Published in: Fractals, Volume: 18, Issue: 1 (2010) pp. 101-110
DOI: 10.1142/S0218348X10004683

FUNDAMENTAL STATISTICAL FEATURES AND SELF-SIMILAR PROPERTIES OF OIL PRICE

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Abstract. The stochastic nature of oil price fluctuations is investigated over a twelve-year period, borrowing feedback from an existing database (USA Energy Information Administration database, available online). We evaluate the scaling exponents of the fluctuations by employing different statistical analysis methods, namely rescaled range analysis (R/S), scale windowed variance analysis (SWV) and the generalized Hurst exponent (GH) method. Relying on the scaling exponents obtained, we apply a rescaling procedure to investigate the complex characteristics of the probability density functions (PDFs) dominating oil price fluctuations. It is found that PDFs exhibit scale invariance, and in fact collapse onto a single curve when increments are measured over microscales (typically less than 10 days). The time evolution of the distributions is well fitted by a Lévy-type stable distribution. The relevance of a Lévy distribution is made plausible by a simple model of nonlinear transfer. Our results also exhibit an intermittent multifractal scaling in the higher-order statistics.

Key words. Horst exponent, Self-similar, Fractality, Lévy distribution

AMS subject classifications. 37Fxx, 28A80, 91B84

1. Introduction. A wide range of dynamical phenomena, such as turbulence flows, financial stock market fluctuations, seismic activity, internet traffic, climate change, etc., are characterized by randomness, or stochasticity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The analysis of non-stationary stochastic processes, involving quantities which fluctuate widely and are random-valued, has long been identified as a problem of fundamental interest. Fossil oil (petrol) market is a particularly good example of a random system whose (pricing) fluctuation bears a significant socio-political impact in modern society. Over the past two decades, oil price has increased sharply, rising from \$25 per barrel in January 1986 to a peak of close to \$122 per barrel in the last week of July 2008, yet featuring a significant degree of abruptness and erratic behaviour in shorter time windows (intervals). The effects of oil price fluctuations on the world economy are undeniable and particularly evident from global reports. Oil price data as a time series is a highly nonlinear system which exhibits complex patterns.

The behavior of oil price fluctuations can be efficiently modelled by standard statistical-analytical models, such as the Ising model, earlier proposed for stock-price fluctuations [11], or the cascade model developed on fractal concepts, which was employed in hydrodynamics and magnetohydrodynamic turbulence [12, 13]. In the following, we employ the cascade technique to characterize the statistical properties of oil price time series, taking into account (and distinguishing among) the self-similarity and multi-fractality features arising in our time series of interest. Our model is based on two-point increments of oil price, which provide a comprehensive and scale-dependent characterization of the statistical properties of the system via an associated probability density function (PDF). It is necessary to stress that the data series is represented by a finite number of records which do not constitute a stationary process. The effect of non-stationarity on the detrended fluctuation analysis has been investigated in Ref. [14]. The detrended fluctuation analysis (DFA) method intro-

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duced by Peng et al. [15] has became a widely-used technique for the determination of (multi-)fractal scaling properties and the detection of long-range correlations in noisy, non-stationary time series [14, 15]. We ought to keep in mind that, if the aim is to analyze fractality properties of fluctuations, one need not necessarily consider a stationary time series. The fact is that stationary stochastic systems often show scaling in a statistical sense, in consistence with non-Gaussian leptokurtic (heavy-tailed) statistics. Such a distribution is characterized by an enhanced probability of large events, in comparison with a Gaussian behavior. Importantly, once the characteristic scaling exponents have been identified, one is able to interpret and estimate the behavior of the fluctuations, as well as to detect long-range correlations.

Numerous studies indicate the possibility that a stochastic time series may exhibit self-similarity (and/or self-affinity) at short time scales, and yet this property may break down at longer times. Such a behavior is efficiently modeled by a statistical distribution with a truncated tail. A self-similar Brownian walk with Gaussian PDF, which has a scaling exponent $1/2$, is a good example of a process which has independent increments over all temporal scales. Upon estimating the scaling exponents of the fluctuations via different methods, one finds that the results are in agrement with scaling exponents as determined through computing structure functions. It is often stated that the fluctuations are self-similar (monofractal) if the scaling exponents depend linearly on the order of moments. Nevertheless, a nonlinear dependence points towards multifractal scaling, which is a signature of the intermittent structure of oil price fluctuations.

