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Relativistic laser pulse compression in plasmas with a linear axial density gradient

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Abstract
The self-compression of a relativistic Gaussian laser pulse propagating in a non-uniform plasma is investigated. A linear density inhomogeneity (density ramp) is assumed in the axial direction. The nonlinear Schrödinger equation is first solved within a one-dimensional geometry by using the paraxial formalism to demonstrate the occurrence of longitudinal pulse compression and the associated increase in intensity. Both longitudinal and transverse self-compression in plasma is examined for a finite extent Gaussian laser pulse. A pair of appropriate trial functions, for the beam width parameter (in space) and the pulse width parameter (in time) are defined and the corresponding equations of space and time evolution are derived. A numerical investigation shows that inhomogeneity in the plasma can further boost the compression mechanism and localize the pulse intensity, in comparison with a homogeneous plasma. A 100 fs pulse is compressed in an inhomogeneous plasma medium by more than ten times. Our findings indicate the possibility for the generation of particularly intense and short pulses, with relevance to the future development of tabletop high-power ultrashort laser pulse based particle acceleration devices and associated high harmonic generation. An extension of the model is proposed to investigate relativistic laser pulse compression in magnetized plasmas.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The interaction of ultraintense very short laser pulses with plasmas [1–7] has attracted a great deal of attention for fundamental research and technological applications, such as particle acceleration, inertial confinement fusion, high harmonic generation and x-ray lasers [8–14].

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The standard approach to produce an ultrashort, ultraintense multiterawatt laser pulse is the chirped-pulse-amplification (CPA) technique [15], in which a laser pulse is stretched, amplified and recompressed. The CPA scheme has shown the ability to generate subpicosecond petawatt laser pulses with up to 500 J per pulse. This approach is limited by the finite bandwidth of the active millimeter amplifiers used in lasers. Ross *et al* [16] investigated a new scheme of parametric amplification to produce ultrashort and powerful pulses. Superradiant amplification of an ultrashort laser pulse was observed by Shvets *et al* [17] who considered an electromagnetic (em) beam colliding with a long counterpropagating low-intensity pump in the plasma. The methods reported by [15–17] need at least two counterpropagating laser pulses. This fact makes the practical realization of these methods difficult.

At high laser intensities the nonlinear interaction between the plasma and the laser becomes important, giving rise to a variety of novel physical effects [18–20] which are not observed in the linear regime. These include relativistic optical guiding, harmonic excitation, wake-field generation, laser pulse frequency shifting and pulse compression. Akhiezer and Polovin [18] investigated analytically the propagation of very intense em radiation in overdense plasmas customarily begining with the exact traveling wave solution. Dawson [19] investigated the nonlinear longitudinal electron oscillations in a cold plasma considering a plasma of infinite extent and free from static fields. Sprangle *et al* [4] developed the nonlinear one-dimensional theory that describes some important aspects (e.g. nonlinear plasma wake-field generation, relativistic optical guiding and coherent harmonic radiation production) of intense laser–plasma interactions. Pukhov [20] discussed in his elegant review some of the important physical effects emerging at relativistic laser intensities. He also considered the possibility of using plasma as a medium for short laser pulse amplification in the colliding beam configuration.

A fundamental nonlinear effect occurs in the intense laser field, due to electrons oscillating at relativistic velocities which exceed 10^{11} V cm^{-1}, resulting in a significant electron relativistic mass increase. In particular, the laser pulse's spatial extension in both transverse and axial dimensions can be modified by relativistic self-focusing (RSF) and relativistic self-phase modulation (RSPM). In the former case (RSF), the transverse spot size may decrease when a transverse gradient of the index of refraction causes the wavefronts to bend, so that energy is focused radially inward. In the latter case (RSPM), the laser pulse length can be compressed by a frequency chirp that is induced by the nonlinear dependence of the axial phase velocity. This leads to an axial chirp of the group velocity where the back of the pulse progresses faster (higher frequency) and the front of the pulse moves at a slower speed (lower frequency), causing the pulse to (self-) compress. The pulse compression and focusing mechanisms are dynamically correlated through the change in the pulse intensity. RSF [21] was theoretically predicted more than 25 years ago.

Relativistic mass variation during laser–plasma interaction is the origin of longitudinal self-compression of a laser pulse down to a single laser cycle in length, with a corresponding increase in intensity. The main source of nonlinearity is the relativistic mass increase due to the quiver motion of the electrons in the field of the laser. In the last few years, several scenarios have been proposed for the self-focusing [22] and self-compression [23] of a laser pulse in plasma.

