Interaction between two initially non-drifting electromagnetic solitary waves

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The field of laser plasma interactions has seen a resurgence after the invention of Chirped Pulsed Amplification (CPA) technology, thanks to Mourou and co-workers. Modern lasers boast of ultra-high intensities ($I \geq 10^{18} W/cm^2$) which can drive electrons to relativistic jitter speeds. Interaction of such intense laser pulses with a plasma exhibits a rich variety of nonlinear physics. One such interesting phenomenon, namely the formation of electromagnetic solitons, has drawn much attention due to the extremely interesting physics it involves as well as to its potential applications. These solitons, while being coherent electromagnetic entities, are considered to be an integral part of electromagnetic turbulence and therefore are of fundamental interest. On the other hand, the main applications envisaged of these coherent structures are in inertial confinement fusion and in particle acceleration, both of which rely on stable energy transport via pulse propagation in a plasma background. This has motivated several theoretical as well as experimental investigations in the field.

Among numerous theoretical investigations on the electromagnetic solitary waves in plasmas, Esirkepov *et al.* [1] have obtained a mathematical form of a non-propagating soliton solution. They investigated the stability of these solitons by using particle-in-cell (PIC) simulations, and also demonstrated the existence of an upper bound of amplitude a standing soliton can possess based on the physical argument of non-negativity of the fluid density. The question of accessibility of these solitons was partially addressed in [2, 3] where the interaction of an externally imposed circularly polarized intense electromagnetic wave with a plasma was studied numerically by PIC simulations as well as fluid simulations, wherein a significant part of the laser energy was found to be trapped in non-propagating soliton like pulse structures. State of the art experiments also provide evidence of the formation of almost standing localized structures with electromagnetic radiation trapped within a density cavity [4]. All of these observations make standing (zero group-velocity) soliton solutions an object of extreme research interest. Although the stability of these standing structures has been tested by Esirkepov *et al.* [1] by using one dimensional PIC simulations, there is no report on the interactions among two or more such structures, while being in the neighborhood of one another.

In the present work, we report a series of fluid simulation results regarding the evolution of a pair of two such standing solitons placed next to each other with partially overlapping field profiles. We study cases of small as well as large amplitudes. Our numerical investigation relies on a one dimensional cold plasma fluid-Maxwell model which has been extensively used in the existing literature on laser plasma interactions. The ions are considered to form a stationary neutralizing background for the cold electron fluid. The model equations are, the electron continuity equation, the longitudinal electron momentum equation, Poisson's equation, and electromagnetic wave equation,

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0. \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)(\gamma u)=\frac{\partial \phi}{\partial x}-\frac{1}{2\gamma}\frac{\partial A_\perp^2}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n-1 \tag{3}$$

$$\frac{\partial^2 \vec{A}_\perp}{\partial x^2}-\frac{\partial^2 \vec{A}_\perp}{\partial t^2}=\frac{n\vec{A}_\perp}{\gamma} \tag{4}$$

with all notations being standard. The perpendicular electron momentum has been eliminated by using the relation $u_\perp - \vec{A}_\perp/\gamma = 0$, which is an outcome of the conservation of the transverse canonical momentum. Here γ is the relativistic factor defined as $\gamma = \sqrt{\frac{1+A_\perp^2}{1-u^2}}$. The fluid densities, lengths and time are respectively normalized by the background plasma density n_0, the corresponding skin depth c/ω_{pe0} (where $\omega_{pe0} = \sqrt{4\pi n_0 e^2/m_e}$) and the inverse of the plasma frequency ω_{pe0}^{-1}. The scalar and vector potentials are normalized by $m_e c^2/e$.

The coupled set of nonlinear equations (1)- (4) permit a variety of coherent equilibrium solutions. We focus here on the non-propagating soliton solutions obtained by Esirkepov et al.[1]. The solution with zero group velocity is given by :

$$R=\frac{2\sqrt{1-\omega^2}\cosh\left(\xi\sqrt{1-\omega^2}\right)}{\cosh^2\left(\xi\sqrt{1-\omega^2}\right)+\omega^2-1}, \tag{5}$$

$$n=1.0+\frac{(1-\omega^2)^2\left(\cosh\left(4\xi\sqrt{1-\omega^2}\right)-2\left(2\omega^2-1\right)\cosh\left(2\xi\sqrt{1-\omega^2}\right)-3\right)}{\left(\cosh^2\left(\xi\sqrt{1-\omega^2}\right)+\omega^2-1\right)^3}, \tag{6}$$

$$u=0, \tag{7}$$

and ϕ is the solution of Eq.(3) using the density expression (6). The forementioned EM solitary waves exhibit excellent stability properties during the course of their time evolution, as was shown by Esirkepov et al.[1] and has been verified in our fluid simulations. Here, we investigate the evolution of a pair of standing solitons which are placed in the vicinity of one another, so as

