Superluminal Electromagnetic Solitary Waves in Electron-Positron Plasmas

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Abstract – The propagation of an electromagnetic wavepacket in an electron-positron plasma, in the form of coupled localized electromagnetic excitations, is investigated, from first principles. By means of the Poincaré section method, a special class of superluminal localized nonlinear stationary solutions, existing along a separatrix curve, are proposed as intrinsic electromagnetic modes in a relativistic electron-positron plasma. The ratio of the envelope time scale to the carrier wave time scale of these envelope solitary waves critically depends on carrier's phase velocity. In the strongly superluminal regime, $v_{ph}/c \gg 1$, the large difference between the envelope and carrier time scales enables us to carry out a multiscale perturbative analysis resulting in an analytical form of the solution envelope. The analytical prediction thus obtained is shown to be in agreement with the solution obtained via a direct numerical integration.

Introduction. – Electron-positron (e-p) plasmas occur in various astrophysical environments, including pulsar magnetospheres [1–4], in bipolar outflows (jets) in active galactic nuclei (AGN) [5, 6]. Such plasmas have been argued to occur at the center of our own galaxy [7], and are thought of as a first state of matter in the early universe [8]. Experimental efforts are being made to create such plasmas in the laboratory in order to mimic astrophysical conditions, thanks to modern day laser techniques that promise to deliver pulse intensities exceeding 10^{22} W/cm^2 in the near future [9]. e-p plasma dynamics arises as a relevant paradigm in the ultra-high intensity laser plasma interaction [10]. The feasibility of multiphoton production of e-p pairs in a plasma by electromagnetic waves has been discussed in Ref. [11]. Nonrelativistic pair plasmas have also been created in experiments [12]. Finally, the possibility of pair production in large magnetic plasma confinement fusion devices (*tokamaks*) [13] due to collisions between multi-MeV runaway electrons and thermal particles has been suggested [14]. From a fundamental modeling viewpoint, localized excitations in electron-positron plasmas may occur either in the presence of an external magnetic field (e.g., in pulsar magnetospheres), or in unmagnetized plasmas [15].

The propagation of electromagnetic waves in electron-positron plasmas has been an active area of research in the recent past. Many theoretical investigations have been carried out to understand the instabilities a light wave might suffer while passing through an electron-positron plasma [16, 17]. Since localized energy lumps (in the form of solitary waves) are considered to be an intrinsic element in turbulent plasmas, the occurrence of coherent electromagnetic modes in an electron-positron plasma is a topic of fundamental interest. The formation of such nonlinear coupled modes as a final state of fully developed modulational instability have been discussed in many earlier works, both in plasmas [17–20] and, e.g., in condensed matter physics [21]. Moreover, a series of earlier works have addressed the problem of linearly polarized electromagnetic modes in electron-ion plasmas [22–26] and in electron-positron plasmas [27]. The case of circularly polarized dark solitons in an unmagnetized electron-positron plasma has been investigated by Farina and Bulanov [28] and that of circularly polarized bright-type envelope solitary waves in an unmagnetized electron-positron-ion plasma has been studied by Berezhiani and Mahajan [18]. In another work, Berezhiani et al. [17] have investigated large amplitude bright electromagnetic solitary waves in a magnetized electron-positron plasma. In a recent work by Saxena *et al.* [29], superluminal solitary solutions were shown to exist for the case of an electron plasma with immobile ions. There, the solitary solution was found to correspond to a separatrix curve on the Poincaré section surface plot. An approximate mathematical form for the envelope of the solitary wave solution was found for large velocities by using a multiple scale perturbative expansion. To our best knowledge, the case of linearly polarized electromagnetic solitary waves in a cold unmagnetized pure electron-positron plasma has not yet been studied and forms the subject of present manuscript.