Shorokhov *et al* [24] have employed a 3D PIC simulation to show that a 30 fs long laser pulse is efficiently compressed to 5 fs by using a periodic plasma–vacuum structure to damp filamentation. Tsung *et al* [25] reported a scheme to generate single-cycle laser pulses based on photon deceleration in underdense plasmas. This robust and tunable process is ideally suited for lasers above critical power because it takes advantage of the RSF of these lasers and the nonlinear features of the plasma wake. Ren *et al* [26] demonstrated the compression and focusing of a short laser pulse by a thin plasma lens. A set of analytical formulae for the spot size and for the length evolution of a short laser pulse were derived in their model. Shibu *et al* [27] also proposed the possibility of pulse compression in relativistic homogeneous plasma and reported the interplay between transverse focusing and longitudinal compression.

It was recently demonstrated numerically [28] that microwave pulses can be compressed (with or without a frequency shift) inside magnetized plasma by changing the magnitude or the direction of the magnetic field uniformly in space and adiabatically in time. Balakin *et al* [29] investigated the self-focusing of a few optical cycle pulses recently. They showed that the wave-field self-focusing proceeds with overtaking the steepening of the pulse longitudinal profile, leading to shock-wave formation. Consequently, a more complex singularity is formed where an unlimited field increase is followed by wave breaking with a broad power-law pulse spectrum.

A uniform plasma has been considered in most of the studies on pulse self-compression cited above [23–28]. Few investigations, to our knowledge, have been devoted to the effect of plasma inhomogeneity on pulse propagation [30–33] (yet these were limited to transverse beam focusing). An axial inhomogeneity has been considered in earlier studies, addressing among other effects the RSF of intense laser radiation in relation to the fast ignitor scheme [30] and em beam penetration in overdense plasma [31]. Interestingly, third harmonic generation was investigated in [33], considering the case of a plasma inhomogeneity scale larger than the em wave wavelength [33].

The injection of a short-pulsed intense laser into a radially inhomogeneous underdense plasma was studied in [34], and shown to sustain a self-trapped photon channel. The authors observed that under appropriate conditions the laser and plasma fiber system can provide a slow em wave structure that is suitable for energy acceleration. The RSF of ultraintense laser pulses in inhomogeneous plasma was investigated in [35] (considering transverse beam focusing), where inhomogeneity was argued to play a major role in the self-focusing process. The authors pointed out that the self-focusing is due to the combined effect of the relativistic electron mass increase and plasma inhomogeneity, while the ponderomotive force plays a secondary role in the process [35]. The nonlinear, oblique (non-paraxial) propagation of an intense short Gaussian laser pulse in a preformed plasma channel having a parabolic density profile was analyzed in [36]. The authors observed that the presence of the parabolic channel leads to compression of pulse length.

Reference [37] investigated the higher order paraxial theory of the propagation of an initially Gaussian em beam in an inhomogeneous plasma with an overdense region. Higher order terms (up to r^4) in the expansion of the dielectric constant and of the eikonal have been taken into account. The authors observed that the higher order paraxial theory significantly affects the dependence of em beam width on the distance of propagation.

Modern lasers (e.g. Vulcan Petawatt Upgrade at the Rutherford Appleton Laboratory Central Laser Facility and the Gekko Petawatt Laser at the Gekko XII facility in the Institute of Laser Engineering at Osaka University) rely on CPA techniques for amplifying an ultrashort laser pulse to extremely large intensities. Because of limitations, e.g., in gain bandwidth, high-power CPA systems are currently limited from below to pulses of order 30 fs. The physical reason for this limitation is the finite bandwidth of the active medium amplifiers used in the lasers. The advantage of plasma as an 'active' medium for pulse compression is that it sustains extremely high intensities. Nonlinearity becomes significant only close to the relativistic threshold and thus high power can be achieved. Filling the gap in existing theoretical research in longitudinal pulse compression in inhomogeneous relativistic plasmas, from first principles, is our scope here, and has motivated the theoretical and numerical study presented below.

In this paper, we investigate the longitudinal self-compression of a Gaussian laser pulse, propagating in relativistic plasma with an axial inhomogeneity. We show that inhomogeneity of the plasma medium can boost the compression mechanism and lead to an increase in pulse intensity (in comparison with the same mechanism occurring in homogeneous plasma). We compare our numerical results showing the longitudinal pulse compression in plasma, both in the presence and in the absence of inhomogeneity. The simulation results point out the role of inhomogeneity for pulse amplification and compression. We rely on an earlier approach [38–40], also recently employed [41] in a study of em pulse profile dynamics in collisional plasmas, and proceed by introducing a set of trial functions via the intensity profile of the laser pulse, and following their evolution in space/time in the plasma. As a first step, we adopt a one-dimensional (1D) model, relying on the em wave equation as derived from Maxwell's equations. A nonlinear Schrödinger equation (NLSE) is obtained and solved by using the paraxial formalism, to demonstrate the occurrence of longitudinal pulse width compression and energy localization. The role of plasma inhomogeneity is incorporated via a quantitative plasma non-uniformity ansatz. The analysis is then extended to a three-dimensional (3D) pulse profile description. A pair of appropriate trial functions is defined, accounting for the beam width parameter (in space) and the pulse width parameter (in time), whose evolution determines the dynamics of the pulse. Both longitudinal and transverse self-compression is examined for a finite extent Gaussian laser pulse through this model. These functions are determined by a system of coupled nonlinear differential equations, which are integrated numerically to yield the spatiotemporal laser pulse profile.

