Proc. 43rd Eur. Phys. Society Conf. on Plasma Physics in Leuven, Belgium, 4-8 July 2016
Paper P5.097
Available at www.kourakis.eu

# Analytical model for dissipative shocks in pair plasmas under the combined effect of collisionality and kinematic viscosity

I. S. Elkamash[1,2] and I. Kourakis[1]

[1] *Centre for Plasma Physics, Queen's University Belfast, BT7 1NN Northern Ireland, UK*

[2] *Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt*

**Abstract**

An analytical model is introduced for shock excitations in pair plasmas, taking into account collisionality and kinematic (fluid) viscosity. The description embraces pair-ion (e.g. fullerene) plasmas, in the presence of a third component (electrons, dust), but also electron-positron plasmas (disregarding annihilation, for simplicity). A hybrid Korteweg de Vries/Burgers equation (KdV-B) is derived, and the effect of relevant plasma configuration parameters, in addition to dissipation, is investigated.

**Introduction.** Pair plasmas (*p.p.*), comprising particles with equal masses and equal absolute charge of opposite signs [1], exist in various environments. Electron-positron (e-p) plasmas, widely occuring in astrophysical environments such as pulsar magnetosphere and neutron stars [1], but also in the laboratory [2], have been the prototypical physical system to be studied as *p.p.* Recent experimental techniques have enabled electron-positron plasma production in the laboratory, via a sophisticated laser-plasma setup [3], and lie in the motivation of our work. Interestingly, fullerene (pair-ion) plasmas have been produced in the lab [4] providing new inspiration, essentially mimicking the dynamics of e-p plasma, without recombination effects.

This work aims at providing a first analytical model for shocks and dissipative solitary waves (pulses) in *asymmetric* pair plasmas, i.e. in the presence of a third species, thus extending earlier studies of ion-acoustic type shock waves in "pure" (two-component) pair-ion plasmas [5, 6]. *Inter alia*, we aim at providing a comprehensive description of fullerene plasmas or, e.g., dusty pair-ion plasmas [7] in particular, where dissipative effects (due to collisions) may be relevant.

**Theoretical model.** We consider a multicomponent plasma comprising two ion populations with equal masses and opposite charge, denoted by indices $+$ and $-$, viz. $q_+ = -q_- = +ze$, $m_+ = m_- = m$. The existence of a third species is taken into account, typically taken to be a neutralizing background of thermal electrons.

Interactions among the plasma components include: electrostatic interactions, taken into account via a self-consistent generated electric field, but also interparticle collisions and viscous drag. The electron inertia is neglected. At equilibrium, we have $n_{e0} = z(n_{+0} - n_{-0})$, i.e.

$\frac{n_{e0}}{zn_{+0}} = 1 - \frac{n_{-0}}{n_{+0}} = 1 - \delta$, where $n_{j0}$ is the unperturbed number density of the particle species $j$ ($j = (1,2) = (+,-)$ for the two ions, respectively, or $j = 3$ for the third species, i.e. electrons) and $\delta = n_{-0}/n_{+0}$ is the positive-to-negative component number density ratio. Considering electrons as 3rd species, we assume: $n_3 = n_e = n_{e0}\exp(e\phi/k_BT_e) \approx n_{e0}(1 + c_1\phi + c_2\phi + \ldots)$ where $c_1 = 2c_2 = 1$, e is the magnitude of the electron charge, $\phi$ the electrostatic (ES) potential, $k_B$ is the Boltzmann constant and $T_e$ the electron temperature.