The two-fluid plasma-dynamical model (for electrons and positrons) coupled with the Maxwell equations provide a working paradigm for understanding the nonlinear interaction of an intense electromagnetic wave with an unmagnetized cold electron-positron plasma. Such a nonlinear model admits a wide range of solutions which have not been fully explored yet. Our work aims at elucidating a special class of nonlinear localized superluminal solitary solutions which have not been investigated in the past. To this effect, we extend the approach proposed in Ref. [29] for electron-ion plasma with immmobile ions. First, we derive the fluid-Maxwell set of equations for an electron-positron plasma. Adopting a transformation of variables, we derive a simpler set of equations for superluminal pulses in the traveling reference frame. A constant of motion (a "pseudo-Hamiltonian") is then obtained, analogous to a classical Hamiltonian with two degrees of freedom. The solutions of the coupled nonlinear system of equations are shown on Poincaré surface section plots for different values of the phase velocity variable, in the regime $\beta > 1$ ($\beta = v_{ph}/c$). It is found that the island structures on the Poincaré section plots, which correspond to amplitude modulated waves, disappear as the phase velocity is increased from the weakly superluminal ($\beta - 1 \ll 1$) to the strongly superluminal ($\beta \gg 1$) regime, for a given value of the total energy of the system (Hamiltonian). We proceed by showing that a non-chaotic separatrix curve exists in the regime $\beta - 1 \sim 1$, corresponding to a localized electromagnetic solitary wave. The amplitude of the solitary wave in this regime is well in the relativistic regime and the spatiotemporal scales of the solitary wave envelope and the carrier wave are not too far apart. It is therefore difficult to obtain an analytical form for the solution in this parameter region. However, for substantially high velocities $\beta \gg 1$, the amplitudes are weakly relativistic and the envelope and carrier wave scales differ by orders of magnitude. This allows one to obtain an approximate analytical form for the envelope by means of a multiple scale perturbative analysis. The analytical envelope solution is shown to nicely match the envelope of the numerically obtained solution.

The paper is organized as follows. In the next section we discuss the model equations and their reduction to a Hamiltonian system with two degrees of freedom. In Section III, the possible solutions in different phase velocity regimes are discussed using Poincaré section surface plots. The existence of a solitary wave solution along the separatrix curve is shown and the differences in the characteristics of solitary solutions in different carrier phase velocity regimes are investigated. In Section IV, a multiple scale perturbative analysis is presented in the $\beta \gg 1$ regime and a comparison of the thus obtained analytical envelope solution with the numerical solution is made. We then discuss the characteristics of the novel solitary envelope solutions presented here. Finally, our results are summarized in the last section.

Model equations and reduction to a Hamiltonian system. – The interaction of a relativistic laser pulse with an electron-positron plasma is described by the following one dimensional fluid-Maxwell model:

$$\mathbf{A}_{xx} - \mathbf{A}_{tt} = \left(\frac{n_e}{\gamma_e} + \frac{n_p}{\gamma_p}\right)\mathbf{A}\,, \tag{1}$$

$$\phi_{xx} = n_e - n_p\,, \tag{2}$$

$$(p_{e,p})_t = (\pm\phi_x - \gamma_{e,p})_x\,, \tag{3}$$

$$(n_{e,p})_t + \left(\frac{n_{e,p}\,p_{e,p}}{\gamma_{e,p}}\right)_x = 0\,. \tag{4}$$

The relativistic factor $\gamma_{e,p}$ is given by $\gamma_{e,p} = \sqrt{1 + |\mathbf{A}|^2 + p_{e,p}^2}$. The indices e and p distinguish the dynamical relations for the electrons and the positrons, respectively, while the subscripts x and t denote differentiation with respect to space and time, respectively. Here, $\mathbf{A}$ and ϕ represent the electromagnetic vector potential and the scalar potential both normalized by $m_e c^2/e$ where m_e and e are the mass and electric charge of the electron respectively; $n_{e/p}$ stands for the electron/positron density normalized by the background plasma density n_0; $p_{e/p}$ the electron/positron longitudinal momentum and are normalized by $m_e c$. The length is normalized by the electron skin depth c/ω_{pe0} (where $\omega_{pe0} = \sqrt{4\pi n_0 e^2/m_e}$ is the electron plasma frequency) and time by the inverse of the plasma frequency ω_{pe0}^{-1}.