In this paper, we employ data from an exact real oil-price database [16] to study oil price fluctuations over two distinct time windows/regimes, thus distinguishing among a "micro-" and a "macroscopic" regime. At micro scales (typically shorter than 10 days), fluctuations are self-similar and exhibit characteristic scale similarity properties. This suggests that they obey similar physical laws. As a matter of fact, a similar behavior has also been found in hydrodynamic and magnetohydrodynamic (MHD) turbulence, which is associated with lack of any characteristic spatial scale within the inertial range, as was predicted by Kolmogorov [13, 17, 18, 19]. On the other hand, the characteristics of macro scales (typically larger than 10 days) appear to be uncorrelated and converge towards a Gaussian distribution.

Our results, to be exposed below, suggest that there exist various distributions which can be fitted to the oil price fluctuations. However, the most obvious difference between the different models involves the wings of the distribution. Distinguishing among random processes by comparing the distribution wings can be quite difficult because data sets are limited. According to the Generalized Central Limit Theorem (GCLT) which can be surprisingly robust when it comes to the condition of independent and identically distributed random variables, a Lévy process is proposed [20]. Lévy processes have been identified for example in biological systems [21], financial markets [3, 22], and physical systems [23, 24]. We find that the oil price fluctuations are remarkably well described by a Lévy stable symmetric distribution, exception made for most rare events.

This article is structured as follows. In Section II we describe our data set. In Section III, we review the rescaled range analysis (R/S) and the scale windowed variance analysis (SWV). In Section IV we employ a recently developed technique [13, 17, 18] that sensitively distinguishes between self-similarity and multifractality in a time series. Moreover, the scaling exponent of oil price fluctuations is computed by using generalized structure functions and the peaks of PDFs, and the results are

TABLE 2.1
Mean value, standard deviation, skewness, and kurtosis of the oil price increments.

Mean	Standard Deviation	Skewness	Kurtosis
0.0124274	0.729173	-0.606749	9.55225

thus compared with estimated values in Section IV. The micro-scale PDF resembles a leptokurtic Lévy distribution which will be discussed in Section V. Finally, in Section VI we summarize our results.

2. The data. Over a twenty-two-year period, approximately, the price of oil has increased from $25 per barrel in January 1986 to a peak of close to $ 122 per barrel in the last week of July 2008. Oil price, as recorded in international markets [16], offers us a unique possibility to gain information on the stochastic dynamics state in a very large scale range, say from one day up to 200 days. Fig. 6.1 presents the daily fluctuations in oil price $p(t)$ in the period 1986-2008. It is evident from the figure that the fluctuations do not constitute a stationary process; for instance, one may show that the variance of the signal in some window does not remain stable upon increasing the window size. Let us introduce the increments $\delta p(t,\tau)$ defined by, $\delta p(t,\tau) = p(t+\tau) - p(t)$. The resulting series for $\delta p(\tau)$ is shown in the inset graph of Fig. 6.1. It is straightforward to show, by measuring the variance of $\delta p(t,\tau)$ in a moving window, that $\delta p(t,\tau)$ is stationary. Upon initiating the analysis of the distribution of oil price increments, the mean, standard deviation, skewness, and kurtosis of the return series are calculated (see Table 2.1). Throughout this paper we have used day as the unit of time.

The skewness of a Gaussian distribution is zero, therefore the negative value of skewness, here $\lambda = -0.606749$, is viewed as a hallmark of departure of the PDF from the Gaussian distribution and may thus here be attributed to a stable Lévy distribution. On the other hand, the large value of kurtosis, $\kappa = 9.55225$, with respect to Gaussian kurtosis ($\kappa = 3$), suggests that the tails of the return distribution are fatter than the Gaussian ones, which confirms the existence of intermittency in the fluctuations.