We have investigated the self-compression of a laser pulse in a narrow window of plasma density values from 0.25 n_c to slightly below n_c, where $n_c = m_e\omega^2/4\pi e^2$ is the critical plasma density for a laser pulse with the frequency $\omega = 2\pi c/\lambda$. In this density region, the Raman instability [42] that otherwise destroys the pulse is impeded. In particular, the Raman instability, most simply characterized as the resonant decay of an incident photon into a scattered photon and an electron plasma wave (or plasmon), relies on the frequency and wave number matching conditions are $\omega_0 = \omega_s + \omega_{pe}$; $k_0 = k_s + k$, where subscript 0 and s denote the incident and scattered light wave, respectively. Since the minimum frequency of a light wave in a plasma is ω_{pe} (the electron plasma frequency), it is clear that Raman Instability requires strongly underdense conditions, such that $\omega_0 \geqslant \omega_{pe}$ i.e. $n \leqslant n_{cr}/4$, where n is the plasma density and n_{cr} is the critical density. When an ultrashort laser pulse ($L < \lambda_p$ where L and λ_p are the pulse length and plasma wavelength; $\lambda_p = 2\pi c/\omega_p$) propagates in the plasma, the relativistic nonlinear effect is balanced by the axial ponderomotive effect [43]. The axial ponderomotive force accumulates the electrons ahead of the laser pulse, thus increasing the plasma frequency [42, 44], which counteracts the effect of RSF and self-compression of the laser pulse. In this work, we have considered pulse lengths longer than the plasma wavelength; we shall neglect the ponderomotive nonlinearity below.

2. 1D laser pulse compression: the analytical framework

The electric field of a circularly polarized laser pulse propagating in the z-direction can be written as

$$E(z,t) = \tfrac{1}{2}A(z,t)(e_x + \mathrm{i}e_y)\exp[-\mathrm{i}(\omega t + kz)]. \tag{1}$$

The electric field of the laser pulse satisfies the wave equation in 1D,

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J}{\partial t}, \tag{2}$$

where $\boldsymbol{J}$ is the current density and is given by

$$J = n_0 e v, \tag{3}$$

where

$$v = -\frac{\mathrm{i}eE}{m\omega\gamma} \tag{4}$$

denotes the electron quiver velocity,

$$\gamma = \left(1 + \frac{e^2|A|^2}{m^2\omega^2c^2}\right)^{1/2} \tag{5}$$

is the relativistic factor, e is the electronic charge, n_0 is the background plasma density, ω_p is the background plasma frequency and ω is the frequency of the laser beam.

The dielectric constant of plasma in the relativistic regime [21, 24–28] can be written as

$$\varepsilon = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2}\frac{1}{\gamma}, \tag{6}$$

where $\omega_\mathrm{p} = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency.

One can formally express the effective permittivity as

$$\varepsilon = \varepsilon_\mathrm{o} + \varphi(|A|^2), \tag{7}$$

where

$$\varepsilon_\mathrm{o} = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \tag{8}$$

is the classical (non-relativistic) part and [27, 45]

$$\varphi(|A|^2) = \frac{\omega_p^2}{\omega^2}\frac{\gamma - 1}{\gamma} \tag{9}$$

For a slowly time varying envelope A,

$$\frac{\partial J}{\partial t} = -n_0 e\frac{\partial v}{\partial t}. \tag{10}$$

Combining equations (1), (7) and (10) into the wave equation (equation (2)), one is led to

$$2\mathrm{i}k\left(\frac{\partial A}{\partial z} + \frac{1}{v_\mathrm{g}}\frac{\partial A}{\partial t}\right) - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \frac{k^2\varphi(|A|^2)}{\varepsilon_0}A = 0. \tag{11}$$

The laser pulse propagates at the group velocity $v_\mathrm{g} = c^2k/\omega$, where $k = 2\pi/\lambda = \omega\varepsilon_0^{1/2}/c$ is the wave number given by the plasma dispersion relation, $c^2k^2 = \omega^2 - \omega_\mathrm{p}^2$.

We shall now introduce the new dimensionless variables $\zeta = \omega z/c$ and $\tau = (z/v_\mathrm{g} - t)\omega$, and the normalized laser field $a = eA/m\omega c$.

We note that

$$a = 0.85 \times 10^{-9}\sqrt{I}\lambda, \tag{12}$$

where I is expressed in $\mathrm{W\,cm^{-2}}$ and λ is expressed in μm.