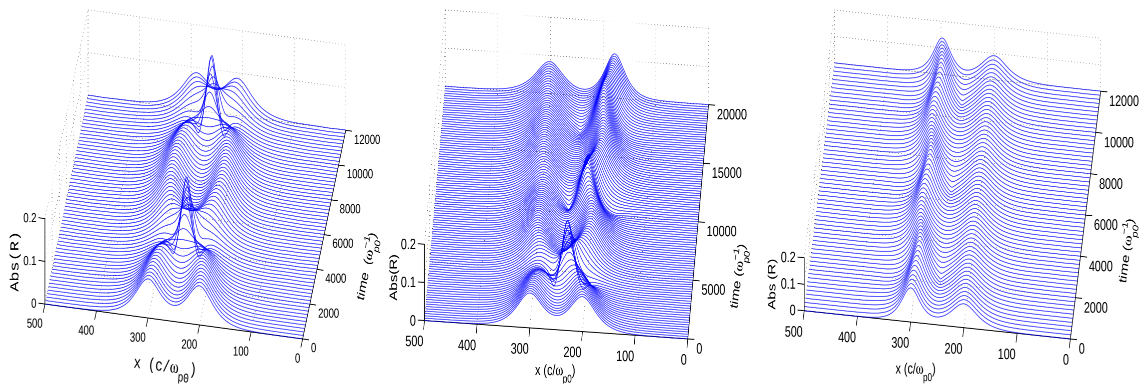

Figure 1: *The temporal evolution of a pair of standing solitary solutions corresponding to (i)* $\omega_1 = \omega_2 = 0.999$*,* $\Delta\phi = 0$*; (ii)* $\omega_1 = \omega_2 = 0.999$*,* $\Delta\phi = 0.1\pi$ *and (iii)* $\omega_1 = 0.999, \omega_2 = 0.998$*,* $\Delta\phi = 0$*.*

to have a spatial overlapping. We numerically study the cases of small amplitude as well as of large amplitude and discuss the respective results in the following.

In the small-amplitude case we have chosen a standing soliton solution with an indicative value $\omega = 0.999$. A combination of two such solitons with their field profiles partially overlapping is chosen as the initial condition for the time evolution studies. In Fig. 1 we depict the time evolution of such an initial condition for three interesting cases. The left panel in Fig. 1 corresponds to the case of similar amplitudes and phases, the middle one for similar amplitudes but with a phase difference of 0.1π, and the right one for different amplitudes with no phase difference. It is clear from the plots that the two solitons are attracted towards each other in all three cases; however, the end state depends on the difference between their amplitudes and phases. In the first case, the solitons form an oscillating bound state, whereas in the case of finite phase difference the end state is a pair of escaping solitons. In the case of different but small amplitudes and no phase difference, the two structures interact very weakly and perform small amplitude oscillations around their respective initial positions. We have also observed the formation of an oscillatory bound state, for three in-phase solutions of similar (small) amplitudes and no phase difference.

We believe that our results can be generalized to an array of solitons, and that they hint towards the existence of oscillating bound-state multi-soliton solutions of the system. Our results are in qualitative agreement with the results obtained by Gordon [5] for the time evolution of an optical pulse in the form of a bi-soliton solution of the nonlinear Schrodinger equation (NLSE).

Now, we consider two large amplitude standing solitary solutions initially close to each other. For this case, we consider two similar solutions with $\omega = 0.99$ and without any initial phase difference. We observe that unlike the small amplitude case, the two structures do not form a bound state, rather they end up in a pair of escaping solitons. The numerical results have been displayed in Fig. 2. It is clear from the plots that after the collision the structures emerging out are not similar to the original structures. This hints towards the non-existence of a bound state solution for arbitrary amplitudes.

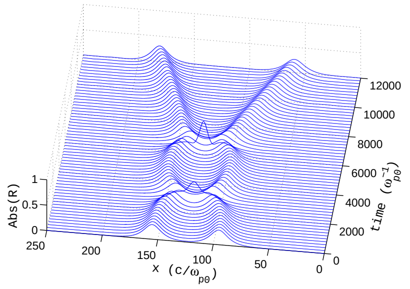

Figure 2: *The time evolution of a pair of standing solitary waves with* $\omega_1 = \omega_2 = 0.99$ *and* $\Delta\phi = 0$

To summarize, we have reported a series of challenging numerical observations of the time evolution of two partially overlapping standing electromagnetic solitary waves. We show that two standing solitary waves with small amplitudes are attracted by each other and form an oscillatory bound state if their amplitudes are equal and there is no phase difference between the two constituent solutions. In the case of a finite phase difference and small (similar) amplitudes, the final state is a pair of escaping solitons, whereas in the case of different (but small) amplitudes with no phase difference, the interaction between the two structures is very weak and they keep oscillating in the vicinity of their respective initial positions. Our observations agree qualitatively with Gordon's [5] results for the time evolution of a bi-soliton solution of a nonlinear Schrodinger equation.

Acknowledgment. The authors gratefully acknowledge funding from the UK EPSRC (Engineering and Physical Sciences Research Council) via grant EP/D06337X/1.

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