3. Scaling analysis.

3.1. Hurst's Rescaled-Range Analysis. The first method for analysis of long records in time series based on random walk theory has been proposed by Harold Edwin Hurst(1880-1978) [25]. Hurst found that the ratio (R/S) is very well described for a large number of natural phenomena by the following empirical relation

$$R/S \sim \tau^H \,, \tag{3.1}$$

where τ and H are defined as the time span and the Hurst exponent, respectively. For our oil price time series, we define R and S by the following steps.
We first divide the profile $(p_k), k = 1, ..., N$ into $N_s \equiv int(N/s)$ non-overlapping segments of size s. In the second step, the profile (integrated data) is calculated in each segment $\nu = 1, ..., N_s - 1$, as

$$Y_\nu(j) = \sum_{i=1}^{j} (p_{\nu s+i} - \langle p_{\nu s+i}\rangle_s) = \sum_{i=1}^{j} p_{\nu s+1} - \frac{j}{s}\sum_{i=1}^{s} p_{\nu s+1} \,. \tag{3.2}$$

Piecewise constant trends in the data are subsequently eliminated upon a simple subtraction of the local average. In the third step, the difference between the minimum and maximum values, R_ν, and the standard deviations S_ν in each segment are calculated, viz.

$$R_\nu(s) = \mathrm{Max}\{Y_\nu(s)\} - \mathrm{Min}\{Y_\nu(s)\} \quad \text{and} \quad S_\nu(s) = \left(\frac{1}{s}\sum_{j=1}^{s} Y_\nu^2(s)\right)^{1/2}. \tag{3.3}$$

Finally, the rescaled range is averaged over all segments to obtain the fluctuation function R/S,

$$(R/S)_s = \frac{1}{N_s}\sum_{\nu=0}^{N_s-1}\frac{R_\nu(s)}{S_\nu(s)} \sim s^H. \tag{3.4}$$

The R/S analysis can be viewed as a special form of coarse-graining for a time series. However, the method was developed long before the *fractality* concept. After having calculated R/S values for a large range of different time horizons s, we plot $\log(R/S)_s$ against $\log s$. By performing a linear least-square fit, we find the slope of the curve which is our estimate of the Hurst exponent H. The Hurst exponent H and the fractal dimension D_f are related as $D_f = H - 2$ [26]. The Hurst exponent is called the scaling exponent or correlation exponent, and its value depends on the correlation properties of the signal. If $H = 0.5$, there is no correlation and the signal is an uncorrelated signal; if $H < 0.5$, the signal is anticorrelated, while if $H > 0.5$, there is positive correlation in the signal. Fig. 6.2 presents a log-log plot of the rescaled range (R/S) fluctuations as a function of s, which results to the value $H = 0.65 \pm 0.029$. This shows that all daily price changes are correlated with future daily price changes.

3.2. Scale Windowed Variance Analysis. The Scaled Windowed Variance (SWV) analysis was developed by Cannon et al. (1997) [27]. For oil price time series, (p_k), we define the profile $Y(i)$ as

$$Y(i) = \sum_{k=1}^{i}(p_k - \langle p\rangle). \tag{3.5}$$

The profile $Y(i)$ is divided into $N_s \equiv int(N/s)$ non-overlapping segments of equal lengths s. The standard deviation is then calculated within each interval by using

$$SWV(s) = \left(\frac{1}{s}\sum_{i=1}^{s}[Y(i) - \langle Y(s)\rangle]^2\right)^{1/2}. \tag{3.6}$$

The average standard deviation of all time intervals of length s is computed. This computation is repeated over all possible interval lengths. The scaled windowed variance is related to s by a power law

$$SWV \sim s^H, \tag{3.7}$$

where H is well-known Hurst exponent. In Fig. (6.3) , we plot in double-logarithmic scale the corresponding SWV fluctuations against the coarse-graining s. Using the above procedure, we obtain the following estimate for the Hurst exponent: $H = 0.68 \pm 0.022$. Since $H > 0.5$, it is concluded that the oil price fluctuations show persistence; i.e., strong correlations between consecutive changes.

4. Statistical self-similarity. A set of time series $\delta p(t, \tau)$ is obtained for each time lag τ. The return of the stochastic variable $\delta p(t, \tau)$ is said to be self-similar with parameter α ($\alpha \geq$), if for any λ

$$\delta p(\tau) \stackrel{EL}{=} \lambda^{\alpha} \delta p(\lambda\tau). \tag{4.1}$$

