Modelling of ion-acoustic shocks in superthermal plasmas

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We have undertaken a theoretical and numerical investigation of ion-acoustic shock wave dynamics in nonthermal plasmas characterized by finite ion viscosity. The non-thermality of the plasma modeled by a κ-type distribution for the electrons. Shock evolution is modeled by a 1-dimensional Korteweg-de Vries – Burgers equation, obtained via a multiscale perturbation technique. The parametric dependence of the shock amplitude and width on plasma non-thermality (via the κ parameter) is investigated. A stability criterion for the shock profile is analytically derived and tested by numerical integration.

1. Introduction

Space plasma observations [1–6] as well as laboratory experiments [7–10] provide abundant evidence for the occurrence of nonthermal (non-Maxwellian) plasmas, mostly due to the presence of accelerated particles or superthermal radiation fields [11]. Plasma distributions involving a population of superthermal particles feature a power-law behavior [1, 11], which is efficiently described by a kappa- (κ-) parametrized distribution function [5,12,13]. The value of κ lies in the range $\kappa \in (3/2, \infty)$: small κ values account for a stronger deviation from the Maxwellian distribution, which is recovered for $\kappa \to \infty$ (see, e.g., Fig. 1 in [13]). The nonlinear propagation of electrostatic excitations is known to be dramatically modified by the presence of a superthermal electron component, as confirmed by experiments [6, 10].

This paper presents an analytical treatment of the propagation of ion-acoustic shock waves in collisionless superthermal plasmas in the presence of ion viscosity, by means of a Korteweg-de Vries – Burgers equation. The geometric characteristics of the shock are found to be significantly affected by the non-thermality of the background plasma, as corroborated by numerical simulations.

2. The model

We consider an unmagnetized electron-ion (e-i) plasma with cold ions ($T_i \ll T_e$) and superthermal electrons, which are described by a $\kappa-$ type distribution function. Since we focus on plasma dynamics at the ion-acoustic scale, electron inertia is neglected by assuming $v_{th,i} \ll v_{ph} \ll v_{th,e}$, where $v_{th,i}$ ($v_{th,e}$) and v_{ph} are the ion (electron) thermal speed and the ion-acoustic phase velocity, respectively. The ion dynamics is thus described, in a one-dimensional geometry, by the fluid evolution equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \, ,$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{q_i}{m_i}\frac{\partial \phi}{\partial x} + \eta_i \frac{\partial^2 u_i}{\partial x^2} \, ,$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e - Z_i n_i) \, , \qquad (1)$$

where nonthermality is taken into account by assuming for the electron density [12]:

$$n_e = n_{e0}\left[1 - \frac{e\phi}{(\kappa - \frac{3}{2}) k_B T_e}\right]^{-\kappa + 1/2} . \qquad (2)$$

An *ad hoc* damping term was introduced in the momentum equation, involving the (ion) kinematic viscosity η_i. In our analysis, we have scaled the ion number density n_i, the velocity u_i and the electrostatic potential ϕ by the equilibrium ion number density n_{i0}, the ion sound speed $c_s = (Z_i k_B T_e/m_i)^{1/2}$ and $\phi_0 = k_B T_e/e$, respectively (lower case symbols n, u and ϕ will be used for the scaled state variables below). Space and time are normalized by the ion plasma Debye length $\lambda_{Di} = [k_B T_e/(4\pi n_{e0} e^2)]^{1/2}$ and the inverse ion plasma frequency $\omega_{pi}^{-1} = (4\pi n_{i0} Z_i^2 e^2/m_i)^{-1/2}$, respectively. Quasi-neutrality is assumed at equilibrium (viz., $n_{e0} = Z_i n_{i0}$). The normalized (ion) kinematic viscosity variable is $\eta = \eta_i/(\omega_{pi}\lambda_{Di}^2)$.