We have assumed linear polarization of the electromagnetic wave, so that $\mathbf{A} = \hat{a}\,e_1$, and focus on the superluminal case i.e. $\beta > 1$. Now, we first make a plane wave *ansatz* and perform a variable transformation $\xi = x - \beta t$, from the laboratory frame to a frame moving at the phase velocity of the carrier electromagnetic wave, β. Then we define the moving coordinate $\xi' = \xi/(\beta^2-1)^{1/2}$ (where the prime in ξ will henceforth be dropped) and also define scaled variables: $\left(\beta^2-1\right)^{1/2} a = X$ and $1 + \phi = -Z$. The above transformations result in the following simplified forms for the scalar and vector potential equations:

$$\ddot{X} + \beta\left[\frac{1}{\sqrt{\beta^2 - 1 + X^2 + Z^2}} + \frac{1}{\sqrt{\beta^2 - 1 + X^2 + (Z+2)^2}}\right]X = 0 \tag{5}$$

and

$$\ddot{Z} + \beta \left[\frac{Z}{\sqrt{\beta^2 - 1 + X^2 + Z^2}} + \frac{(Z+2)}{\sqrt{\beta^2 - 1 + X^2 + (Z+2)^2}} \right] = 0 \,. \qquad (6)$$

The set of equations (5) and (6) admit the following constant of motion

$$H = \frac{1}{2}\dot{X}^2 + \frac{1}{2}\dot{Z}^2 + \beta \left[\sqrt{\beta^2 - 1 + X^2 + Z^2} + \sqrt{\beta^2 - 1 + X^2 + (Z+2)^2} \right] . \qquad (7)$$

This set of equations is formally analogous to those obtained for an electron plasma (keeping ions fixed) in Refs. [24] and [29], and admit a wide class of nonlinear solutions including periodic, quasi-periodic, amplitude modulated and solitary wave solutions. In the next section, we numerically obtain some of the solutions and display them on the Poincaré section surface plots for different parameter regimes. Special emphasis is given on a novel class of solitary wave solutions.

Nonlinear solutions: Poincaré analysis. – The technique of Poincaré surface section diagrams has been very helpful in understanding and analyzing the behavior of dynamical systems, as it allows one to trace the properties of periodic and quasi-periodic orbits of the original higher-dimensional system, as projected on a lower-dimensional space (Poincaré surface). Kaw *et al.* [24] have elegantly shown various classes of solutions of a coupled fluid-Maxwell model describing the electromagnetic wave propagation in cold electron plasmas with fixed ion background, by making use of the Poincaré section method. Saxena *et al.* [29] extended their analysis to obtain localized solitary solutions along the separatrix curve on the Poincaré section plot. Here, we extend this formalism further, in order to obtain similar localized solutions in the more interesting case of an electron-positron plasma. We solve the set of equations (5)-(7) by using a 4^{th} order Runge-Kutta method to obtain possible solutions and to analyze the nonlinear solutions by means of Poincaré section plots.

In Figures 1a and 1b, we show the Poincaré section plots for $\beta = 1.001, H = 10$ and for $\beta = 1.1, H = 10$, respectively. Upon comparing the two plots, one can notice the difference between the characteristics of possible solutions. While in the (weakly superluminal) $\beta = 1.001$ case, there exist interesting island structures corresponding to amplitude modulated waves, such solutions cease to exist for $\beta = 1.1$. Moreover, we notice that the separatrix region in the Poincaré plot is somewhat scattered, whereas the separatrix curve for $\beta = 1.1$ is very clear and corresponds to a nice localized solitary solution. We depict the corresponding solitary solution in Fig. 2, where we find that the coupled solitary wave solution exhibits a dip in electromagnetic potential coupled to a co-propagating hump