The relation (4.1) is interpreted as an *equality in law* (EL), that is the two sides of the equation have the same statistical properties. For the associated cumulative probability distribution $\wp$, it follows that

$$\wp(\delta p(\tau) \leq \rho) = \wp(\lambda^{\alpha} \delta p(\lambda\tau) \leq \rho)\,, \tag{4.2}$$

for any real ρ. This implies for the probability density P

$$P[\delta p(\tau)] = \lambda^{-\alpha} P_s[\lambda^{-\alpha} \delta p_s]\,, \tag{4.3}$$

introducing the master PDF P_s with $\delta p_s = \delta p(\lambda\tau)$. According to Eq. (4.3), there is a family of PDFs which may collapse to a single curve P_s, if α is independent of τ. This is known as monoscaling, in contrast to multifractal scaling often observed. To characterize quantitatively the observed stochastic process, we measure $P(\delta p)$ of the price fluctuations for different τ, i.e. in an interval whose breadth ranges between 1 day up to 200 days. In Fig.(6.4) we show the PDFs (normalized with the variance $\langle \delta p(\tau)^2 \rangle^{1/2}$) for various τ, including micro and macro time scale. A simple comparison to a Gaussian illustrates the highly non-Gaussian nature of the tails of the PDFs over the micro scales, as shown in the left inset panel of Fig. 6.4. For $\tau \leq 10$ days, the distributions expand in an increasing manner without essentially much changing in form, as we can see in Fig.(6.5). The similarity of the $P(\delta p)$ on the micro scales suggests the possibility of monofractality. The monofractality exponent is expected to be scale-independent at the micro scales. Consequently, this allows us to apply rescaling procedure given by Eq.(4.3) over the micro time scales. The distributions lose their leptokurtic shape and converge toward a Gaussian distribution as τ increases, i.e. for $\tau > 10$. We can see in the right inset panel of Fig.(6.4) that a Gaussian curve fits well to $P(\delta p)$ for $\tau = 200$ days. It is worthwhile to note that, for experimental data, an approximate collapse of the PDF is an indicator of a dominant self-similarity trend in the time series, i.e., this method may not be sensitive enough to detect monofractality, which might in fact manifest itself only during short time intervals.

The above-mentioned observations can be a hallmark of statistical intermittency over all time scales. Accordingly, the scaling behavior of the distributions at different time scales present two different regimes. At micro-scales, correlations between successive price changes are dominant. Interestingly, the PDFs at micro-scales, which show similarity, seem to obey the same physical mechanism. On the other hand, long time effects on the oil price seem to indicate a Gaussian distribution for the fluctuations. This is a trace of multifractality in the evolution of oil price. It is remarkable that the micro- time scale regimes can lead to a linear scaling dependence while the entire range may just as well present a nonlinear dependence.

For a quantitative comparison, we fit the tail of the PDFs to the following function as [24]

$$P(|\delta p|) \sim \exp(-A|\delta p|^{\mu})\,, \tag{4.4}$$

with an exponent μ, which describes the distribution shape at the wings. As a result, numerical analysis on positive fluctuations implies that good fits can be obtained in the wings of PDFs and in the interval $[10\sigma, 15\sigma]$, where σ is standard deviation the data. Fig. (6.6) shows the dependence of the exponent μ on τ for PDFs of oil price fluctuations. The exponent μ changes as τ increases. This is a signature of the presence of multifractality in the stretched tails of PDFs. As expected, at large scales, μ remains close to a value around 2 in which the PDFs collapse to a Gaussian distribution.

Let us now consider the scaling as defined by the structure functions. The generalized structure function of order n is simply defined as

$$S^n(\tau; \pm\infty) = \langle |\delta p|^n \rangle = \int_{-\infty}^{+\infty} |\delta p|^n P(\delta p, \tau) d(\delta p). \tag{4.5}$$

The analysis which follows is also valid for the moments; however, structure functions are typically calculated for a data series. The arguments do not apply to structure functions of odd order, which not only may have negative coefficients, but could also even change sign of the scaling range. The proof will, however, remain valid for odd orders when the structure functions are defined with the absolute value of the increments. Using the relation (4.3) we obtain

$$S^n(\tau; \pm\infty) = \lambda^{\zeta_n} S_s^n(\delta p_s; \pm\infty)\,, \tag{4.6}$$

where the linear function $\zeta_n = \alpha n$ reflects the statistical self-similarity, in the monoscaling case. On the contrary, in some cases, one may observe a multifractal scaling, in the sense that a nonlinear dependence is observed on n where $\zeta_n = n\alpha(n)$ is a convex function of n and $\zeta_{n+1} > \zeta_n \quad \forall n$. This deviation from strict self-similarity over all time scales τ, also termed multifractal scaling, is caused by the intermittent structure of turbulence.

