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Electromagnetic beam profile dynamics in collisional plasmas

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Abstract

The compression of a finite extent Gaussian laser pulse in collisional plasma is investigated. An analytical model is employed to describe the spatiotemporal evolution of a laser pulse propagating through the plasma medium. The pulse geometry is modeled via an appropriate ansatz which takes into account both beam radius (in space) and pulse width (in time). Compression and self-focusing are taken into account via appropriated group velocity dispersion and nonlinearity terms. The competition among the collisional nonlinearity in the plasma and the effect of divergence due to diffraction is pointed out and investigated numerically. Our results suggest that laser pulse compression and intensity localization is enhanced by plasma collisionality. In specific, a pulse width compression by an order of magnitude approximately is observed, for typical collisional laser plasma parameters, along with a significant increase in the intensity.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The interaction of ultraintense very short electromagnetic (EM) (laser) pulses with plasmas [1–7] has attracted a great deal of attention in relation with fundamental research and technological applications [8–13], such as particle acceleration, inertial confinement fusion, high harmonic generation and x-ray lasers [14]. The standard approach to produce an ultrashort, ultraintense multiterawatt laser pulse is the chirped-pulse-amplification (CPA)

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technique [15], in which a laser pulse is stretched, amplified and recompressed. The CPA scheme has shown the ability of generating subpicosecond petawatt laser pulses with up to 500 J per pulse. This approach is limited due to the finite bandwidth of the active mm amplifiers used in the lasers. A new scheme of parametric amplification to produce ultrashort and powerful pulses was proposed in [16]. The superradiant amplification of an ultrashort laser pulse was reported in [17] considering an electromagnetic beam colliding with a long counterpropagating low-intensity pump in the plasma.

Short pulses are of particular importance because they could lead to higher peak powers without increasing the laser pulse energy, e.g. without increasing the size and cost of the laser system. In the last few years, several scenarios have been proposed for the self-focusing [18] and self-compression [19] of a laser pulse in plasma. In a three-dimensional (3D) geometry, self-compression in the longitudinal direction competes with transverse self-focusing/filamentation. A 3D PIC simulation reported in [20] has shown that a 30 fs long laser pulse can be efficiently compressed to 5 fs by using a periodic plasma-vacuum structure to damp filamentation. A mechanism for (self-)compression of a single laser pulse, thanks to the interplay between relativistic self-phase modulation, group velocity dispersion and the presence of a high-amplitude nonlinear plasma, was demonstrated theoretically in [21–23]. A recent numerical investigation [24] has shown that microwave pulses can be compressed (with or without a frequency shift) inside magnetized plasma by changing the magnitude or the direction of the magnetic field uniformly in space and adiabatically in time.

An independent approach [25], cited here for completeness, relies on self-focusing of few-optical-cycle pulses. It was shown that the wave-field self-focusing proceeds by overtaking the steepening of the pulse longitudinal profile, leading to shock-wave formation. Consequently, a more complex singularity is formed where an unlimited field increase is followed by wave breaking with a broad power-law pulse spectrum. Furthermore, yet independently, relativistic nonlinearity effects on the evolution of the spot size and of the pulse length of a short laser pulse propagating through a thin uniform plasma lens have been investigated in [26] and elsewhere.

The propagation of an electromagnetic pulse in collisional plasmas may involve a variety of physical phenomena [27–30]. In collisional plasma, the focusing of electromagnetic beams is due to a non-uniform profile of the dielectric function on account of non-uniform ohmic heating and of the associated electron plasma diffusion. Elegant reviews of this phenomenon have been furnished by Sodha *et al* [31] and Sprangle *et al* [32]. A number of earlier works [19–24, 26] have focused on pulse compression in relativistic plasmas, in relation with short and ultraintense pulses. We shall adopt and extend here the earlier approach of [27–30] and [33, 34].

In this paper, we investigate the compression of a finite extent Gaussian laser pulse propagating through collisional plasma. We employ a trial function methodology to model the spatiotemporal beam profile. We rely here on the model introduced by Akhmanov [33] and later extended by Sodha and co-workers [34], advancing it to an efficient description of collision-dominated plasmas. The results obtained are novel and of relevance in laser-ignition experimental schemes. We show that plasma collisionality competes with the effect of pulse divergence due to diffraction. A numerical investigation suggests the possibility of compression and increase in the intensity of a laser pulse in a collision-dominated plasma. From a wider scope, the theory employed here is generic, so it may be adapted to other types of nonlinearities, to study different effects involved in the dynamics of short and intense pulses.

The paper is organized as follows. In section 2 we derive an analytical model for the beam width parameter (in space) and the pulse width parameter (in time). Following the Akhmanov approach [33] (also see [7]), the nonlinear Schrödinger equation is solved to obtain a set of two coupled nonlinear second-order ordinary differential equations (ODEs) for the pulse profile trial functions (pulse width and beam width parameters). The pulse width parameter measures the longitudinal compression of the pulse width in the plasma medium while the beam width parameter shows the transverse focusing of the pulse. In section 3, we derive an analytical form for the plasma dielectric function taking into account collisional nonlinearity. In section 4, the set of evolution equations for the pulse width parameter g and the beam width parameter f are solved numerically for g and f, along with the force and energy balance equations, allowing for an efficient characterization of the pulse profile in collisional plasma. In section 5, we conclude by summarizing our analytical model and the results of our numerical computation.