3. Evolution equation for shock waves

Assuming a weak dissipation, a linear dispersion relation is obtained by linearizing Eqs. (1), in the form:

$$\omega^2 = \frac{k^2\omega_{pi}^2}{k^2 + c_1/\lambda_{Di}^2}\,, \qquad (3)$$

where $c_1(\kappa) = \frac{\kappa-1/2}{\kappa-3/2}$ (> 1). Eq. (3) is essentially the dispersion relation for ion-acoustic waves in a Maxwellian plasma, provided that yet correcting the Debye length by a factor $c_1^{-1/2}$ (< 1). Non-thermality therefore induces a *reduction* of the Debye length $\lambda_{Di}^{(\kappa)} = \lambda_{Di}/\sqrt{c_1}$, as compared to the Maxwellian result (which is recovered in the limit $\kappa \to \infty$; see Fig. 1). The (real) sound speed is also reduced (in comparison to the Maxwellian value) by the same (κ-dependent) factor. In the long wavelength limit (i.e., for $k \ll 1$), Eq. (3) reduces to:

$$v_{ph}^{(\kappa)} = \frac{\omega}{k} \simeq c_1^{-1/2}\; c_s\,. \qquad (4)$$

In order to gain insight into the shock dynamics, we now proceed by considering a weak electrostatic perturbation in the plasma ($\phi \ll k_BT_e/e$): we adopt a multiscale technique [14, 15] by introducing a dependence of the anticipated shock solutions on the stretched (slow) coordinates $\xi = \epsilon^{1/2}(x - V_st)$ and $\tau = \epsilon^{3/2}t$, where V_s denotes the shock propagation speed. Damping is introduced via the ion kinematic viscosity which is assumed to be weak, viz. $\eta = \epsilon^{1/2}\eta_0$. The state variables are expanded near their equilibrium values in a power series of ϵ $(\ll 1)$ as

$$\begin{aligned} n_i &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \cdots\,, \\ u_i &= \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \cdots\,, \\ \phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \cdots\,. \end{aligned} \qquad (5)$$

We now substitute the expansion relations given in (5) into the fluid model above and separate different orders of ϵ. The first-order relations lead to $n_1 = \phi_1/V_s^2$, $u_1 = \phi_1/V_s$ and $V_s = \sqrt{1/c_1}$, in agreement with the linear model (all quantities here and below are in dimensionless form).

Combining the first-order expressions into the second order equations, we obtain a Korteweg–de Vries Burgers (KdVB) type equation in the form:

$$\frac{d\phi_1}{d\tau} + A\phi_1\frac{d\phi_1}{d\xi} + B\frac{d^3\phi_1}{d\xi^3} - C\frac{d^2\phi_1}{d\xi^2} = 0\,, \qquad (6)$$

where the nonlinearity (A), dispersion (B) and

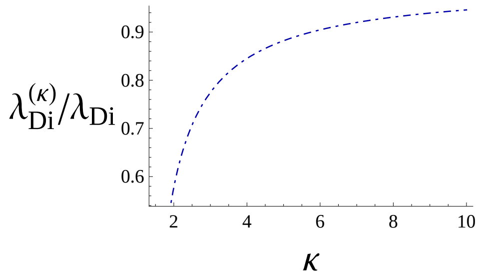

Fig. 1: Ratio between the κ−dependent and related Maxwellian Debye length as a function of κ.

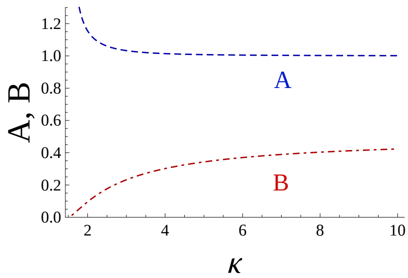

Fig. 2: Variation of the nonlinear coefficient A and the dispersion coefficient B with superthermality parameter κ.

dissipation (C) coefficients read:

$$\begin{aligned} A &= \frac{2(\kappa-1)}{2\kappa-1}\sqrt{1 + \frac{2}{2\kappa-3}}\,, \\ B &= \frac{1}{2}\left(1 + \frac{2}{2\kappa-3}\right)^{-3/2} \quad \text{and} \quad C = \frac{\eta_0}{2}\,. \end{aligned}$$