Equation (11) can thus be cast in the form

$$2\mathrm{i}\varepsilon_0^{1/2}\frac{\partial a}{\partial \zeta} + \frac{\partial^2 a}{\partial \tau^2} + \varphi(|a|^2)a = 0. \tag{13}$$

For a weakly relativistic circularly polarized laser pulse, one has $|a| \ll 1$, i.e. $\gamma \ll 1$ and $\varphi(|a|^2) \cong (\omega_\mathrm{p}^2/\omega^2)|a|^2$.

Equation (13) is a NLSE, where the second term represents the dispersion broadening and the third term representing the nonlinear compression. When the two effects balance each other, one obtains a coherent structure in the form of a solitary wave. The NLSE (13) has been derived in various forms and via different methods earlier by many authors [3, 22–24, 29, 45–47]. We follow here the paraxial approach introduced by Akhmanov in [38], later extended by Sodha and coworkers in [39] and later revisited by Sharma *et al* [41].

We shall consider a laser pulse having an initial Gaussian geometrical (beam spot) profile at $z = 0$, given by

$$a^2(z = 0, t) = a_0^2 \exp(-t^2/\tau_0^2), \tag{14}$$

where τ_0 is the initial pulse width.

We shall now investigate the beam profile (14) in the plasma medium, by substituting into the NLSE (13), and using the paraxial approach ($t \ll \tau_0 g$, where g is the pulse width parameter expressed by equation (16)) [39].

The time/space advanced beam (field) profile is given by

$$a^2(\zeta, \tau) = \frac{a_0^2 T(0)}{T(\zeta)} \exp\left(-\frac{\tau^2}{T(\zeta)^2} - \mathrm{i}\frac{\varepsilon_0^{1/2}}{2}\frac{\tau^2}{T(\zeta)}\frac{\mathrm{d}T(\zeta)}{\mathrm{d}\zeta}\right), \tag{15}$$

where $T(z) = \tau_1 g(z)$ is the pulse width of laser in plasma and $g(z)$ is the pulse width parameter. Retain that $T(0) = \tau_1 = \tau_0\omega$ is the initial dimensionless pulse width (in time).

The pulse width $T(z)$ can be evaluated by solving the second order nonlinear differential equation for the pulse width parameter $g(z)$

$$\varepsilon_0 \frac{\mathrm{d}^2 g}{\mathrm{d}\zeta^2} = \frac{1}{\tau_1^4 g^3} - \frac{2(\omega_\mathrm{p}^2/\omega^2)|a|^2}{\tau_1^2 g^2 (1 + |a|^2/g)^{3/2}}. \tag{16}$$

Equation (16) has been obtained by solving the NLSE (13) in the paraxial approach [38, 39, 41, 48]. The first term in the RHS of equation (16) represents diffraction of the beam while the second term corresponds to the nonlinear part of the relativistic dielectric constant, given by equation (9). Using the paraxial approach we have expanded the expression of $\varphi(|A|^2)$ and obtained the term as a coefficient of r^2, which appears as the second term in RHS (16). We shall now proceed by investigating the dynamic evolution of the beam profile, relying on (16) in combination with (15) (and (8)).

3. Inhomogeneous plasmas: exact results

Let us consider a plasma with a slow variation of the electron density n along the z (variable $\zeta = \omega z/c$) direction:

$$n(\zeta) = n(0)\varphi(\zeta), \tag{17}$$

where

$$\varphi(\zeta) = 1 + b\zeta. \tag{18}$$

The plasma frequency in the presence of a weak density gradient in the plasma can be expressed as [31]

$$\omega_\mathrm{p}^2 = \omega_{\mathrm{p}0}^2 (1 + b\zeta), \tag{19}$$

$\omega_{\mathrm{p}0}$ here being the plasma frequency corresponding to electron density n_0 at $z = 0$, while b is a characteristic inhomogeneity parameter (inverse length).

We have performed a numerical computation for an initially Gaussian pulse of amplitude $a_0 = 0.1$, $\tau_0 = 100$ fs, $n_0 = 3.12 \times 10^{21}$ cm^{-3}, $\omega = 5 \times 10^{15}$ rad s^{-1} and for $b = 0.001$. By integrating equation (16), we obtain the curves depicted in figure 1, for the initial pulse and for the fully compressed pulse in the uniform and non-uniform cases. The plot in figure 1