2. Analysis

We consider the propagation of an EM pulse of intensity

$$\boldsymbol{E}(r, z, t) = \boldsymbol{A}(r, z, t) \exp[-\mathrm{i}(\omega t - kz)] \tag{1}$$

in a plasma of uniform density n_0 (in the absence of the electromagnetic field). Considering a Gaussian envelope for the incident laser pulse, the irradiance is given by

$$EE^* = A_{00}^2 \,\mathrm{e}^{-t^2/\tau_0^2}\,\mathrm{e}^{-r^2/r_0^2}, \tag{2}$$

where $\boldsymbol{E}$ is the amplitude of the electric field, $\boldsymbol{A}_{00}$ is the axial amplitude of the beam, r_0 is the beam radius (in cylindrical coordinates) and τ_0 is the pulse width (in time).

The electric field satisfies the wave equation

$$\nabla^2 \boldsymbol{E} - \frac{\varepsilon}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0. \tag{3}$$

The dielectric function ε can be expressed as

$$\varepsilon = \varepsilon_0 + \phi(\boldsymbol{E}\boldsymbol{E}^*) \tag{4}$$

where

$$\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2} \tag{5}$$

where ω_p is the plasma frequency and ω is the frequency of the incident laser pulse.

Substituting for $\boldsymbol{E}$ from equation (1) in the wave equation, one obtains

$$2\mathrm{i}k\left(\frac{\partial \boldsymbol{A}}{\partial z} + \frac{1}{v_g}\frac{\partial \boldsymbol{A}}{\partial t}\right) + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \frac{\partial^2 A}{\partial z^2} + \frac{k^2\,\phi\,(AA^*)}{\varepsilon_0}\boldsymbol{A} = 0, \tag{6a}$$

where $v_g = \frac{c^2k}{\omega} = c\varepsilon_0^{1/2}$ is the group velocity of the pulse.

We will now introduce the dimensionless variables $\tilde{t} = (t - z/v_g)\omega_p$, $\tilde{z} = (\omega_p/c)z$, $\tilde{\boldsymbol{A}} = (e/m\omega_p c)\boldsymbol{A}$, $\tilde{k} = (c/\omega_p)k$, $\tilde{r} = (\omega_p/c)r$, so that equation (6a) is written in a dimensionless form as

$$2\mathrm{i}\tilde{k}\frac{\partial \tilde{\boldsymbol{A}}}{\partial \tilde{z}} + \left(\frac{\partial^2}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\right)\tilde{\boldsymbol{A}} + \beta\frac{\partial^2 \tilde{A}}{\partial \tilde{t}^2} - \frac{2}{\varepsilon_0^{1/2}}\frac{\partial^2 \tilde{A}}{\partial \tilde{z}\partial \tilde{t}} + \frac{\partial^2 \tilde{A}}{\partial \tilde{z}^2} + \frac{\tilde{k}^2\,\phi\,(AA^*)}{\varepsilon_0}\tilde{\boldsymbol{A}} = 0 \tag{6b}$$

where $\beta = \left(\frac{\omega^2}{\omega_p^2} - 1\right)^{-1} = \frac{\omega_p^2}{\omega^2}\varepsilon_0^{-1}$. The tilde, denoting dimensionless quantities in the latter relation, with henceforth be omitted everywhere. The second term in the above equation (within parenthesis) physically represents diffraction, which is necessary for transverse focusing. The third and fourth terms account for the finite pulse length effect. The third term is known as the group velocity dispersion (GVD) term and it results in pulse compression when combined with (balanced by) nonlinearity. The last term in equation (6b) represents nonlinearity, which is here due to collisional heating of plasma. Since the parallel variation of the amplitude $\boldsymbol{A}$ in terms of z is assumed to be slower than the transverse one (in r), therefore terms in $\partial^2/\partial\tilde{z}\partial\tilde{t}$ and $\partial^2/\partial\tilde{z}^2$ can be neglected.

Considering the above, equation (6*b*) can be simplified as

$$2\mathrm{i}k\frac{\partial \boldsymbol{A}}{\partial z} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\boldsymbol{A} + \beta\frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \frac{k^2\,\phi(AA^*)}{\varepsilon_0}\boldsymbol{A} = 0, \tag{7}$$

where contributions representing diffraction, dispersion and nonlinearity are obvious. The nonlinear Schrödinger equation (NLSE) (7) has been derived in various forms and via different physical approaches in earlier works [3, 18–20, 25, 26, 35, 36]. Here, we follow the paraxial beam profile approach, introduced by Akhmanov in [33] and later extended by Sodha and co-workers in [34].

In the paraxial approximation, the dielectric function can be expressed as

$$\varepsilon(r,z,t) = \varepsilon_0(z) - \left(r^2/r_0^2\right)\varepsilon_{2r} - \left(t^2/\tau_0^2\right)\varepsilon_{2t} = \varepsilon_0(z) + \phi(r,z,t) \tag{8}$$

where we defined

$$\phi(r,z,t) = -\left(r^2/r_0^2\right)\varepsilon_{2r} - \left(t^2/\tau_0^2\right)\varepsilon_{2t} = \phi_r + \phi_t.$$

Here $\varepsilon_0(z)$ is the linear part of the dielectric constant, ε_{2r} is the nonlinear transverse part of the dielectric constant and ε_{2t} is the nonlinear longitudinal part of the dielectric constant. In equation (8) the dimensionless form of beam width $\tilde{r}_0 = (\omega_p/c)r_0$ and pulse width $\tilde{\tau}_0 = \omega_p\tau_0$ is used. Again, tildes will be omitted below.