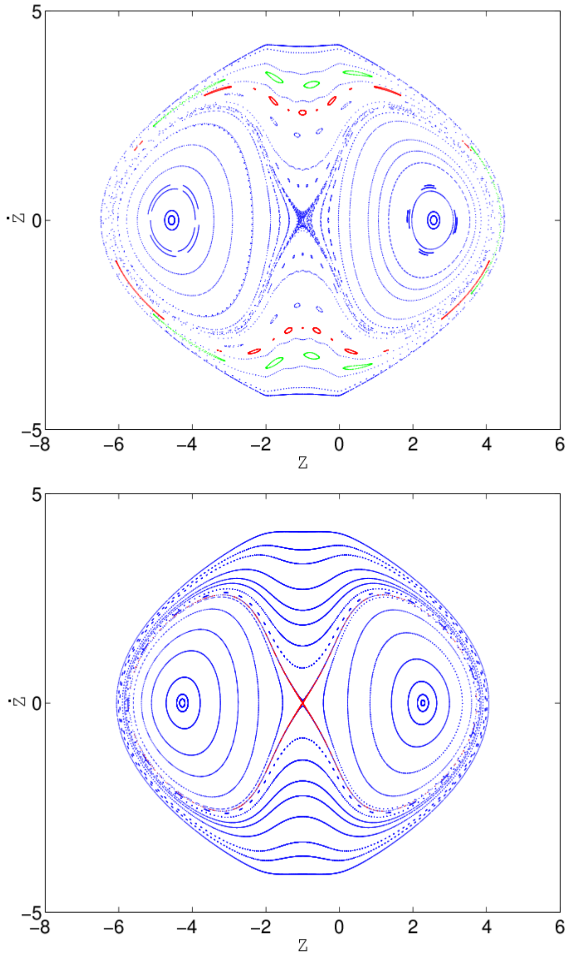

Fig. 1: (color online) Poincaré section surface plot, $\dot{Z}$ vs Z ($X = 0$, $\dot{X} > 0$), for (a) $\beta = 1.001$ and $H = 10$ (upper subplot) and (b) $\beta = 1.1$ and $H = 10$ (lower subplot).

in the electrostatic potential profile. Also, the time scales of the carrier wave and the envelope are not very far apart. The corresponding amplitudes are in the strong relativistic regime, as $a \sim X/\sqrt{\beta^2 - 1} \approx 100$.

In Fig. 3 we depict a solitary wave solution obtain in the (strongly superluminal) case $\beta = 20$. It is worth noting that the amplitudes for this solution lie in the weakly relativistic regime, $a \sim X/\sqrt{\beta^2 - 1} \approx 0.2$, unlike the previous case in (Fig. 2). The electrostatic potential amplitude is in the overcritical regime [27] and hence the frequencies of oscillations in electromagnetic potential and electrostatic potential are equal. The density profiles of electron and positron fluid corresponding to the solution shown in Fig. 3 are displayed in Fig. 4. We would like to point out here that the electron and positron fluid densities have a similar envelope profiles. It can be noticed that in the centre of the structure their oscillations are completely out of phase which indicates a strong charge separation and thus excitation of a strong localized electrostatic wave (represented by Z in the right plot of Fig. 3). Interestingly, the scales of the carrier wave and of the envelope are far apart,

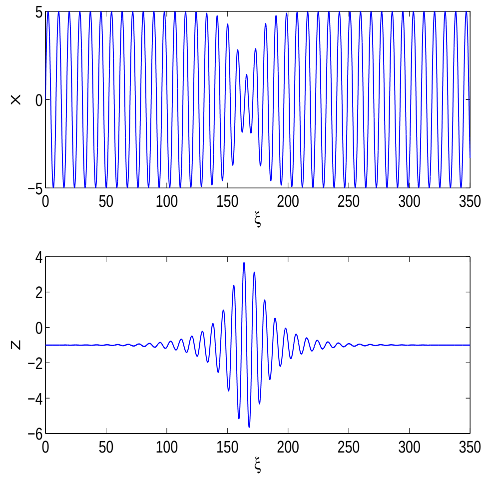

Fig. 2: (color online) Soliton solution corresponding to the separatrix curve shown in Fig. 1(b), for $\beta = 1.1$ and $H = 10$. The upper panel corresponds to the transverse field X, and the lower panel shows the profile of the electrostatic field Z.

