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Electromagnetic Rogue Waves in Beam-Plasma Interactions

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Abstract. The occurrence of rogue waves (freak waves) associated with electromagnetic pulse propagation interacting with a plasma is investigated, from first principles. A multiscale technique is employed to solve the fluid-Maxwell equations describing a weakly nonlinear circularly polarized electromagnetic pulses in magnetized plasmas. A nonlinear Schrodinger (NLS) type equation is shown to govern the amplitude of the vector potential. A set of non-stationary envelope solutions of the NLS equation are considered, as potential candidates for modeling of rogue waves (freak waves) in beam-plasma interactions, namely in the form of the Peregrine soliton, the Akhmediev breather and the Kuznetsov-Ma breather. The variation of the structural properties of the latter structures with relevant plasma parameters is investigated, in particular focusing on the ratio between the (magnetic field dependent) cyclotron (gyro-) frequency and the plasma frequency.

1. Introduction

Reports of extreme wave events abound in ocean seafarer stories: an ultra-high *ghost wave* occurs unexpectedly, propagates for short times destroying everything in its passing and then disappears without a trace [1, 2]. Initially due to their catastrophic effect (involving loss of human lives, beyond material disasters [1, 2]) and subsequently due to the fundamental scientific interest involved, the elucidation of the mechanisms underlying the formation and dynamics of such structures has attracted significant interest recently [3]. Rogue waves (or freak waves, or monster waves, or rogons, or WANDTs, viz. *Waves that appear from nowhere and disappear without a trace* [4]) are now recognized as proper intrinsically nonlinear structures (beyond an initial attempt to identify them as superposed linear modes). Most interestingly, fundamental research has by now gone beyond the standard ocean-surface-dynamical problem, tracing rogue waves in nonlinear optics [5, 6, 7], in superfluidity [8], in hydrodynamics [9], in atmospheric dynamics [10] and even in econophysics [11], where these model extreme events via the quantitative Black-Scholes theory [12]. Of particular interest are studies based on the idea that rogue waves may occur as a result of wave-wave interaction (colliding states) [13, 14]. A recent series of studies of interacting optical pulses has attempted to shed some light in this challenging area [15, 16, 17], thus proposing soliton interactions as a rogue-wave formation mechanism in its own right.

Due to the intrinsically random nature of freak waves and the complex mechanisms involved in their formation, these are most often modeled via computer simulations. Nonetheless, it was shown at a very early stage [18] that exact "breather"-type solutions of the nonlinear Schrödinger equation (NLSE), well-known to govern the nonlinear propagation of modulated wavepackets in various physical contexts [19], reproduced the qualitative characteristics of freak waves to a highly satisfactory extent. The early study by Dysthe and Trulsen [18] relied on earlier pioneering works by Peregrine [20, 21], Kuznetsov [22], Ma [23] and Akhmediev [24], succeeding to construct an analytical toolbox for freak waves whose relevance was later established experimentally in different frameworks [6, 9, 25]. The study at hand builds up on the foundations set by Dysthe and Trulsen.

In large ensembles of charged particles (*plasmas*), rogue waves may be anticipated as isolated events in the form of extreme amplitude electric/magnetic field excitations, which occur, e.g., in laser-plasma interaction experiments, and may also presumably be detected in satellite data-series. Due to their unpredictable and random nature, these may arguably be hard to detect and even harder to identify via rigorous diagnostics. Nonetheless, their fundamental interest (and presumably catastrophic effect) suggest an urge for the investigation of their occurrence in plasmas.

The dynamics of modulated wavepackets in plasmas is long-known to be described by generic NLSE models [26, 27, 28] or their variants, e.g., the derivative NLSE [29, 30]. Despite the striking formal analogy to the aforementioned formalism, it was only recently that a first attempt was carried out to link the rogue wave paradigm to plasma dynamics

[30]. That first heuristic – though arguably pioneering – attempt [30] was followed by a small number of studies, tracing the building blocks for freak wave formation in various plasma configurations. As main representatives, we cite a number of works (mainly by Shukla and coworkers), which investigated the possibility for rogue wave formation in contexts like Langmuir waves [31], Alfvén waves [32], surface plasma waves [33] and dusty plasmas [34]. Interestingly, an experimental investigation of the relevance of the rogue wave paradigm in plasmas was recently reported for the first time [25], to our best knowledge. Rogue waves thus appear as a rising paradigm in plasma dynamics, in the dawn of its exploration.