In one-dimensional planar geometry, the normalized fluid equations read:

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}(n_+u_+) = 0, \tag{1}$$

$$\frac{\partial u_+}{\partial t} + u_+\frac{\partial u_+}{\partial x} = -\frac{\partial \phi}{\partial x} - \nu(u_+ - u_-) + \eta\frac{\partial^2 u_+}{\partial x^2}, \tag{2}$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x}(n_-u_-) = 0, \tag{3}$$

$$\frac{\partial u_-}{\partial t} + u_-\frac{\partial u_-}{\partial x} = \frac{\partial \phi}{\partial x} - \nu(u_- - u_+) + \eta\frac{\partial^2 u_-}{\partial x^2}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -n_+ + \delta n_- + (1-\delta)n_3, \tag{5}$$

In the above set of equations we have used the time scale $t$ and space scale $x$ are normalized in units of the ion plasma frequency $\omega_{p,+} = (4\pi z^2e^2n_{+o}/m)^{1/2}$ and the ion Debye length $\lambda_{Di,+} = \left(4\pi ze^2n_{+o}/k_BT_e\right)^{-1/2}$. The number density $n_j$ and fluid velocity $u_j$ variables ($j = +,-,3$) are scaled by the unperturbed number density $n_{j0}$ and the ion-acoustic speed $c_s = (zK_BT_e/m)^{1/2}$, respectively. The ES potential $\phi$, $\eta$ and $\nu$ are scaled by $k_BT_e/e$, $\omega_{p,+}$ and $\lambda^2_{D,+}\omega_{p,+}$, respectively. Thermal effects have been neglected in the pair components, for simplicity.

**Linear wave analysis.** Linearizing and Fourier transforming ($\sim e^{i(kx-\omega t)}$), one finds the dispersion relation: $\Gamma_3\omega^3 + i\Gamma_2\omega^2 + \Gamma_1\omega + i\Gamma_0 = 0$, where $k$ is the wavenumber, $\omega$ is the frequency, and: $\Gamma_0 = -k^4\eta(1+\delta)$, $\Gamma_1 = -k^2[1+\delta+c_1\eta(1+k^2-\delta)(k^2\eta+2\nu)]$, $\Gamma_2 = 2c_1(k^2\eta+\nu)(1+k^2-\delta)$ and $\Gamma_3 = c_1(1+k^2-\delta)$. Omitting dissipation for a minute, this leads to $\omega^2 = \frac{k^2(1+\delta)/c_1}{1-\delta+k^2}$. This is an acoustic mode, sustained by the +/- component asymmetry (see that $\omega \to \infty$ if $\delta \to 1$).

**Nonlinear wave analysis.** In order to study the small amplitude nonlinear ion acoustic wave in dissipative multicomponent media, the reductive perturbation technique has been employed [8] and the following stretched coordinates has been introduced: $\xi = \varepsilon^{\frac{1}{2}}(x - Vt)$ and $\tau = \varepsilon^{\frac{3}{2}}t$, where $V$ is the phase velocity (to be determined) and $\varepsilon$ ($\ll 1$) is a small (real) expansion parameter. The state variables appearing in Eqs. (1-5) are expanded around their equilibrium values as $n_j = 1 + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \varepsilon^3 n_j^{(3)} + \ldots,$, $u_j = \varepsilon u_j^{(1)} + \varepsilon^2 u_j^{(2)} + \varepsilon^3 u_j^{(3)} + \ldots$ and $\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \ldots$. For the sake of analytical tractability, we we shall also assume that $\nu = \varepsilon^{\frac{3}{2}}\nu_0$ and $\eta = \varepsilon^{\frac{1}{2}}\eta_0$ – where $\nu_0$ and $\eta_0$ are $\simeq O(1)$ – suggesting that collisionality and viscosity related timescales are much slower that the plasma period. Combining into Eqs. (1)-(5) and considering different powers of $\varepsilon$, a tedious but straightforward algebraic procedure leads to a *hybrid Korteweg - de Vries/Burgers* (KdV-B) equation in the form:

$$\frac{\partial \psi}{\partial \tau} + A\psi \frac{\partial \psi}{\partial \xi} + B\frac{\partial^3 \psi}{\partial \xi^3} = C\frac{\partial^2 \psi}{\partial \xi^2} - D\psi, \tag{6}$$

in terms of the leading potential disturbance $\phi^{(1)} = \psi$. The coefficients in this equation read:

$$A = \frac{(2c_2V^4 - 3)(-1+\delta)}{2V(1+\delta)}, \qquad B = \frac{V^3}{2(1+\delta)}, \qquad C = \frac{\eta}{2}, \qquad D = \nu, \qquad V^2 = \frac{(1+\delta)}{c_1(1-\delta)}.$$