The solution of equation (7) can be written as

$$\boldsymbol{A}(r,z,t) = \boldsymbol{A}_0(r,z,t)\exp[-\mathrm{i}kS(r,z,t)], \tag{9}$$

where both amplitude A_0 and eikonal (phase) S are real quantities. Substituting for A from equation (9) in equation (7) and separating real from imaginary parts, one obtains

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \beta\frac{\partial S}{\partial t}\frac{\partial A_0^2}{\partial t} + \beta\, A_0^2\frac{\partial^2 S}{\partial t^2} = 0 \tag{10}$$

and

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 + \beta\left(\frac{\partial S}{\partial t}\right)^2 = \frac{\phi}{\varepsilon_0} + \frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\beta}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial t^2}\right). \tag{11}$$

We anticipate a solution for equations (10) and (11) in the form [31, 33, 34]

$$A_0^2 = \frac{A_{00}^2}{gf^2}\exp\left(-r^2/r_0^2f^2\right)\exp\left(-t^2/\tau_0^2g^2\right) \tag{12}$$

and

$$S(r,z,t) = S_0(z) + r^2S_{2r}(z) + t^2S_{2t}(z), \tag{13}$$

where $g = g\,(z)$ and $f = f\,(z)$ are trial functions, to be determined. Here f represents the beam width parameter, whose form determines the (variation of the) beam width as it propagates through plasma. In an analogous manner, the pulse width parameter g determines the pulse width profile (in time).

Substituting for A_0^2 and S from equations (12) and (13) in equation (10) and equating the coefficients of r^2 and t^2 on both sides of the resulting equations, one obtains

$$S_{2r} = \frac{1}{2f}\frac{\partial f}{\partial z} \tag{14a}$$

and

$$S_{2t} = \frac{1}{\beta}\frac{1}{2g}\frac{\partial g}{\partial z}. \tag{14b}$$

The same procedure, along with equation (8) for ϕ, leads from equation (11) to the coupled equations

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2 r_0^4 f^3} - \frac{\varepsilon_{2r}}{\varepsilon_0}\frac{f}{r_0^2} \tag{15a}$$

and

$$\frac{d^2 g}{dz^2} = \frac{\beta^2}{k^2 \tau_0^4 g^3} - \frac{\varepsilon_{2t}}{\varepsilon_0}\frac{g\beta}{\tau_0^2}, \tag{15b}$$

which govern the joint evolution of the beam width (space) and pulse width (time) parameters, f and g. These two coupled second-order ordinary differential equations given can be further expressed in terms of the dimensionless diffraction length $\left(R_d = k\, r_0^2\right)$ as

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{\varepsilon_{2r} f}{\varepsilon_0}\frac{R_d^2}{r_0^2} \tag{16a}$$

and

$$\frac{d^2 g}{d\xi^2} = \frac{\beta^2}{g^3}\frac{r_0^4}{\tau_0^4} - \frac{\varepsilon_{2t} g\beta}{\varepsilon_0}\frac{R_d^2}{\tau_0^2}, \tag{16b}$$

where $\xi = z/R_d$. For an initial plane wave, the boundary conditions on equations (16*a*) and (16*b*) are taken to be $f = g = 1$ and $df/d\xi = dg/d\xi = 0$ at $\xi = z = 0$ (implying certain initial conditions on ε_{2r} and ε_{2t} from equations (16*a*) and (16*b*)).

Equations (16*a*) and (16*b*) can be numerically integrated by using appropriate boundary conditions to evaluate the beam width parameter f and the pulse width parameter g as functions of ξ, provided that the functional forms of ε_0, ε_{2r} and ε_{2t} are known. Results for collisional plasmas with a specific expression for ε are presented in the following sections.

3. Dielectric function of collisional plasma

The drift velocity $\boldsymbol{v}_e$ of electrons in collisional plasma is governed by

$$m\frac{d\boldsymbol{v}_e}{dt} = -eE - m\,\nu_e \boldsymbol{v}_e \tag{17}$$

where m is the electron mass and ν_e is the effective frequency of mutual electron collisions. Considering an alternating uniform electric field of frequency ω (namely $\boldsymbol{E} = \boldsymbol{E}_0 \exp(i\omega t)$) switched on at $t = 0$, one obtains from the above equation

$$\boldsymbol{v}_e = \frac{-e\boldsymbol{E}_0}{m(\nu_e + i\omega)}(e^{i\omega t} - e^{-\nu_e t}). \tag{18}$$

At times longer than the characteristic (inverse) collision frequency ($t \gg 1/\nu$), one has

$$\boldsymbol{v}_e = \frac{-e\boldsymbol{E}_0\, e^{i\omega t}}{m(\nu_e + i\omega)}. \tag{19}$$

The energy balance for the electrons in the plasma may be expressed as [37]

$$\frac{d}{dt}\left(\frac{3}{2}k_B T_e\right) = -e\,\mathrm{Re}(\boldsymbol{E}\cdot\boldsymbol{v}_e) - \frac{3}{2}k_B\,\delta\,\nu_e\,(T_e - T_0)\,, \tag{20}$$

where the first term on the right-hand side represents the rate of ohmic heating per electron and the second term represents the rate of loss of energy by an electron in collisions with the heavy particles. Here, T_e denotes the electron temperature in the presence of the field, k_B is Boltzmann's constant, T_0 is the temperature of the plasma (electrons and ions) in the absence of the pulse and δ is the fraction of excess energy lost in an electron collision with heavy particles ($\sim 2m/M$); M is the effective mass of the scatterer (ion and neutral particle).