Inspired by the ubiquity of this challenging phenomenon, we have undertaken an investigation, from first principles, of the occurrence of rogue waves associated with electromagnetic pulse propagation interacting with a plasma. A multiscale technique is employed to solve the fluid-Maxwell equations describing a weakly nonlinear *circularly polarized electromagnetic* (CPEM) pulses in magnetized plasmas. A nonlinear Schrodinger type equation is shown to govern the amplitude of the vector potential. A set of non-stationary envelope solutions of the NLSE is presented, and the variation of their structural properties with the magnetic field have been investigated. Finally, an *ad hoc* numerical study of interacting pulses is briefly presented.

2. A fluid-plasma/Maxwell model for electromagnetic excitations

Let us consider a plasma consisting of singly ionized ions and electrons, permeated by a uniform magnetic field $\vec{B} = B\hat{x}$. The massive ions are assumed to be stationary, thus providing a neutralizing background. Based on the principles of the seminal paper by Akhiezer and Polovin [37], the propagation of electromagnetic (EM) waves in magnetized plasma is governed by the (relativistic) fluid-dynamical equations for the electrons, coupled to Maxwell's laws [35, 36]. The evolution of the plasma state variables is thus described by a closed system of scalar equations in the form:

$$\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_y}{\partial t^2} = \frac{n}{\gamma} p_y, \quad (1)$$

$$\frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial t^2} = \frac{n}{\gamma} p_z, \quad (2)$$

$$\frac{\partial}{\partial t}(p_y - A_y) + u\frac{\partial}{\partial x}(p_y - A_y) = -\frac{\Omega p_z}{\gamma} \quad (3)$$

$$\frac{\partial}{\partial t}(p_z - A_z) + u\frac{\partial}{\partial x}(p_z - A_z) = \frac{\Omega p_y}{\gamma} \quad (4)$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \quad (5)$$

$$\frac{\partial(\gamma u)}{\partial t} = \frac{\partial}{\partial x}(\phi - \gamma) + \frac{1}{\gamma}\left[p_y \frac{\partial}{\partial x}(p_y - A_y) + p_z \frac{\partial}{\partial x}(p_z - A_z)\right], \quad (6)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n - 1\,, \tag{7}$$

where n is the electron fluid number density, and $\mathbf{A}$ and ϕ are the vector and scalar potentials, respectively. For a circularly polarized EM pulse, the electron momentum is expressed as

$$\mathbf{P} = \mathbf{p}_\perp(x,t) + \gamma u(x,t)\hat{x} \tag{8}$$

where $\mathbf{p}_\perp = p_x\hat{x} + p_y\hat{y}$ is the transverse component of the electron momentum and $u(x,t)$ is the longitudinal component of the electron (fluid) speed, while γ is the relativistic factor. We use the notation $\alpha = +1$ ($\alpha = -1$) for left- (right-) hand circularly polarized electromagnetic waves (henceforth denoted by the acronyms LCP and RCP, respectively). In the above relations, we have normalized the scalar and vector potentials by mc^2/e, the electric field $\mathbf{E}$ by $mc\omega_{pe}/e$, the magnetic field by $\mathbf{B}$ by $m\omega_{pe}/e$, the momentum by mc, the density by the n_0, the electron velocity by the light velocity c; furthermore, the length is normalized by the *skin length* c/ω_{p0}, and time is scaled by the plasma period (inverse plasma frequency) (ω_{pe}^{-1}), where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ (here n_0 denotes the equilibrium electron density).

The above equations may essentially be viewed as a closed system of scalar equations describing the evolution of the different components of a state vector, say $\mathbf{S} = (n, u, \phi; A_y, A_z, p_y, p_z)$, which describes the state of the system at a given position x and time t. We note the parametric dependence on the magnetic field via the parameter Ω, which is the ratio between the (electron) cyclotron frequency $\omega_c = eB/m_ec$ and the (electron) plasma frequency ω_{pe}, viz. $\Omega = \omega_c/\omega_{pe}$.