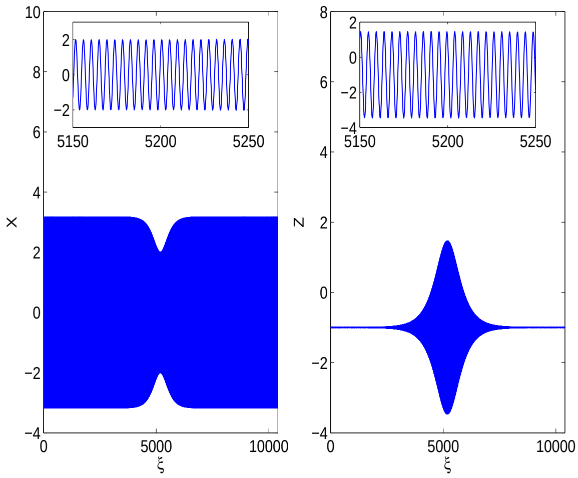

Fig. 3: (color online) Soliton solution for $\beta = 20$ and $H = 410$. The left plot shows the spatial profile of scaled electromagnetic potential (X) whereas right plot displays the profile of electrostatic potential (Z). In the inset of each plot, expanded view around the central part of the corresponding profile is shown.

by orders of magnitude. At this stage, one may exploit the smallness of field amplitudes as well as the large separation between the scales (ratio $\sim \beta^2 - 1 \sim 400$), in order to to obtain an approximate mathematical form of the solitary wave envelope by performing a multiple-scale perturbative analysis. We present such an analysis in the next Section, and then provide a comparison of the analytical envelope solution with the envelope of the solitary wave solution obtained via numerical simulation of the original fluid system.

Multiple scale perturbative analysis: envelope solution in the $\beta \gg 1$ regime. – As a matter of fact, the set of coupled nonlinear equations (5)-(7) can be solved numerically, for any values of the relevant parameters. Nonetheless, it is certainly beneficial to dispose of analytical insight on possible nonlinear solutions. A handy analytical expression for these coupled electromagnetic modes will be useful to have, for a better theoretical understanding of the coupling between the longitudinal and transverse waves. In principle, having an analytical solution at hand should also provide a platform for further investigating the stability of the localized solutions (this is, however, outside the scope of present investigation). In this Section, we follow a similar methodology as used by Saxena *et* al. for electron-ion plasma with fixed ions [29], to find an analytical form of the envelope of small amplitude solitary solution in strong superluminal regime.

In the limit $\beta \gg 1$ and for $\left[X^2 + (Z+2)^2\right]/(\beta^2 - 1) \ll 1$, we may Taylor-expand the nonlinear terms in equations (5)-(7), thus reducing them in the lowest nonlinear approximation to

$$\frac{d^2 X}{d\xi^2} + \omega_0^2 X - \frac{\epsilon\omega_0^2}{2} X\left(X^2 + Z^2 + 2Z + 2\right) = 0 \qquad (8)$$

and

$$\frac{d^2 Z}{d\xi^2} + \omega_0^2 (Z+1) - \frac{\epsilon\omega_0^2}{2}\left[Z\left(X^2 + Z^2\right) + X^2 + 3Z^2 + 6Z + 4\right] = 0\,. \qquad (9)$$

where $\omega_0 = \sqrt{2\beta/\sqrt{\beta^2 - 1}}$ is the frequency of the linearized equations, and $\epsilon = 1/(\beta^2 - 1)$ is a (real) smallness parameter.

Following the standard procedure of multiple time scale perturbation analysis [30], we define new time variables $\xi_0 = \xi$ and $\xi_1 = \epsilon\xi$, and proceed to obtain, in successive orders

$$\frac{d}{d\xi} = \frac{\partial}{\partial\xi_0} + \epsilon\frac{\partial}{\partial\xi_1}$$
$$\frac{d^2}{d\xi^2} = \frac{\partial^2}{\partial\xi_0^2} + 2\epsilon\frac{\partial^2}{\partial\xi_1\partial\xi_0} + O(\epsilon^2)\,. \qquad (10)$$