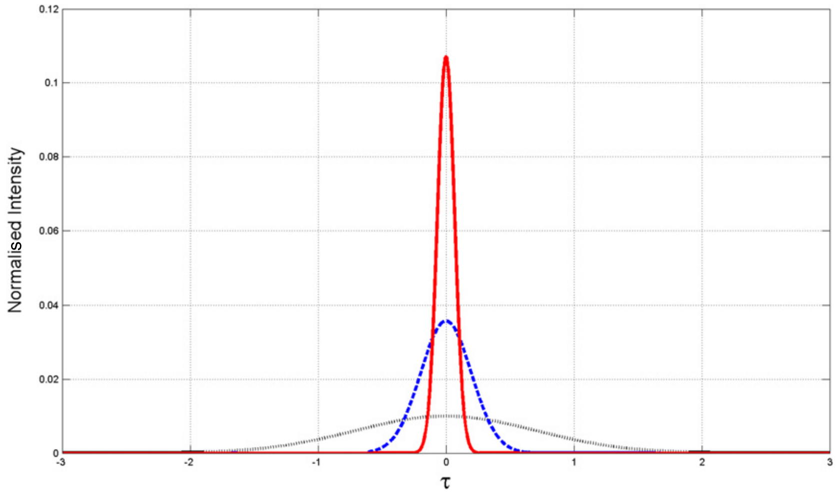

Figure 1. The dependence of the normalised laser pulse intensity (I/I_0) with τ is depicted: initially (at $z = 0$), (dotted curve, black), after compression in homogeneous plasma (at $z = 841\,\mu$m), (dashed blue curve) and after compression in inhomogeneous plasma (at $z = 440\,\mu$m), (solid red curve for $b = 0.001$). The initial laser and plasma parameters: $a_0 = 0.1$, $\tau_0 = 100$ fs, $n_0 = 3.12 \times 10^{21}$ cm^{-3}, $\omega = 5 \times 10^{15}$ rad s^{-1} and for $b = 0.001$. (Color online.)

shows how the pulse form changes during its propagation in the plasma in the presence (solid curve) and in the absence (dashed curve, respectively) of inhomogeneity. As a reference point, we stress that the dashed curve in figure 1 indicates a strong pulse compression in homogeneous plasma, in agreement with earlier results [24]. The solid curve in figure 1 shows that the axial inhomogeneity intensifies the compression of pulse width and pulse intensity. These numerical results suggest the possibility for compression of a real 100 fs laser pulse in relativistic inhomogeneous plasma by a factor thirty or higher.

We have seen that the inhomogeneity in the medium strengthens the nonlinear compression, in a shorter length of the plasma, in comparison with homogeneous plasma. We have numerically integrated equation (16) together with equation (19) to delineate the dependence of the compression length on the inhomogeneity factor. The compression length in the plasma (i.e. the minimum length in the plasma where the pulse becomes compressed (cf figure 1(*b*) in [49])), decreases as inhomogeneity (as expressed by b in (18)) increases, as illustrated in figure 2. It is clear from figure 2 that the compression length is inversely proportional to the inhomogeneity factor b. In simple terms, if the slope is larger, the pulse gets compressed on a shorter distance of propagation. In the absence of density gradient (homogeneous plasma), the relativistic mass variation due to laser–plasma interaction would be the sole origin for longitudinal self-compression of a laser pulse, and for the corresponding increase in intensity. In a non-uniform plasma, though, the source of pulse compression and amplification is the combined density gradient and relativistic nonlinear effect. Due to a weak density gradient, the pulse sees an increasing plasma density as it starts propagating along the z-direction, and the laser beam is thus compressed and amplified in an enhanced manner (in comparison with the uniform plasma case), as shown in figure 1. We point out that the pulse already gets compressed and amplified at an early stage after entering the plasma, where the plasma frequency is low in comparison with the laser frequency. However, the overdense regime of plasma is approached after some time. When the plasma density reaches a critical value, i.e. when the pulse starts to see an overdense plasma, then the high intense field of the

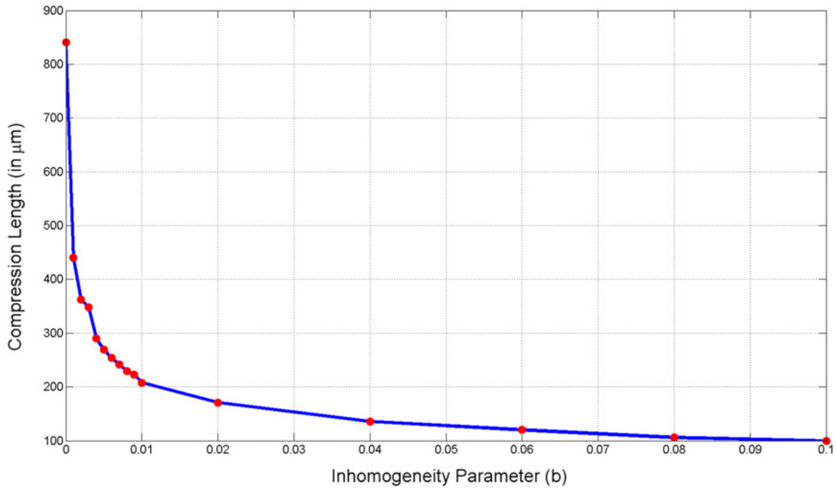

Figure 2. The dependence of the compression length in the plasma on the inhomogeneity factor b. The initial laser and plasma parameters are $a_0 = 0.1$, $\tau_0 = 100\,\text{fs}$, $n_0 = 3.12 \times 10^{21}\,\text{cm}^{-3}$, $\omega = 5 \times 10^{15}\,\text{rad s}^{-1}$. (Color online.)

amplified pulse enhances the nonlinear relativistic effects causing the pulse to penetrate deeper into the plasma.