We note that, in the limit $\delta \to 1$ (for *symmetric p.p.*, i.e., in the absence of 3rd species), the phase speed $V \sim (1-\delta)^{-1}$ diverges, while $A$ and $B$ diverge like $A \sim \sqrt{2}c_2 c_1^{-3/2}(1-\delta)^{-1/2}$ and $B \sim (2c_1^3)^{-1/2}(1-\delta)^{-3/2}$. The model therefore collapses for symmetric ("pure") pair plasmas, and no excitations may then be sustained, as predicted earlier by Verheest [9]. We note that, as expected, $V = \lim_{k\to 0}(\omega/k)$ essentially denotes the true sound speed in asymmetric *p.p.* ($\delta \neq 1$).

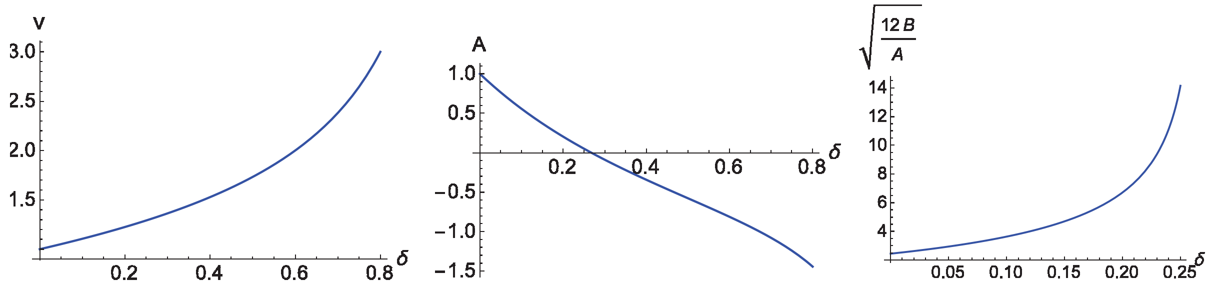


Figure 1: Left panel: the phase speed (scaled by $(zk_BT_e/m)^{1/2}$) is depicted. Middle panel: The nonlinearity coefficient $A$ in Eq. (6) is shown. Right panel: the factor $L(0)\,\psi(0)^{1/2} = (12B/A)^{1/2}$, representing the pulse width $L(0)$ for fixed amplitude $\psi(0)$ (cf. the analytical pulse solution below) is depicted. The $x-$axis represents the density ratio $\delta = n_{-0}/n_{+0}$ in all 3 plots.

A number of comments are in row, considering special cases. First of all, *in the absence of dissipation* ($\nu = \eta = 0$, or $C = D = 0$), one recovers the KdV (pulse) soliton:

$$\psi(\xi,\tau) = \psi_0(0)\,\mathrm{sech}^2\left(\frac{\xi - U(0)\tau}{L(0)}\right) \tag{7}$$

where the pulse velocity and width are given by $U(0) = A\psi_0(0)/3$ and $L(0) = \{12B/[A\psi_0(0)]\}^{1/2}$. We note that, for a higher negative-ion concentration (i.e., increasing $\delta$), solitary waves are faster and wider (see Fig. 1a, c), for fixed maximum amplitude $\phi(0)$. Importantly, $A$ (and thus $\psi(0)$) changes sign (suggesting polarity reversal) at a critical value of $\delta \approx 0.268$ (see Fig. 1b). *In the absence of viscosity* ($\eta = 0$, i.e. $C = 0$, $D \neq 0$), simple perturbative analysis leads to a (time-dependent) damped soliton solution of the dissipative KdV Eq. (6) in the form [10]

$$\psi(\xi,\tau) = \psi_0(\tau)\,\mathrm{sech}^2\sqrt{\frac{A\psi_0(\tau)}{12B}}\left(\xi - \frac{A}{3}\int_0^\tau \psi_0(\tau')d\tau'\right) \tag{8}$$

where the (decaying) amplitude is $\psi_0(\tau) = \psi_0(0)\exp\left(-\frac{4D}{3}\tau\right)$, the (decreasing) velocity is $U(\tau) = \frac{A\psi_0(0)}{3}\exp\left(-\frac{4D}{3}\tau\right)$, and the width of the (spreading) pulse is $L(\tau) = \sqrt{\frac{12B}{A\psi_0(0)}}\exp\left(\frac{2D}{3}\tau\right)$.