Upon substituting for v_e from equation (19) in equation (20), one obtains

$$\frac{\mathrm{d}T_e}{\mathrm{d}t} + \delta\ \nu_e(T_e - T_0) = \frac{e^2\nu_e EE^*}{3m\omega^2 k_B}. \tag{21}$$

Let us express the electron temperature as

$$T_e = T_0 + T_s, \tag{22}$$

where T_s is the variation in electron temperature due to the field. We shall assume a constant mean free path for the electron collisions, so the collision frequency becomes

$$\nu_e = \nu_0(T_e/T_0)^{1/2} = \nu_0\left(1 + \frac{T_s}{T_0}\right)^{1/2}, \tag{23}$$

where ν_0 represents the collision frequency in the absence of the field. It is appropriate to stress that this expression assumes a constant value for the field-free collision frequency, i.e. a collision cross section which is velocity independent. The situation we aim at describing here presupposes electron-neutral collisions dominating over electron-ion ones, so that the dependence such as $\nu_e \propto T_e^{1/2}$ is found to be more appropriate than the Spitzer formula $\nu_e \propto T_e^{-3/2}$ (valid for highly ionized or fusion plasmas) (see e.g. NRL Plasma Formulary [38]). Such a plasma exists in many cases [37, 39] e.g. in the lower ionosphere, weakly ionized laboratory plasma (dominated by *e*–*n* collisions) and semiconductors dominated by acoustical-phonon scattering (e.g. Ge at low temperature). In particular, the dependence of the electron density (as given by equations (27–30)) on the electron temperature is significant only at very high irradiances, as encountered in ionospheric modification experiments [40, 41].

Substituting for T_e and ν_e as above and $EE^* = A_0^2$ from equation (12) into equation (21), one obtains the energy balance equation in the form

$$\frac{\mathrm{d}T_s}{\mathrm{d}t} + \delta\nu_0\left(1 + \frac{T_s}{T_0}\right)^{1/2} T_s = \frac{e^2\nu_0}{3m\omega^2 k_0}\left(1 + \frac{T_s}{T_0}\right)^{1/2}\frac{A_{00}^2}{gf^2}\,\mathrm{e}^{-r^2/r_0^2 f^2}\,\mathrm{e}^{-t^2/\tau_0^2 g^2}. \tag{24}$$

By using the dimensionless variables $\tilde{t} = (t - z/v_g)\omega_p$ and $T_s/T_0 = \theta$ the above equation can be written as

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} + \delta\frac{\nu_0}{\omega_p}(1+\theta)^{1/2}\theta = \frac{\nu_0}{\omega_p}\frac{e^2}{3m\omega^2 k_B T_0}(1+\theta)^{1/2}\frac{A_{00}^2}{gf^2}\,\mathrm{e}^{-r^2/r_0^2 f^2}\mathrm{e}^{-t^2/\tau_0^2 g^2}. \tag{25}$$

In the paraxial approximation the electron temperature can be cast in the form

$$\theta \approx \theta_0 - \frac{r^2}{r_0^2}\theta_r - \frac{t^2}{\tau_0^2}\theta_t \tag{26}$$

where θ_0 is the axial temperature, θ_r is the transverse part of electron temperature and θ_t is the longitudinal part of electron temperature. Substituting θ from equation (26) in equation (25), one obtains the compatibility conditions

$$\frac{\mathrm{d}\theta_0}{\mathrm{d}t} + \delta\frac{\nu_0}{\omega_p}\frac{\theta_0}{(1+\theta_0)^{1/2}} = \frac{\alpha A_{00}^2}{gf^2}(1+\theta_0)^{1/2}, \tag{27}$$

$$\frac{\mathrm{d}\theta_r}{\mathrm{d}t} + \left[\delta \frac{\nu_0}{\omega_p} \frac{\theta_r}{2} \frac{(2+3\theta_0)}{(1+\theta_0)^{1/2}}\right] = \frac{\alpha A_{00}^2}{g f^2 (1+\theta_0)^{1/2}} \left[\frac{\theta_r}{2} + \frac{(1+\theta_0)}{f^2}\right], \tag{28}$$

and

$$\frac{\mathrm{d}\theta_t}{\mathrm{d}t} + \left\{\delta \frac{\nu_0}{\omega_p} \frac{\theta_t}{2} \frac{(2+3\theta_0)}{(1+\theta_0)^{1/2}}\right\} = \frac{\alpha A_{00}^2}{g f^2 (1+\theta_0)^{1/2}} \left(\frac{\theta_t}{2} + \frac{(1+\theta_0)}{g^2}\right), \tag{29}$$

where $\alpha = e^2(\nu_0/\omega_p)/3m\omega^2 k_B T_0$.

A redistribution of electrons can result from a nonuniform electron temperature distribution:

$$n_e = n_0 \frac{2T_0}{T_e + T_0} = n_0 \frac{2}{2+\theta_0} \left(1 + \frac{\theta_r}{2+\theta_0} \frac{r^2}{r_0^2} + \frac{\theta_t}{2+\theta_0} \frac{t^2}{\tau_0^2}\right), \tag{30}$$

where n_e and n_0 are the electron densities in the presence of and absence of the pulse, respectively. A redistribution of electrons can result from a nonuniform electron temperature distribution, where one may use the steady state equation [31, 34] as given by equation (30). When the pulse duration τ is greater than the collision frequency ν^{-1}, say of the order of $(\delta\nu)^{-1}$; the current density has almost the steady state value, corresponding to the instantaneous electron temperature. Since the electron density distribution corresponding to a given electron temperature distribution approaches the steady state value in a period of the order of ν^{-1}, it is an excellent approximation to assume steady state electron density distribution, corresponding to instantaneous electron temperature.

In a neutral collision-dominated plasma $\nu_{im} \gg \delta_{ei}\nu_{ei}$ and hence the heating of ions is not significant. The duration in which the steady state current density corresponding to a given electron temperature gets established, namely $1/\nu$, is much smaller than the duration $(\delta\nu)^{-1}$, for the temperature to change significantly. Hence the steady state current density, corresponding to instantaneous electron temperature, is a good approximation.