Furthermore, the fields can be expanded as

$$X = X^{(0)} + \epsilon X^{(1)}\;,\; Z = Z^{(0)} + \epsilon Z^{(1)}\,. \qquad (11)$$

Following a similar methodology used in Saxena *et* al. [29], the analytical expressions of the envelope are obtained as

$$A(\xi) = \left\{\frac{M_3 + \sqrt{M_3^2 - 3M_2M_4}}{3M_4} - \frac{\sqrt{M_3^2 - 3M_2M_4}}{M_4}\right.$$
$$\left.\operatorname{sech}^2\left[\frac{\left(M_3^2 - 3M_2M_4\right)^{1/4}}{2}(\epsilon\xi + d_1)\right]\right\}^{1/2} \qquad (12)$$

and

$$B(\xi) = \left\{-\frac{L_3 + \sqrt{L_3^2 + 3L_2L_4}}{3L_4} + \frac{\sqrt{L_3^2 + 3L_2L_4}}{L_4}\right.$$

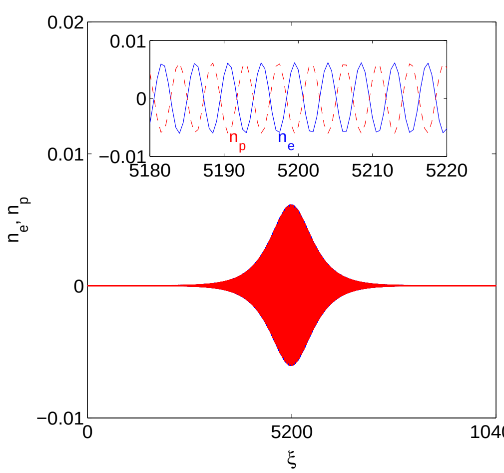

Fig. 4: (color online) The profiles of electron and positron fluid densities associated with the solution shown in Fig. 3. The envelopes of the two species densities overlap with each other, however, on expanding the central region of the profiles, one can see that electron density (solid line) and positron density (dashed curve) are completely out of phase indicating a strong space charge separation.

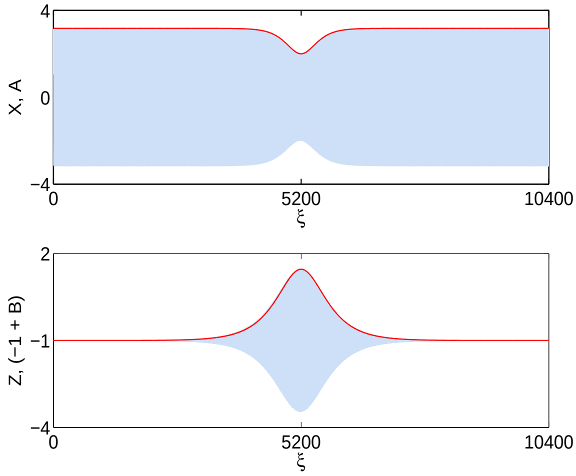

Fig. 5: (color online) Comparison of the analytically obtained envelopes with the numerically obtained exact solution, for $\beta = 20$ and $H = 410$. The analytical envelope curves drawn in solid curves are seen to fit the envelope quite nicely.

$$\left.\operatorname{sech}^2\left[\frac{\left(L_3^2+3L_2L_4\right)^{1/4}}{2}\left(\epsilon\xi+d_2\right)\right]\right\}^{1/2}, \quad (13)$$

where d_1 and d_2 are constants of integration, which can be calculated by using the initial values of A and B. Constants M_1, M_2, M_3, M_4, L_1, L_2, L_3 and L_4, have the following definitions

$$M_1 = \frac{\omega_0^2 C_1^2}{16}, \qquad M_2 = \frac{\omega_0^2}{16}\left(C_1C^2+8C_1\right)$$
$$M_3 = \frac{\omega_0^2}{16}\left(C_1+4C^2+16\right), \qquad M_4 = \frac{\omega_0^2}{4}$$
$$L_1 = \frac{\omega_0^2 C_2^2}{16}, \qquad L_2 = \frac{\omega_0^2}{16}\left(C_2C^2-8C_2\right)$$
$$L_3 = \frac{\omega_0^2}{16}\left(C_2-4C^2+16\right), \qquad L_4 = \frac{\omega_0^2}{4}.$$