The coefficients A and B, both positive, are thus functions of the superthermality parameter κ: in fact, the former (A) increases, while the latter (B) decreases, if one considers stronger superthermality (i.e., for lower κ): see Fig. 2. The expected Maxwellian limit $A = 1$ and $B = 1/2$ is recovered for $\kappa \to \infty$, in agreement with Eq. (28) in Ref. [16] (considering $\alpha = A$, $\beta = B$, $\delta = n_{i0}/n_{e0} = 1$ and $\sigma = 0$ therein, to bridge the notation).

4. Travelling wave solutions

Different types of solutions are possible, depending on the interplay between the coefficients in the KdVB equation (6). A monotonic, kink-like shaped, solution is obtained via the hyperbolic tangent method [17, 18] in the form:

$$\phi_1(\xi, \tau) = \frac{V}{A} - \Phi_0\left[(1 + \tanh\zeta)^2 - 2\right]\,, \qquad (7)$$

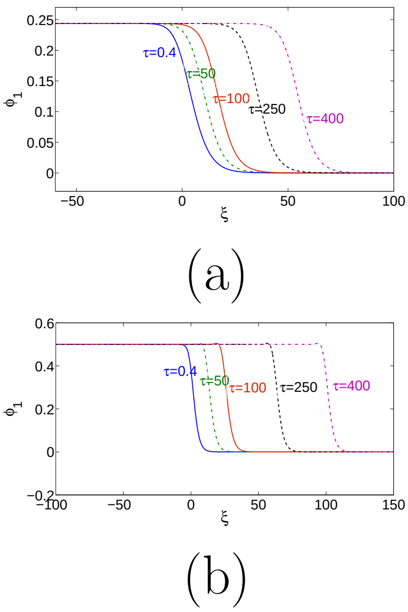

Fig. 3: Evolution of an ion-acoustic shock wave, as given by Eq. (6), propagating in (a) a "superthermal" plasma ($\kappa = 3$) [where Eq. (7) is taken as an initial condition with $V = 0.1$]; and (b) a "Maxwellian" plasma ($\kappa = 100$) [where Eq. (7) is taken as an initial condition with $V = 0.25$]. In both cases, $C = 0.5$ is considered.

where we have performed a transformation of the coordinates $\zeta = \alpha(\xi - V\tau)$ and $\tau = \tau$; here V represents the excitation propagation velocity (in a frame moving at the sound speed), i.e., the increment in speed above the sound speed, α^{-1} represents the shock width and Φ_0 represents the shock amplitude.

As a preliminary result, it is interesting to note that both the spatial extension (width) of the shock $L = \alpha^{-1}$ and its maximum amplitude Φ_0

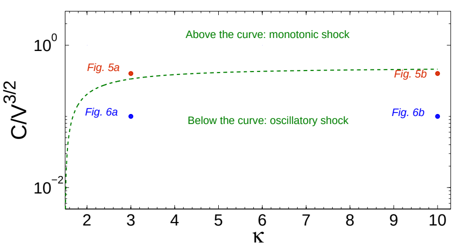

Fig. 4: Threshold for monotonic shock profiles as given in Eq. (10).

are now expressed as functions of κ:

$$\alpha^{-1} = \frac{10}{\eta_0}\left(1 + \frac{2}{2\kappa - 3}\right)^{-3/2},$$

$$\Phi_0 = \frac{3\eta_0^2}{100} \frac{(2\kappa - 1)^2}{(\kappa - 1)(2\kappa - 3)}. \quad (8)$$

We point out that this type of solution bears a strictly monotonic form, regardless of the values of the relevant parameters.