Equation (30) considers the plasma as quasi-neutral (so that one can neglect the electron-ion charge separation force compared to the electron pressure force in the electron momentum equation). For the plasma to be quasi-neutral the pulse duration should be much larger than the time required for the ion-acoustic sound wave to travel a distance equal to the laser pulse width. This is

$$\tau_s = r_0/c_s = r_0/\sqrt{T_e/M} = r_0(M/m)^{1/2}/V_{T_e} = (r_0\omega_p/c)\omega_{pi}^{-1}(c/V_{T_e}),$$

where c_s is the ion-acoustic sound speed. Therefore the condition required for quasi-neutrality is

$$\tau_0 \gg \tau_{pi}(c/V_{T_e}) \gg \tau_{pi},$$

where $\tau_{pi}\left(\sim\omega_{pi}^{-1} \sim \omega_p^{-1}\delta^{-1/2}\right)$ is the ion plasma period.

Using equation (30) the dielectric function of the plasma can be expressed as

$$\varepsilon = 1 - \frac{4\pi n_e e^2}{m\omega^2} = \varepsilon_0 - \left(r^2/r_0^2\right)\varepsilon_{2r} - \left(t^2/\tau_0^2\right)\varepsilon_{2t}, \tag{31}$$

where

$$\varepsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega^2} \frac{2}{2+\theta_0}, \tag{32}$$

$$\varepsilon_{2r} = \frac{\omega_{p0}^2}{\omega^2} \frac{2\theta_r}{(2+\theta_0)^2} \tag{33}$$

and

$$\varepsilon_{2t} = \frac{\omega_{p0}^2}{\omega^2} \frac{2\theta_t}{(2+\theta_0)^2}. \tag{34}$$

ω_{p0} is the plasma frequency in the absence of the pulse.

4. Numerical results and discussions

We shall consider an incident laser pulse whose (field) Gaussian envelope is given by

$$EE^* = A_{00}^2\,\mathrm{e}^{-t^2/\tau_0^2}\,\mathrm{e}^{-r^2/r_0^2}. \tag{35}$$

In order to investigate the pulse evolution in plasma, we have used the paraxial approximation. We have found a solution in the form (combining equations (1), (9), (12) and (13))

$$EE^* = \frac{A_{00}^2(T(0)F(0)^2)}{T(z)F(z)^2} \times \exp\left[-\frac{t^2}{T(z)^2} - \frac{\mathrm{i}k}{2}\frac{t^2}{T(z)}\frac{\mathrm{d}T(z)}{\mathrm{d}z} - \frac{r^2}{F(z)^2} - \frac{\mathrm{i}k}{2}\frac{r^2}{F(z)}\frac{\mathrm{d}F(z)}{\mathrm{d}z}\right], \tag{36}$$

where $T(z) = \tau_0\, g(z)$ is the pulse width (in time) in plasma and $F(z) = r_0\, f(z)$ is the beam width (in space) in the radial direction of the plasma; $T(0) = \tau_0$ is the initial pulse width and $F(0) = r_0$ is the initial beam width.

The pulse compression process for a finite extent Gaussian laser pulse in plasma may be investigated by calculating the pulse length $T(z)$ and the beam width $\mathrm{F}(z)$. Recall that the pulse length and beam width are proportional to the pulse width parameter $g(z)$ (given by equation (16)) and the beam width parameter $f(z)$ (given by equation (15)) respectively.

We have numerically solved the sets of equations (15), (16) and (27)–(29) for the beam width parameter f, pulse width parameter g, θ_0, θ_r and θ_t, in combination with (31)–(34) for the dielectric function, in order to investigate the focusing and compression of the pulse. We have performed numerical computation for the following dimensionless parameters:

$$\alpha A_{00}^2 = 0.10, \qquad \delta\nu_0/\omega_p = 1.1\times 10^{-5}, \qquad \tilde{r}_0 = 3.34, \qquad \tilde{\tau}_0 = 10^6, \qquad \frac{\omega_{p0}^2}{\omega^2} = 0.5,$$

corresponding to given laser and plasma parameters, i.e. $A_{00} = 10^4$ esu, $\omega = 1.4\times 10^{15}\,\mathrm{s}^{-1}$, $r_0 = 1\,\mu$m, $\tau_0 = 1$ nsec, $\omega_p = 10^{15}\,\mathrm{s}^{-1}$, $\nu_0 = 1\times 10^{13}\,\mathrm{s}^{-1}$. These parameters for a laser plasma system have been considered by keeping in mind the validity of equations (23) and (30). Equation (23) requires that ion neutral collision frequency should be less than electron neutral collision frequency which is true from the following:

$$\nu_i \approx \nu\,(m/M)^{1/2}.$$

where ν_i and ν are the ion neutral collision frequency and electron neutral collision frequency respectively.

Applicability of equation (30) requires that the pulse duration τ_0 should be greater than the collision frequency ν_0^{-1} and of the order of $(\delta\nu_0)^{-1}$, as satisfied by the above parameters.

Figure 1 represents the redistribution of the electron temperature and the density distribution scaled by their equilibrium (unperturbed) values. During the simulation we observe a redistribution of the electron temperature due to the non-uniform Gaussian intensity distribution. We find that the maximum electron temperature lies in the high-field region and gradually decreases toward the low-field region. However, the relative heating (increase in ratio) in the electron temperature does not exceed a factor 2. The associated electron density distribution establishes a gradient in the dielectric constant, which leads to compression and focusing of beam. We have to add for rigor that, at higher electron temperature, the electron density can also change due to additional ionization and thus equation (30) may be violated. Admittedly, the ionization effect was not incorporated in our physical model. However, the observed increase in electron density (figure 1) suggests that ionization appears to be ruled out under the conditions considered.

