q||\psi_0|^2)$ at $\tilde{k} = \tilde{k}_{cr}/\sqrt{2}$. Eq. 7 leads to an inter-grain-interaction dependent (via $T_d^{(eff)}$) growth rate

$$\Gamma = \alpha x\left(2\frac{q}{\alpha} - x^2\right)^{1/2}\,, \qquad (8)$$

where $\Gamma = \tilde{\omega}/(Q_m|\psi_0|^2)$, $\alpha = P/P_m$, $q = Q/Q_m$, and $x = \tilde{k}/(Q_m/P_m)^{1/2}|\psi_0|$ with $P_m = P(T_d^{(eff)} \to 0)$ and $Q_m = Q(T_d^{(eff)} \to 0)$. A numerical study reveals that the instability growth rate increases with the dust-to-ion density ratio p (see Fig. 2, right panel), i.e. dust enhances the instability. For comparison, the warm-dust model (including T_{eff}) increases the instability ratio (the maximum for $T_{eff} = 0$ would occur at $(1.4, 1)$, in Fig. 2).

In order to test our prediction for (in)stability as determined by Fig 2 (left), we have performed a numerical simulation of (5) (employing Runge-Kutta 4 method, with spatial grid size and time interval 0.15 and 5×10^{-4}, respectively). It is seen in Fig. 3 that a localized pulse-shaped wavepacket is stable (constant amplitude while propagating) for a carrier wave with wavenumber $k = 0.7$, while it is unstable (amplitude decreases in time) for $k = 0.6$, in complete agreement with Fig. 2 (left).

Our findings aim at inspiring purpose designed experiments, meant to test and confirm our stability predictions.

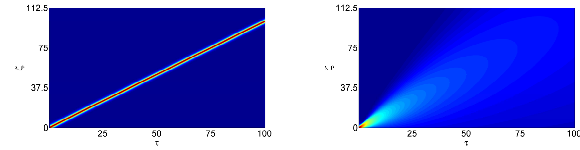

FIGURE 3. Propagation of a localized modulated pulse (bright-type envelope soliton [5]) propagating in a plasma for $d_0 = 0.6, \sigma = 0.009, \kappa_0 = 2.5,\ \tau = 100$: the pulse is stable for a carrier wavenumber value $k = 0.7$ (left panel) and unstable for $k = 0.6$ (right panel).

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