4. 3D pulse compression: extended model and results

In the previous section, we have considered a quasi-1D pulse, i.e. one whose lateral EM field variation is negligible, so that longitudinal dynamics may essentially be considered. We shall now extend the analysis to a 3D geometry, by assuming that the transverse extent of the laser pulse is finite. The Gaussian pulse profile propagating in the plasma can be written as

$$a^2(r, z = 0, t) = a_0^2 \exp(-r^2/r_0^2) \exp(-t^2/\tau_0^2), \tag{20}$$

where $r = x^2 + y^2$ refers to a cylindrical polar coordinate system and r_0 is the initial spot size of laser. The NLS equation as given by equation (13), in addition to the radial dependence of em field, can be written as

$$2\mathrm{i}\varepsilon_0^{1/2} \frac{\partial a}{\partial \zeta} + \frac{\partial^2 a}{\partial \tau^2} + \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \right) a + \varphi(|a|^2) a = 0, \tag{21}$$

where $\rho = r\omega/c$.

Equation (21) is the simple equation including both longitudinal compression and transverse self-focusing. The second term in equation (21) is known as the group velocity dispersion (GVD) term and it results in pulse compression when combined with (balanced by) nonlinearity. The third term in the above equation (within parentheses) physically represents diffraction, which is necessary for transverse focusing. The last term in equation (21) represents the nonlinear effect, which arises from the relativistic motion of the electrons in the intense laser field. Equation (21) recovers with equation (2.3) of [26] in the weakly relativistic case, $|a| \ll 1$, i.e. $(1 + |a|^2/g)^{3/2} \approx 1$. The evolution of a pulse (relying on equation (21), in the

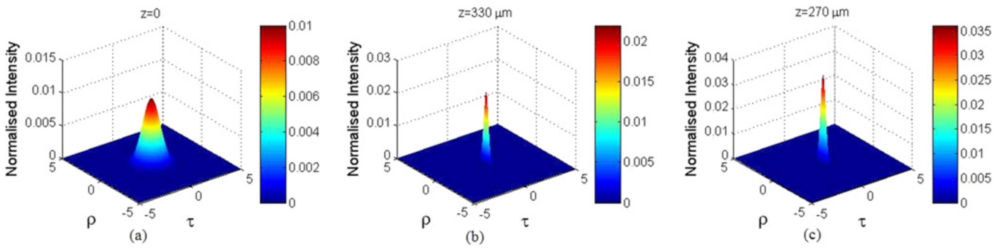

Figure 3. The pulse profile dynamics are depicted at a fixed distance (equivalent to fixed time) in the plasma. (*a*) Initial (at $z = 0$), profile, (*b*) after compression in homogeneous plasma (at $z = 330\,\mu$m) and (*c*) after compression in inhomogeneous plasma (at $z = 270\,\mu$m) for $b = 0.01$. The initial laser and plasma parameters: $a_0 = 0.1$, $\tau_0 = 100$ fs, $n_0 = 3.12 \times 10^{21}$ cm^{-3}, $\omega = 5 \times 10^{15}$ rad s^{-1}. The beam width (spatial extension) and the pulse width (in time) are represented by ρ and τ, respectively. The bar shows the variation in the normalized intensity. (Color online.)

paraxial approximation) in plasma can be expressed as

$$a^2(\zeta,\tau) = \frac{a_0^2(T(0)R(0)^2)}{T(\zeta)R(\zeta)^2}\exp\left[-\frac{\tau^2}{T(\zeta)^2} - \mathrm{i}\frac{\varepsilon_0^{1/2}}{2}\frac{\tau^2}{T(\zeta)}\frac{\mathrm{d}T(\zeta)}{\mathrm{d}\zeta} - \frac{\rho^2}{R(\zeta)^2} - \mathrm{i}\frac{\varepsilon_0^{1/2}}{2}\frac{\rho^2}{R(\zeta)}\frac{\mathrm{d}R(\zeta)}{\mathrm{d}\zeta}\right], \tag{22}$$

where $T(\zeta) = \tau_1 g(\zeta)$ is the pulse width (time) in plasma and $R(\zeta) = \rho_1 f(\zeta)$ is the beam width (space) in the radial direction in plasma. Here, $T(0) = \tau_1 = \tau_0\omega$ is the initial dimensionless pulse width (at $\zeta = 0$) and $R(0) = \rho_1 = r_0\omega/c$ is the initial dimensionless beam width.