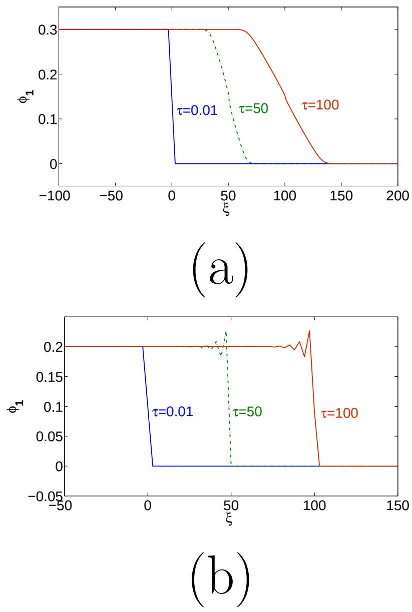

Fig. 5: Evolution of the shock solution given in Eq. (9) for (a) a superthermal plasma ($\kappa = 3$, with $B = 0.23$) and (b) a Maxwellian plasma ($\kappa \to \infty$, with $B = 0.5$). We have assumed $C = 0.4$ and $V = 1$.

In the dispersionless limit ($B \ll C$), a shock-like solution is exactly obtained – upon setting $B = 0$ in (6) – in the form [19]

$$\phi_1(\xi, \tau) = \Phi_1\Big(1 - \tanh[(\xi - V\tau)/L_1]\Big). \quad (9)$$

Eq. (9) represents a shock structure with speed V, amplitude $\Phi_1 = V/A$ and width $L_1 = 2C/V$.

Considering a small periodic perturbation around the solution in Eq. (9), and investigating the stability of the solution against dispersion, an explicit condition is obtained

$$\frac{C}{V^{3/2}} \ll \frac{\kappa - 3/2}{\sqrt{2(\kappa - 1)(2\kappa - 1)}}, \quad (10)$$

for an oscillatory profile to dress the shock in response to an external perturbation (linear stability; see Fig. 4). In the opposite lime, a purely monotonic shock front occurs.

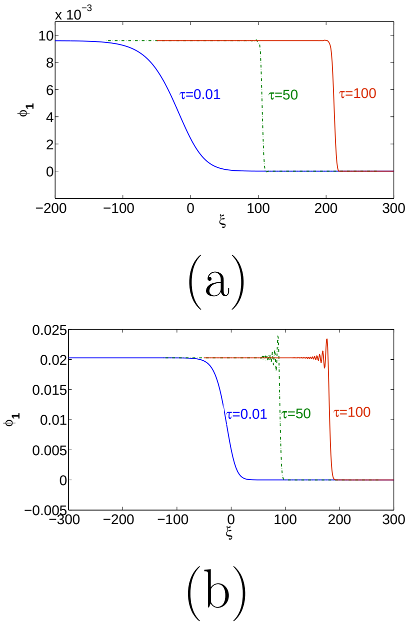

Fig. 6: Evolution of the shock solution given in Eq. (9) for (a) a superthermal plasma ($\kappa = 3$) and (b) a Maxwellian plasma ($\kappa \to \infty$). We have taken $C = 0.4$ and $V = 1$.

We have studied the shock behavior by numerical simulation of Eq. (6), taking as initial condition Eq. (9). Our results, depicted in Figs. 5 and 6, confirm the above condition.

5. Conclusions

We have investigated the nonlinear propagation of ion-acoustic shock waves in a plasma characterized by a superthermal (non-Maxwellian) electron population. Our study relies on a KdV/Burgers equation, obtained via a multiscale technique, which models the evolution of a weak perturbation in the electrostatic potential. The intrinsically competing plasma nonlinearity and dispersion mechanisms were shown to be modified due to plasma superthermality, entailing a significant influence on the dynamics of electrostatic shocks. The shock profile is seen to increase its amplitude and narrow its width with superthermality. Numerical simulations suggest that a shock wave maintain its robustness while crossing the interface between a Maxwellian and a superthermal plasma. In the inverse case (superthermal to Maxwellian), oscillatory structures are generated in its downstream part in agreement with analytical findings.

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