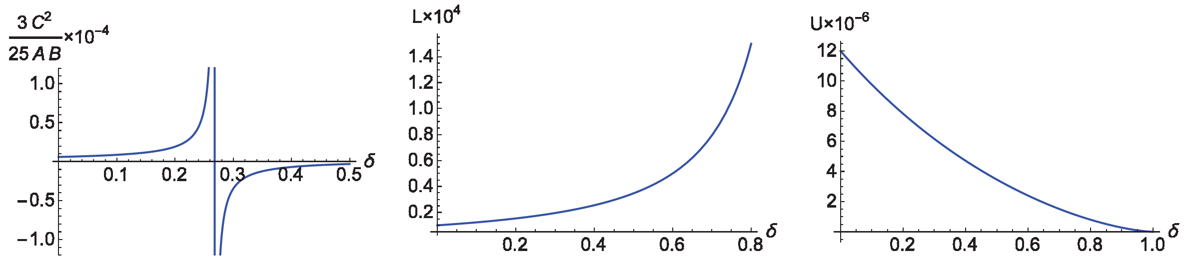


Figure 2: The structural features of the shock solution (9) are shown (left panel: the amplitude $(3C^2/(25AB))$; middle panel: the speed $U$; right panel: the width $L$) versus the density ratio $\delta$.

*In the absence of collisions* ($\nu = 0$, i.e. $D = 0$, $C \neq 0$), shock-type solutions exist [11]

$$\psi(\xi,\tau) = \frac{3C^2}{25AB}\left\{4 - \left[1 + \tanh\left(\frac{\xi - \mathrm{U}\tau}{\mathrm{L}}\right)\right]^2\right\}, \tag{9}$$

with speed $U = 6C^2/(25B)$. The shock width is $L = 10B/C$. Higher negative-ion concentration (larger $\delta$) leads to wider and slower shocks; see Figs. 2b,c. A polarity switch is also observed here, as pointed out above, at the root of $A$: see Fig. 2a. Interestingly, for $\delta \to 1$ (near-symmetric p.p.), the shock width reduces to nil, while its width spreads infinitely: physically speaking, charge neutrality (in the symmetric case) collapses the shock.

Our investigation should yield a richer dynamical profile in the warm (+/-) fluid model.

**Acknowledgments.** I.S.E. is financially supported from an Egyptian Government fellowship.

## References


[1] G. P. Zank and R. G. Greaves, Phys. Rev. E **51**, 6079 (1995); N. I. Iwamoto, Phys. Rev. E **47**, 604 (1992).

[2] H. Boehmer, M. Adams and N. Rynn, Phys. Plasmas **2**, 4369 (1995).

[3] G. Sarri *et al*, Nat. Commun. **6**, 6747 (2015).

[4] W. Oohara and R. Hatakeyama, Phys. Rev. Lett. **91**, 205005 (2003).

[5] S. Ghosh, A. Adak, and M. Khan, Phys. Plasmas **21**, 012303 (2014).

[6] A. Adak, S. Ghosh, and N. Chakrabarti, Phys. Plasmas **22**, 102307 (2015).

[7] B. Atamaniuk and A.J. Turski, AIP Conf. Proc. **1397**, 259 (2011).

[8] H. Washimi, and T. Taniuti, Phys. Rev. Lett.**17**, 996 (1966).

[9] F. Verheest, Phys. Plasmas **13**, 082301 (2006).

[10] S. Sultana and I. Kourakis, Phys. Plasmas **22**, 102302 (2015).

[11] I. Kourakis , S. Sultana, and F. Verheest, Astrophys. Space. Sci. **338** 245 (2012).