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CONCURRENT SIMULATIONS OF THERMAL RADIATION IN PLASMAS

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ABSTRACT

We employ a novel simulation scheme that is based on new a-priori estimates on partial differential equations to numerically study the fully non-linear heat equation in a plasma model for 2+1 and 3+1 dimensions. The tools we create for that purpose are designed to perform in multicore/distributed clusters, using parallel and purely functional data structures in an event-driven concurrent fashion.

1 INTRODUCTION

The thermal radiation in a plasma is described by a nonlinear parabolic equation that is derived by the energy balance (Zeldovich and Raizer 2002) $t > 0, \mathbf{x} \in \mathbf{R}^3, m \sim 6$:

$$\Theta_t = \nabla \cdot (\Theta^m \nabla \Theta), \qquad \Theta(0,\cdot) = \Theta_0(\cdot) \in L^1(\mathbf{R}^n), \qquad \text{supp}(\Theta_0) \subset B_R(0),$$

where $\Theta(0,\cdot)$ is the initial thermal distribution. This equation has strong solutions that are bounded and Hölder continuous (Friedman 2010),(Caffarelli and Friedman 1980) for $\mathbf{x} \in \mathbf{R}^n, t > \tau_0 > 0$ where τ_0 is the initial time and n is the dimension of the problem (usually 2 or 3). The preceding authors as well as (Lee and Vazquez 2003) study the asymptotics to the following special solution, known as Barenblatt-Prattle solution. Let

$$\sigma = 1 - \frac{\kappa(m-1)}{mn} \frac{|\mathbf{x}|^2}{t^{\frac{2\kappa}{n}}}$$

then

$$\Theta(t,\mathbf{x}) = \begin{cases} t^{-\kappa} \sigma^{\frac{1}{m-1}}, & \text{if } \sigma > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\kappa = \frac{n}{(m-1)n+2}$. They determined the regularity properties of the interface, or front,

$$\mathcal{F} = \{\mathbf{x} \in \mathbf{R}^n, t > \tau_0 / \Theta(t,\mathbf{x}) = 0\}$$

Customarily setting $v = \frac{m}{m-1} \Theta^{m-1}$ one arrives at

$$v_t = (m-1) v \Delta v + |\nabla v|^2 .$$

The usual proof of the existence of weak solutions employs a specific fixed point theorem modelled on an Euler scheme and potential theory estimates. We set up an Euler type scheme for the solution of the above equation combined with new localized in space a-priori estimates (Pliakis 2013). Specifically for domains of the form

$$N(\psi;\epsilon,\eta,\theta)=\{(t,\mathbf{x})\in\mathbf{R}\times\mathbf{R}^n/0<t<\epsilon,\ \ \eta\theta<\psi<\eta(1+\theta)\}$$

where η and θ are the lower and upper bound of the function ψ in the domain. Then we derive the following estimates for the 3 dimensional case where $\mu>0$ is a parameter determined by initial data and for $j=0,1,2,\ldots$

$$\sup_{N(\psi;\epsilon,\eta,\theta)}|\nabla^j\Theta|\leq\frac{(36\pi)^{\frac{1}{3}}\mu^{3/2(j+1)}\theta^{3/2}}{(|\Omega|-c\eta t)^2}\left(\int_{N(\psi;\epsilon,\eta,\theta)\bigcap\{t=0\}}\Theta_0^2\right)\ .$$

2 IMPLEMENTATION

We develop software that solves these partial differential equations in an event driven and concurrent manner, designed from the ground up to perform in parallel and to scale. As design patterns we employ the Actor Model (Agha 1986) and our own purely functional versions of performant data structures like Kd-trees (Bentley 1975) and skip-lists (Pugh 1990). The framework consists of a simulation manager that controls tree workers via events. The latter create or delete points that track the phenomenon being simulated, in order to ensure accurancy and speed. Our code is written in the GO programming language (Google 2008) which is designed to provide primitives for message passing therefore eliminating the need for third party libraries like OpenMPI, OpenMP or TBB for that purpose. The combination of golangs lockless synchronization based on channels (Hoare 1981) and the purely functional nature of our data structures leads to less and much simpler code. As for the underlying cluster architecture, any *x86*, *amd64* or *arm* linux system will be able to run our software.

3 RESULTS

We present a comprehensive series of graphical results in both 2D and 3D heat conduction setups. The nonlinear character of the problem manifests itself in blowups of the temperature gradient. Our software is able to identify those regions and using the a-priori estimates refine the space/time discretization. Therefore we are able to study the evolution of the interface (zero temperature regime) and also, using a variety of signals, the merging of interfaces.

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