To test if the above-mentioned interesting observations in oil price are a phenomenon related to inherent properties of stochastic processes, structure functions $S^n(\tau)$ for different τ, given by Eq. (4.5), are computed. A difficulty that can arise in the experimental determination of the ζ_n is that for a finite length times series, the integral Eq. (4.5) is not sampled over the range $(-\infty; +\infty)$, rather the outlying measured values of y determine the limit, $[-y; +y]$. The structure functions for the data interval studied here are shown in Fig. (6.7). On this *log-log* plot the slopes as shown give estimates of the scaling exponents, ζ_n. Importantly, the higher-order structure functions progressively capture the more intermittent, larger fluctuations. The micro and macro ranges of scaling are well-defined with a sharp transition at the break point at ~ 10 days. We plot ζ_n vs. n for the micro range in the main panel of Fig. (6.8), and for the macro range in the inset. Surprisingly, the micro range is monoscaling i.e. globally scale-invariant; in contrast to the macro range which is multifractal.

To apply the rescaling procedure given by Eq. (4.3) the exponent α is extracted from the underlying data by two independent technique[17, 13]. First, the standard deviation which is defined by the root of the second-order structure function, $\sigma(\tau) = [S^2(\tau)]^{1/2}$ and has the minimum of statistical error, exhibits power-law behavior with respect to the increment distance, $\sigma(\tau) \sim \tau^{\alpha}$ as depicted in Fig. 6.9. A linear least-square fit is carried out to obtain α. The characteristic exponent deduced in this way is $\alpha = 0.66 \pm 0.065$, which is called the *generalized Hurst exponent* (GH). Second, in the micro scales, $\tau \leq 10$, the characteristic exponents can be obtained via the amplitude of $P(0, \tau) \sim \tau^{-\alpha}$, to profit from the fact that the peaks of the PDFs are statistically the least noisy part of the distributions. The logarithmic plot of $P(0, \tau)$ versus τ is

Table 4.1
The values of the Hurst exponent for an oil price time series, as obtained via different approaches.

R/S	SWV	GH	$P(0, \delta\tau)$
0.65 ± 0.029	0.68 ± 0.022	0.66 ± 0.065	0.66 ± 0.012

similar to Fig. 6.9, and is omitted here. The scaling exponent obtained by using this method is in good agreement with the value of α obtained via the PDF variance and values estimated in Section III (see Table 4.1). Fig. 6.10 shows the rescaled PDFs according to Eq. (4.3) over the micro scales. As expected, the PDFs collapse for up to 3σ with weak scattering on the master PDF, P_s, when using the characteristic exponents given above. These rescaled PDFs are leptokurtic rather than Gaussian and are thus strongly suggestive of an underlying nonlinear process. We may model this PDF by a Lévy distribution, which thus turns out to be a successful fit to the distribution of oil price fluctuations. On the other hand, the PDFs over all time scales do not collapse onto a single curve when rescaling Eq. (4.3) is applied (see Fig.(6.11)). The lack of monoscaling is evident and indicates a multifractal process.

5. Lévy distribution model. Lévy stable laws are a rich class of probability distributions (non-Gaussian) that comprise heavy tails and have many intriguing mathematical properties. They have been proposed as models for many types of physical and economic systems which exhibit heavy tails. A Lévy process is a time-dependent or position-dependent process that at an infinitesimal interval has the Lévy distribution of the process variable. The characteristic function of the Lévy process is

$$K_\mu(q, s) = \exp(-cs \mid q \mid^\mu) \,, \tag{5.1}$$

where s can be a characteristic time or space scale and $\alpha = 1/\mu$ is equal to Hurst exponent. If $\mu = 2$ the Lévy collapses to the Gaussian distribution. If $\mu = 1$ the Lévy becomes a Cauchy distribution. The original Lévy process is given by its inverse Fourier transform, i.e.

$$P_\mu(x, s) = \int dq e^{iqx - cs|q|^\mu} \,, \tag{5.2}$$

and the symmetric Lévy distribution becomes

$$L_\mu(\delta x_{\Delta s}) \equiv \frac{1}{\pi} \int_0^\infty \exp(-\gamma \Delta s q^\mu) \cos(q \delta x_{\Delta s}) dq \,, \tag{5.3}$$

where the increment is $\delta x = x_s - x_{s-\Delta s}$; here, $0 < \mu < 2$, and $\gamma > 0$ is a scale factor. The maximum event probability leads to

$$P(0) = L_\mu(0) = \frac{\Gamma(1/\mu)}{\pi \mu (\gamma \Delta s)^{1/\mu}} \,. \tag{5.4}$$