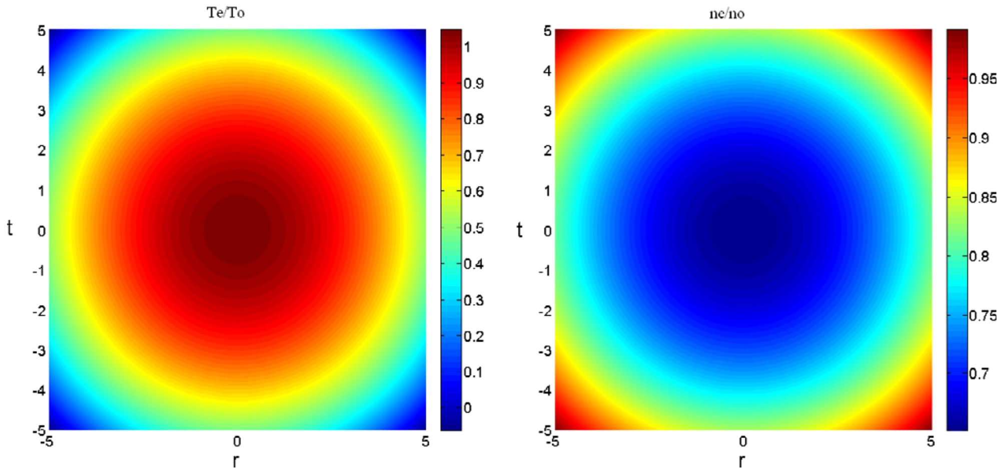

Figure 1. The redistribution of the electron temperature (left panel), and of the electron density (right panel), scaled by their unperturbed values. The initial laser and plasma parameters here are $\alpha A_{00}^2 = 0.10,\ \delta\nu_0/\omega_p = 1.1 \times 10^{-5},\ \tilde{r}_0 = 3.34,\ \tilde{\tau}_0 = 10^6,\ \frac{\omega_{p0}^2}{\omega^2} = 0.5.$

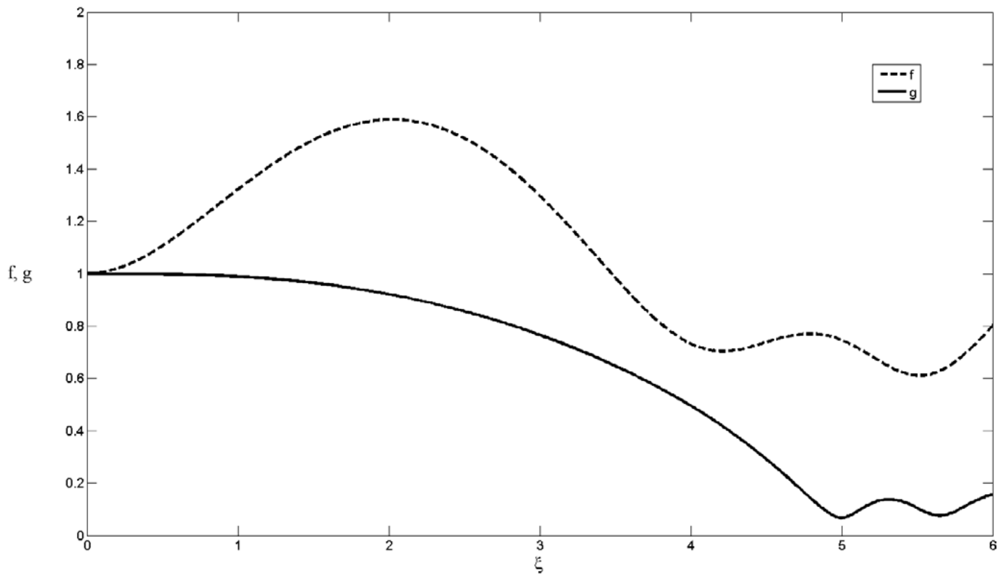

Figure 2. The variation of the beam width (in space) parameter f (upper dashed blue curve), and the pulse length (in time) parameter g (lower solid red curve), both defined as dimensionless quantities, are depicted versus the dimensionless distance of propagation ξ. The initial laser and plasma parameters here are $\alpha A_{00}^2 = 0.10,\ \delta\nu_0/\omega_p = 1.1 \times 10^{-5},\ \tilde{r}_0 = 3.34,\ \tilde{\tau}_0 = 10^6,\ \frac{\omega_{p0}^2}{\omega^2} = 0.5.$

Figure 2 represents the dependence of the beam width parameter f and of the pulse length parameter g on the dimensionless distance of propagation ξ. The analytical behavior of the curves suggests that the pulses undergoes both focusing and compression. The focusing of the pulse results from the nonuniform distribution of the electron density and thus of the