where C is given by $A^2+B^2 = \text{constant} = C^2$, whereas $C_1 = A^2\left(C^2-A^2\right)\sin^2\left(\phi_2-\phi_1\right)+4A^2$, and $C_2 = B^2\left(C^2-B^2\right)\sin^2\left(\phi_2-\phi_1\right)-4B^2$ are constants of integration. The complete approximate solution can then be written as

$$X = A\left(\xi_1\right)\cos\left(\omega_0\xi_0+\phi_1\left(\xi_1\right)\right) \quad (14)$$

$$Z = B\left(\xi_1\right)\cos\left(\omega_0\xi_0+\phi_2\left(\xi_1\right)\right)-1 \quad (15)$$

In Fig. 5 we compare the profile of the analytically obtained envelope solutions for A and B from Eqs. (12) and (13), with the exact numerical solutions. As can be seen, the numerical solutions are in perfect agreement with our analytical predictions.

Discussion. – The solutions discussed in this letter for a cold unmagnetized electron-positron plasma are new. The frequencies of oscillations both in the electromagnetic as well as in the electrostatic field profiles are equal, and correspond to the overcritical amplitude regime ($\phi > 1$) discussed by Shiryaev [27]. This provides a lower bound on the amplitude of the localized superluminal coupled solitary wave. It is important to note that, unlike in the case of an electron-ion plasma with immobile ions [29], in the present case we have localized solutions even for phase velocities nearing the speed of light $\beta - 1 < 1$. Moreover, for a given phase speed, the carrier frequency ω_0 in the electron-positron plasma case is $\sqrt{2}$ times the carrier frequency in the electron-ion plasma with immobile ions.

The superluminal phase speed $c/V_{ph} \ll 1$ of these solutions, corresponds to plasma densities very close to critical density. Such a near-critical density case has recently been discussed by Pesch and Kull [31–33] for electron-ion plasmas with immobile ions. In our opinion, the coupled envelopes of the electromagnetic wave as well as the electrostatic wave move with a velocity equal to the phase velocity (and so does the density perturbation), however, the definition of the group velocity for such coupled structures is not clear, and appears to be physically close to the phase speed (since the envelope excitations are static in the moving frame).

It is important to mention here that superluminal solutions are known to be unstable [34] and possess a finite lifetime $\sim L/(U-c)$, where L is the typical spatial extent of the structure, U is its phase speed and c is the speed of light. However, in our case, we notice that the life time of these localized structures is of the order of several plasma periods and therefore they will have sufficient time to excite wake fields by Vavilov-Cerenkov effect [34] which can be detected by existing diagnostic techniques.

Summary. – We have considered the propagation of a relativistically intense electromagnetic wave in cold collisionless electron-positron plasmas, in the form of a strongly modulated dark solitary wave like structure of electromagnetic component coupled with a localized plasma wave excitation. These coupled modes travel with superluminal phase velocities and constitute a special class of nonlinear solutions of the fluid-Maxwell system of equations describing the coupling of a linearly polarized light wave to a cold relativistic electron-positron plasma. We have numerically shown such specialized solutions to exist for a wide range of amplitudes, however, an analytical form of their envelope can be obtained only in the low amplitude case, using multiple scale perturbative analysis. Our results provide a first prediction for the existence of localized hybrid electrostatic/electromagnetic envelope excitations in cold unmagnetized electron-positron plasmas. This may be an efficient scenario for energy localization in e-p plasmas, of potential relevance in ultra-intense laser-plasma interactions in the laboratory, as well as in astrophysical environments. Moreover, the availability of an explicit expression for the solution envelope as well as the gained knowledge of the parameter regimes of existence of localized solutions will be useful for investigating the stability of solutions. This will help in finding the parameter regime of relevance in future laser plasma experiments where such localized nonlinear excitations involving positron dynamics can be seen.

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