The exponent μ of the best fits is constant at the micro time scales and amounts approximately to $\mu \sim 1.51$ or $\alpha = 0.66$ (see Fig. 6.9). Similar findings have been reported, for example, in financial systems, e.g. the Tehran price stock market, where $\mu \sim 1.36$ [2], and in physical systems such as the solar wind, where $\mu \sim 3.3$ [23]. From Fig. 6.10, we conclude that the central region of the distribution is well described by Lévy stable distributions. On the other hand, the tail of distributions deviates from a

Lévy stable distribution and is approximately exponential as discussed at the previous section, ensuring that the variance of the distribution is finite. These observations might at first sight seem to contradict the Lévy process which has an infinite variance for $\mu < 2$. But, there is no contradiction, as a recent study finds that the Lévy process may hold over a long period of time for dynamics of "quasi-stable" stochastic processes having a finite variance [28].

6. Summary. In this paper, we have presented a statistical analysis of oil price fluctuations for the period of January 1986 to July 2008. For oil price time series we have obtained a Hurst exponent greater than 0.5, indicating that the series has long term dependence (persistence). However, the scaling properties that we have found via analyzing of the PDFs allow us to detect the occurrence of mono-(multi-) fractality features in the distribution. Oil price fluctuations are found to be self-similar, and exhibit a leptokurtic nature for micro time scales, $\tau \leq 10$. Fluctuations on the macro temporal scales, $\tau > 10$, are uncorrelated, in that their PDFs converge toward a Gaussian distribution. We found that the PDFs have exponential tails and the associated exponent μ is not constant as time scale τ change, ranging from micro to macro time scales. This behavior can be interpreted as an indication of the presence of multifractality in the system. We have also obtained a good collapse, according to the rescaling procedure (4.3), onto a single curve over at least three standard deviations for micro scales. The closeness of the PDFs to Lévy stable distributions is made plausible by a simple model mimicking nonlinear spectral transfer.

Acknowledgments. This work was supported by a grant from Shahrood University and Technology. M.M. would like to thank the University for this financial support. The work of IK was supported by the UK EPSRC via a Science and Innovation award (to Centre for Plasma Physics, Queen's U. Belfast).

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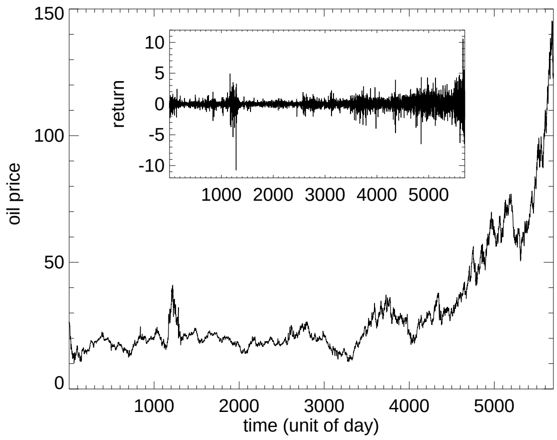

FIG. 6.1. *Semi-log plot of oil price series over the period 1986-2008. Inset: daily return of the oil price index.*

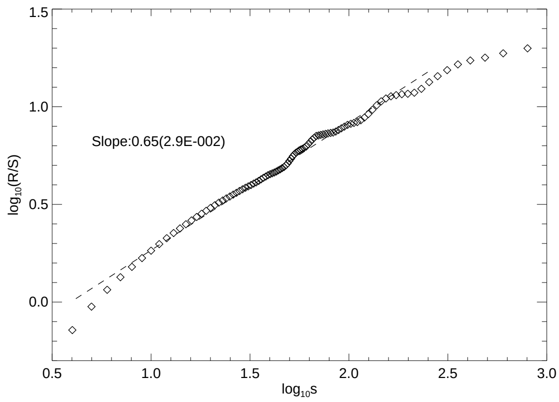

FIG. 6.2. *Rescaled Range* (R/S) *of fluctuations versus box size* s*.*

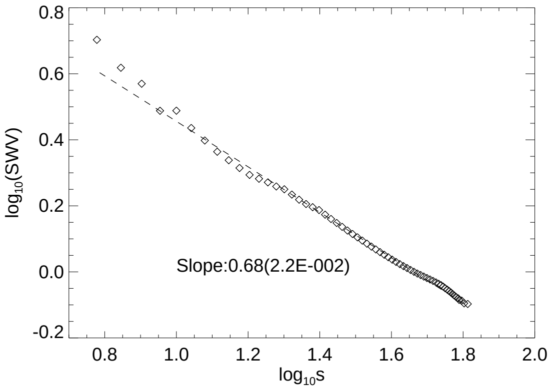

FIG. 6.3. *Behavior of Scaled Windowed Variance (SWV) of fluctuations as a function of s.*