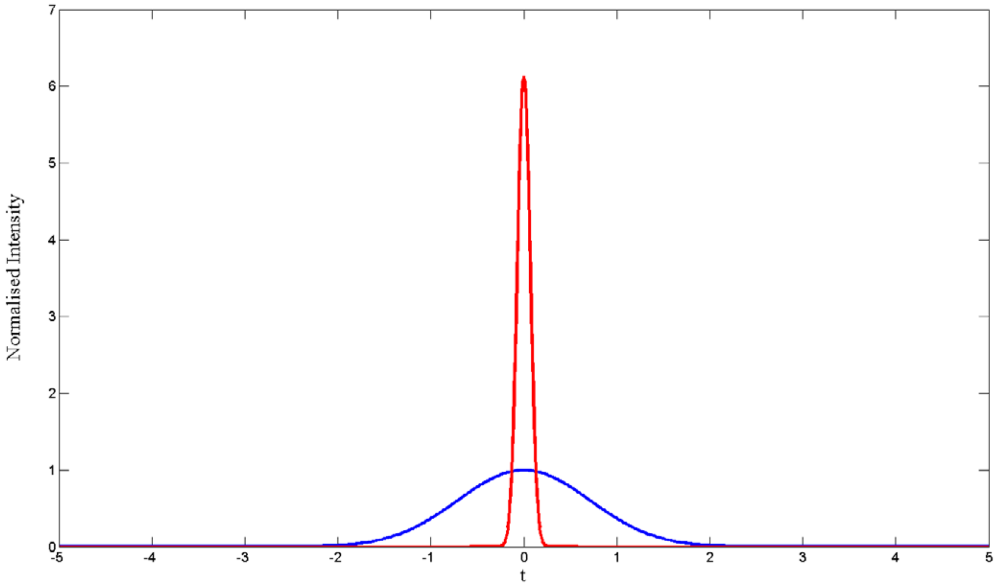

Figure 3. The collisional pulse compression of a finite extent laser pulse along time t, at $\xi = 0$ (blue wide curve) and at $\xi = 5$ (red narrow curve). The initial laser and plasma parameters here are $\alpha A_{00}^2 = 0.10$, $\delta\nu_0/\omega_p = 1.1 \times 10^{-5}$, $\tilde{r}_0 = 3.34$, $\tilde{\tau}_0 = 10^6$, $\frac{\omega_{p0}^2}{\omega^2} = 0.5$.

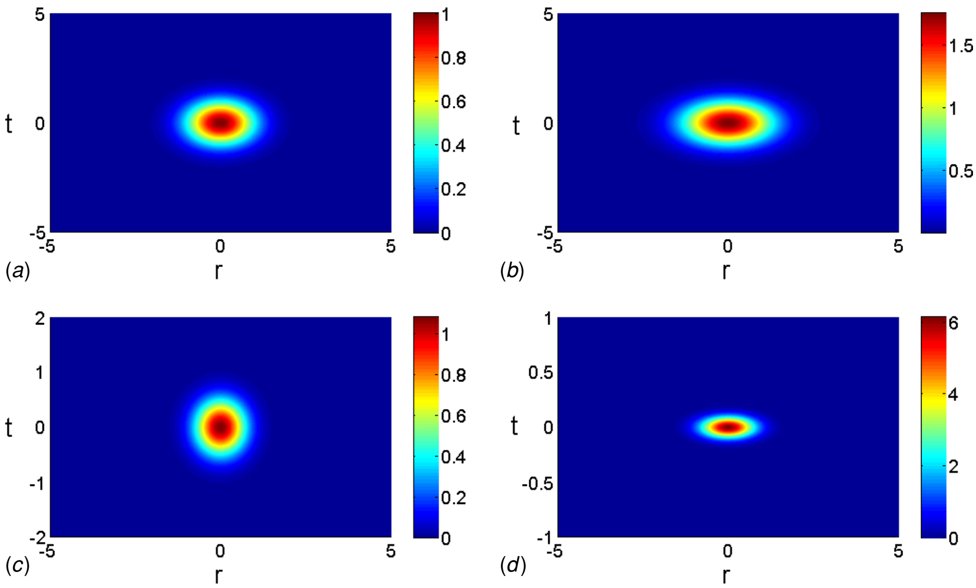

Figure 4. The spatiotemporal profile of the normalized laser field intensity at different points: (*a*) $\xi = 0$, (*b*) $\xi = 1$, (*c*) $\xi = 4$ and (*d*) $\xi = 5$. The initial laser and plasma parameter values are $\alpha A_{00}^2 = 0.10$, $\delta\nu_0/\omega_p = 1.1 \times 10^{-5}$, $\tilde{r}_0 = 3.34$, $\tilde{\tau}_0 = 10^6$, $\frac{\omega_{p0}^2}{\omega^2} = 0.5$.

dielectric function, due to the nonuniform electron temperature. We shall attempt to estimate quantitatively the shortening of pulse length as a consequence of the focusing. We note that regions of lower intensity within the pulse would tend to focus less rapidly. Therefore, over a distance of the order of the focusing length, the intensity of the peak is greatly enhanced whereas the regions away from it are less so; this results in a compression of the pulse.

Figure 3 shows the compression of the pulse and the increase in pulse intensity during its propagation. We observe a compression in the full-width-at-half-maximum of the laser pulse by an order of magnitude approximately, while the pulse intensity is increased six times from its initial value.

To verify the interdependence among compression and focusing we have represented the spatiotemporal dynamics of the pulse irradiance (field intensity) profile at different values of z. Figure 4(*a*) shows the initial irradiance of a Gaussian pulse at $\xi = 0$. Figures 4(*b*)–(*d*) depict the irradiance profile of the compressed pulse, at different points ($\xi = 1$, $\xi = 4$ and $\xi = 5$) in the collision-dominated plasma. The surface plots of the spatiotemporal irradiance distribution confirm the occurrence of pulse compression and of an increase in pulse intensity.

5. Conclusions

In this paper we have investigated the compression of an electromagnetic (laser) pulse in collisional plasma, by considering a finite extent Gaussian pulse profile. We have derived an analytical model for the dynamics of the beam width (in space) and of the pulse duration (in time), starting from a wave equation which incorporates diffraction, dispersion and nonlinear terms due to collisions. We have defined a pair of appropriate trial functions f (in space) and g (in time) whose evolution determines the dynamics of the beam profile in space and time coordinates. These functions were determined by a system of coupled nonlinear differential equations for the beam width parameter f and the pulse width parameter g.

To complement our analytical results, we have solved the system of dynamical equations numerically, along with the equations for the axial, transverse and longitudinal parts of the electron temperature and the dielectric function, to investigate the shortening of the pulse width parameter and hence pulse compression as a function of the distance of propagation. The numerical simulation confirms the compression of the pulse and increase in the pulse intensity. The group velocity dispersion in combination with the collisional nonlinearity leads to the compression of laser pulse. It is also observed that for a finite extent laser pulse, transverse focusing strongly interferes with the process of pulse compression.