The evolution of a finite extent Gaussian laser pulse in plasma can be investigated by calculating the beam width $R(z)$ and the pulse length $T(z)$. These quantities are proportional to the beam width parameter $f(z)$ (given by equation (23)) and the pulse width parameter $g(z)$) (given by equation (24)), respectively. Therefore, the spatiotemporal profile of the laser pulse can be obtained by numerically solving the following two coupled second order differential equations for the beam width and pulse width:

$$\varepsilon_0\frac{\mathrm{d}^2 f}{\mathrm{d}\zeta^2} = \frac{1}{\rho_1^4 f^3} - \frac{2(\omega_\mathrm{p}^2/\omega^2)}{\rho_1^2}\frac{|a|^2}{f^3 g(1+|a|^2/f^2 g)^{3/2}} \tag{23}$$

and

$$\varepsilon_0\frac{\mathrm{d}^2 g}{\mathrm{d}\zeta^2} = \frac{1}{\tau_1^4 g^3} - \frac{(\omega_\mathrm{p}^2/\omega^2)}{\tau_1^2}\frac{2|a|^2}{f^2 g^2(1+|a|^2/f^2 g)^{3/2}}. \tag{24}$$

For an initial plane wave the boundary conditions on equation (23) and (24) are taken at $\zeta = 0$ as $f = g = 1$. Equations (23) and (24) can be numerically integrated using the initial boundary conditions to evaluate the beam width parameter f and pulse width parameter g as a function of z. The nonlinear terms appearing in the above equations are obtained by expanding the expression of $\varphi(|A|^2)$ in the paraxial approach and then collecting the terms in r^2.

In this case we perform the numerical computation for the following laser–plasma parameters $a_0 = 0.1$, $\tau_0 = 100$ fs, $n_0 = 3.12 \times 10^{21}$ cm^{-3}, $\omega = 5 \times 10^{15}$ rad s^{-1} and for $b = 0.01$. In figure 3, the pulse profile is depicted at a fixed distance (corresponding to a specific time) in the plasma. Figure 3(*a*) shows the initial normalized intensity of the Gaussian laser pulse at $z = 0$. Figure 3(*b*) shows the normalized pulse intensity after compression in

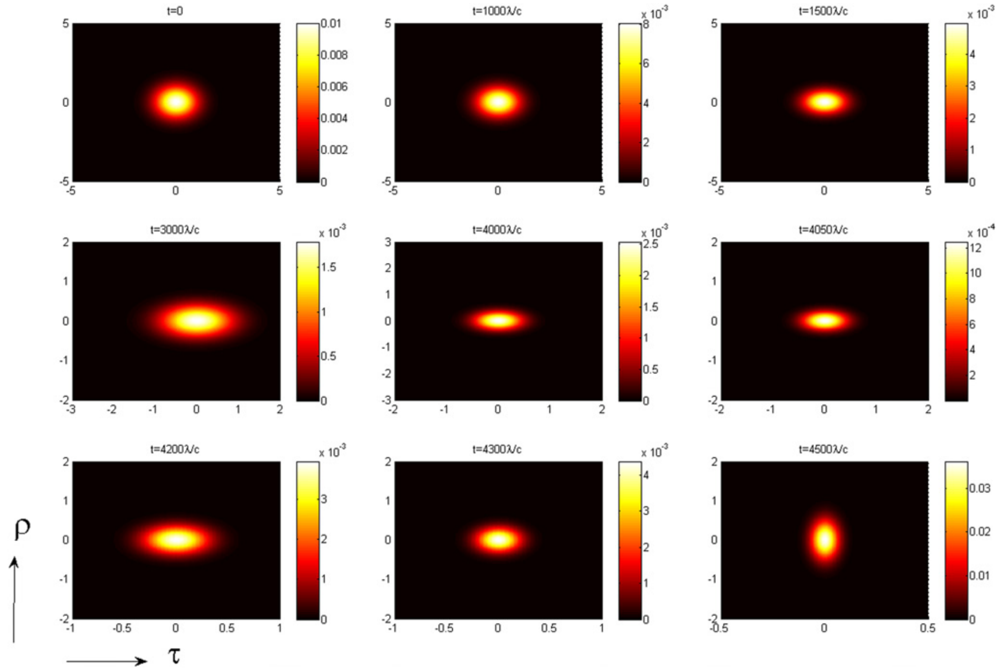

Figure 4. The spatiotemporal plots of the normalized pulse intensity (I/I_0) are depicted at different fixed times (equivalent to fixed distances). The initial laser and plasma parameters: $a_0 = 0.1$, $\tau_0 = 100$ fs, $n_0 = 3.12 \times 10^{21}$ cm^{-3}, $\omega = 5 \times 10^{15}$ rad s^{-1} and $b = 0.01$. The x and y axes, respectively, present the pulse width in time (τ) and beam width in space (ρ). The bar represents the variation of the normalized intensity. (Color online.)