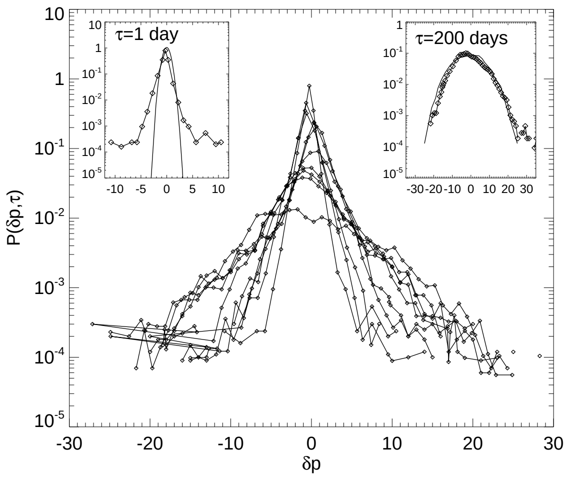

FIG. 6.4. *The PDFs of oil price fluctuations δp for various τ; here, $\tau = 1, 10, 20, 40, 60, 80, 120, 140, 160$ or 200 days. Examples of the PDFs for the smallest ($\tau = 1$ day) and largest ($\tau = 200$ days) time lags are shown in the left and right inset panels, respectively. The solid line is a Gaussian PDF for comparison.*

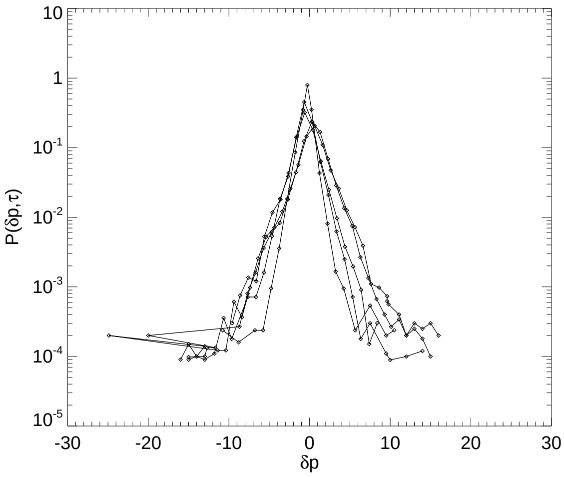

FIG. 6.5. *The PDFs of oil price fluctuations at micro scales $\tau \leq 10$. We can see that the shape of the PDFs does not change fundamentally as a result of monofractality.*

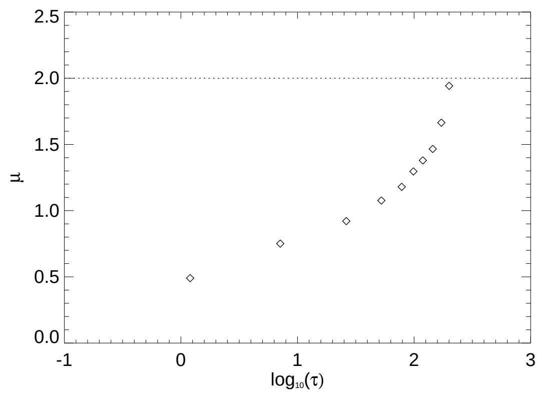

FIG. 6.6. *Exponent μ of the positive tails of the PDFs for different time scales, $\tau = 1, 10, 20, 40, 60, 80, 120, 140, 160, 200$ days.*