In conclusion, our analytical and numerical investigations suggest that a laser pulse can be efficiently compressed in the collision-dominated plasma. We have numerically observed compression in the full-width-at-half-maximum of the laser pulse, along with a significant increase in intensity. The evolution of a short and intense pulse in collisional plasma would be of great interest because they could lead to higher peak powers. We believe our results to be of relevance in achieving short and ultraintense laser pulses.

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References

[1] Shukla P K, Rao N N, Yu M Y and Tsintsadze N L 1986 *Phys. Rep.* **138** 1
Yu M Y, Yu W, Chen Z Y, Zhang J, Yin Y, Cao L H, Lu P X and Xu Z Z 2003 *Phys. Plasmas* **10** 2468
[2] Dieckmann M E, Bret A and Shukla P K 2008 *New J. Phys.* **10** 013029

[3] Silva L O, Marti M, Davies J R, Fonseca R A, Ren C, Tsung F and Mori W B 2004 *Phys. Rev. Lett.* **92** 015002
[4] Mendonca J T, Norreys P, Bingham R and Davies J R 2005 *Phys. Rev. Lett.* **94** 245002
[5] Marklund M, Eliasson B and Shukla P K 2006 *Phys. Plasmas* **13** 083102
[6] Faisal M, Verma M P and Sodha M S 2008 *Phys. Plasmas* **15** 102301
[7] Shibu S, Parashar J and Pandey H D 1998 *J. Plasma Phys.* **59** 91
[8] Wilks S C 1992 *Phys. Rev. Lett.* **69** 1383
[9] Tabak M 1994 *Phys. Plasmas* **1** 1626
[10] Caruso A and Pais V A 1996 *Nucl. Fusion* **36** 745
[11] Atzeni S and Ciampi M L 1997 *Nucl. Fusion* **37** 1665
[12] Pukhov A and Mayer-Ter-Vehn J 1998 *Phys. Plasmas* **5** 1880
[13] Key M H 1998 *Phys. Plasmas* **5** 1966
[14] Eder D C, Amendt P and Wilks S C 1992 *Phys. Rev.* A **45** 6761
[15] Mourou G, Barty C and Perry M D 1998 *Phys. Today* **51** 22
[16] Ross I N, Matousek P, Towrie M, Langley A J and Collier J L 1997 *Opt. Commun.* **144** 125
[17] Shvets G, Fisch N J, Pukhov A and Meyer-ter-Vehn J 1998 *Phys. Rev. Lett.* **81** 4879
[18] Boyd R W, Lukishova S G and Shen Y R 2009 *Self-Focusing: Past and Present* vol 114 (New York: Springer)
[19] Karle Ch and Spatschek K H 2008 *Phys. Plasmas* **15** 123102
[20] Shorokhov O, Pukhov A and Kostyukov I 2003 *Phys. Rev. Lett.* **91** 265002
[21] Tsung F S, Ren C, Silva L O, Mori W B and Katsouleas T 2002 *Proc. Natl Acad. Sci. USA* **99** 29
[22] Gordon D F and Hafizi B 2003 *Phys. Rev. Lett.* **90** 215001
[23] Malka V *et al* 2002 *Science* **298** 1596
[24] Avitzour Y and Shvets G 2008 *Phys. Rev. Lett.* **100** 065006
[25] Balakin A A, Litvak A G, Mironov V A and Skobelev S A 2008 *Phys. Rev.* A **78** 061803
[26] Ren C, Duda B J, Hemker R G, Mori W B, Katsouleas T, Antonsen T M Jr and Mora P 2001 *Phys. Rev.* E **63** 026411
[27] Sodha M S, Sharma A and Agarwal S K 2006 *Phys. Plasmas* **13** 083105
[28] Keskinen M J and Basu S 2003 *Radio Sci.* **38** 1095
[29] Kip D, Kratzig E, Shandarov V and Moretti P 1998 *J. Opt. Soc. Am.* **23** 343
[30] Johnson L C and Chu T K 1974 *Phys. Rev. Lett.* **32** 517
[31] Sodha M S, Ghatak A K and Tripathi V K 1976 *Prog. in Optics* vol 13 ed E Wolf (Amsterdam: North-Holland) p 169
[32] Sprangle P, Esarey E and Krall J 1996 *Phys. Rev.* E **54** 4211
[33] Akhmanov S A, Sukhorukov A P and Khokhlov R V 1968 *Sov. Phys. Uspekhi* **10** 609
[34] Sodha M S, Ghatak A K and Tripathi V K 1968 *Self Focusing of Laser Beams in Dielectrics, Plasmas and Semiconductors* (New Delhi: Tata McGraw Hill) chapter 10
[35] Vlasov S N and Talanov V I 1997 *Wave Self Focusing* (Nizhny Novgorod: Inst. of Appl. Phys. of the Russian Academy of Science) p 220
[36] Sulem C and Sulem P L 1999 *Nonlinear Schrödinger Equation: Self Focusing Instability and Wave Collapse* (New York: Springer)
[37] Gurevich A V 1978 *Nonlinear Phenomena in the Ionosphere* (Berlin: Springer) p 20
[38] Huba J D 2000 *NRL Plasma Formulary* (Washington , DC: Naval Research Laboratory)
[39] Kasperczuk A, Pisarczyk T, Borodzuik S, Ullschmied J, Krousky E, Masek K, Rohlena K, Skala J and Hora H 2006 *Phys. Plasmas* **13** 062704
[40] Stubbe P 1996 *J. Atmos. Sol. Terr. Phys.* **58** 349
[41] Hadi A, Janabi Al, Kumar Atul, Sharma R P and Tripathi Y K 1994 *IEEE Trans. Plasma Sci.* **22** 65