a homogeneous (uniform density) plasma, at $z = 330\,\mu$m. To compare, we have traced the pulse dynamics in the presence of axial inhomogeneity. The results, depicted at a propagation distance $z = 270\,\mu$m in figure 3(c), clearly confirm the occurrence of pulse shortening (compression), in addition to an increase in pulse intensity (localization). The transverse focusing of the pulse results on account of relativistic mass effect in combination with non-uniform distribution of the electron density and hence the inhomogeneity of the dielectric functions. The shortening of the pulse length can be interpreted as a consequence of the focusing phenomenon. The less intense portions of the pulse would focus less rapidly; hence, over a distance of the order of a focusing length, the intensity of the peak is considerably enhanced whereas the portions away from it are less significantly enhanced; this results in longitudinal pulse compression.

In order to investigate the interplay (and also distinguish) between the simultaneously occurring longitudinal self-compression and transverse self-focusing effects, we have depicted the spatiotemporal dynamics of the normalized pulse intensity profile, initially (at $t = 0$) and at a fixed time (equivalent to fixed distance), as the pulse propagates in the plasma medium. The results are depicted in figure 4. At times $t = 1000$, 1500 and 3000 (in units of λ/c), when inhomogeneity in the plasma medium is weak, the transverse focusing of the laser pulse dominates over the longitudinal pulse compression. Later on, at times $t = 4000$, 4050, 4200, 4300 and 4500 (time unit λ/c), when the density gradient in the plasma starts to build up strongly and the electric field of the laser is strong enough to cause longitudinal pulse compression to dominate over the transverse focusing phenomena. Hence, we observe that when the inhomogeneity is weak, the beam width (in space) reduces from its initial width leaving unaffected the pulse width (in space) and as the inhomogeneity starts to increase; the beam width (in space) gets its original size while the compression in pulse width (in time) becomes more effective. The plots in figure 4 depict the transverse focusing of the laser pulse, which is followed by a longitudinal pulse compression due to the combined effect of relativistic mass variation and density gradient present in the plasma. The numerical results suggest that transverse focusing somehow competes with the process of longitudinal self-compression.

5. Conclusions

We have investigated the evolution of a relativistic laser beam propagating in a non-uniform plasma, by focusing on the longitudinal compression (1D) and on the lateral-and-longitudinal compression of a finite-radius em pulse (3D). Relying on nonlinear paraxial theory (recently extended by Sharma *et al* [41]), we have developed an analytical model and then proceeded by numerical computation. Our results strongly recommend a significant enhancement of relativistic pulse compression in an axially inhomogeneous plasma.

We have first analyzed the longitudinal pulse compression in 1D geometry, assuming a uniform transverse distribution of the irradiance profile. We have followed the pulse dynamics, witnessing the compression of the pulse (by tracing the full-width-half-maximum variation), along with a significant increase in intensity (localization mechanism). Although the effect was qualitatively similar with and without plasma non-uniformity taken into account, including an axial inhomogeneity was shown to lead to a significant enhancement of the compression and localization mechanisms, in comparison with the same effect in uniform (homogeneous) plasma. We have observed a length compression by more than ten times, for real laser parameter values.

We have extended the description to a 3D (cylindrical) geometry, by considering the longitudinal self-compression and transverse self-focusing of a finite extent Gaussian laser pulse. We have observed the interplay between longitudinal self-compression and transverse self-focusing, as the pulse propagates in the plasma. Figures 3 and 4 demonstrate a significant longitudinal relativistic pulse compression in an inhomogeneous plasma (higher than in a uniform one).

From a fundamental point of view, the elucidation of the physical mechanisms underlying the evolution of short intense pulses propagating in plasma is of outmost importance. The qualitative aspects of our model are generic, and may be adapted to other types of nonlinearity (e.g., collisional and ponderomotive). Regarding applications, it is challenging to envisage the achievement of higher peak power short and ultraintense laser pulse regimes in millimeter-scale plasma. A future advancement in power level and pulse duration achieved could enable the construction of a new generation of compact, low cast ultrahigh-intensity laser system, of interest in many areas of science and a potential replacement for particle accelerators.

The model employed in this paper may be extended further to investigate laser pulse compression in magnetized plasmas. In principle, a magnetic field is expected to affect the spatial compression mechanism of the incident wave. Furthermore, the propagation characteristics (group velocity, frequency) of the wave can be affected by magnetic field inhomogeneity, e.g., considering a uniform in space, yet adiabatic in time, variation of the magnitude and/or direction of the magnetic field. A shift in the group velocity could result in a shortening of the temporal duration and a frequency shift of the outgoing compressed pulse. The possibility of a dynamical manipulation of laser radiation through a plasma medium opens new avenues for manipulating high-power pulses for industrial applications.

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