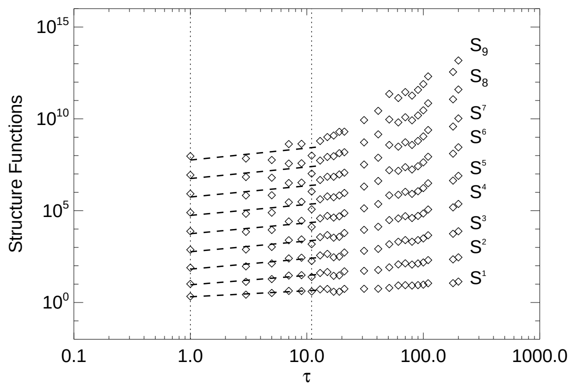

FIG. 6.7. *The structure functions are depicted, as computed from Eq. (4.5). In order to obtain the scaling exponents, we consider the logarithmic slope via a sequence of linear least-square fits. The vertical dashed lines show the micro range.*

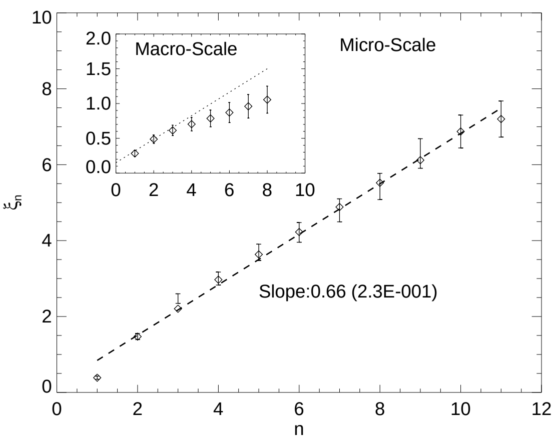

FIG. 6.8. *The scaling exponents ζ_n of the structure functions are depicted versus the corresponding order n. Main plot: a linear relationship between ζ_n and n on this plot indicates monoscaling behavior within the micro scales. Inset: ζ_n vs. n for the macro range, this is concave , consistent with the multifractal nature of the macro range.*

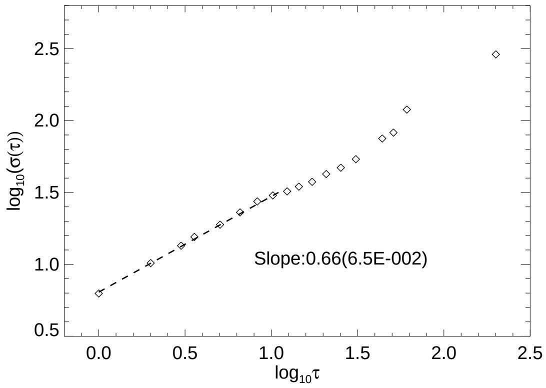

Fig. 6.9. *Standard deviation of oil price increments within the desired range. The dashed line shows a good fit over the micro scales, i.e. for* $\tau \leq 10$.

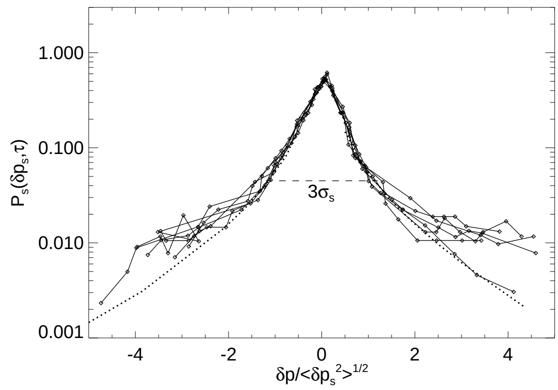

Fig. 6.10. *Rescaled PDFs of oil price fluctuations at micro-scales,* $\tau \leq 10$. *The Lévy distribution with* $\mu = 1.51$ *is represented(dashed line).*

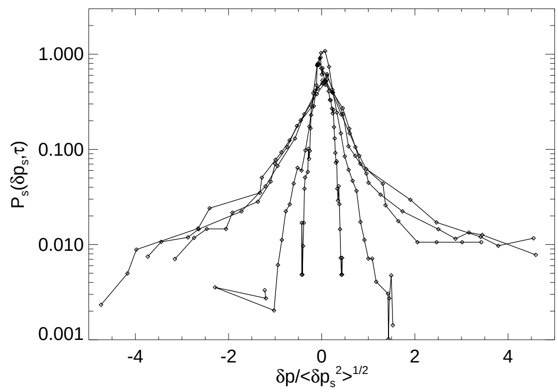

Fig. 6.11. *Rescaled PDFs of oil price fluctuations over some micro-macro time scales. This shows that the collapse of the PDF's to a single curve is broken.*