3. Multiscale perturbation theory for weakly nonlinear wavepackets

Relying on the multiscale analysis proposed by Taniuti and coworkers [38] (also see [39]), the state vector $\mathbf{S}$ can be decomposed as

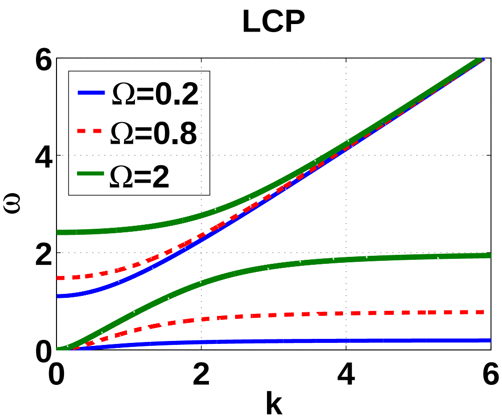
$$\mathbf{S} = \mathbf{S}^{(0)} + \sum_{n=-\infty}^{n} \epsilon^n \mathbf{S}^{(n)} \tag{9}$$

where $S^{(0)} = (1,0,0;0,0,0,0)$ is the equilibrium state of the system and $\epsilon \ll 1$ is a (dimensionless) small real parameter. We introduce new independent spatial and temporal variables, $x_m = \epsilon^m x$, $t_m = \epsilon^m t$ ($m = 0,1,2,\cdots$), and accordingly expand the space and time derivative operators as $\partial_x = \partial_{x_0} + \epsilon\partial_{x_1} + \ldots$ and $\partial_t = \partial_{t_0} + \epsilon\partial_{t_1} + \ldots$.

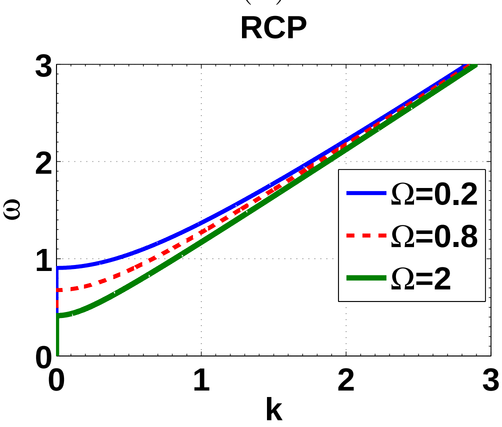
Anticipating harmonic generation in the state variables, we shall seek a solution of the system of Eqs. (1-7) in the form:

$$\mathbf{S}^{(n)} = \sum_{\ell=-\infty}^{\ell=\infty} \mathbf{S}^{(n\ell)}(x_{m\geq 1}, t_{m\geq 1}) e^{i\ell(kx_0 - \omega t_0)}, \tag{10}$$

where n and ℓ in the superscript(s) denotes the order (in expansion in ϵ) and the phase-multiple ($\ell = 0,1,2,...$) for a given harmonic. We have assumed that the perturbed state depends on the fast variables via the fundamental carrier phase $kx_0 - \omega t_0$ only,



(a)



(b)

Figure 1. (Color online) The dispersion relation showing the normalized frequency ω as a function of the normalized wave number k for LCP-(top panel) and RCP-(bottom panel) EM waves. The thin solid (blue), dashed (red) and bold (green) lines show the dispersion relation for different values of Ω, i.e., $\Omega = 0.1$, $\Omega = 0.8$, and $\Omega = 2$, respectively.

where k and ω are the frequency and wavenumber of the carrier wave (here normalized by ω_{pe} and $k_{pe} = \omega_{pe}/c$, respectively). On the other hand, the (slowly varying) harmonic amplitudes $\mathbf{S}^{(n\ell)}(x_{m\geq 1}, t_{m\geq 1})$ depend on the slow coordinates x_m, t_m (for $m = 1, 2, ...$).

The calculation is extremely tedious yet perfectly straightforward. Omitting details (to be reported elsewhere [40]), we shall limit ourselves to summarizing the main results of the analysis in what follows. Analytical predictions for EM rogue waves will then be presented, and a parametric investigation of the role of Ω will subsequently be carried out.

3.1. Linear analysis

At first order ($n = 1$), we obtain the following relations for the first harmonic amplitudes:

$$p_z^{(11)} = i\alpha p_y^{(11)}\,, \qquad A_z^{(11)} = i\alpha A_y^{(11)}\,, \qquad p_y^{(11)} = (\omega^2 - k^2)A_y^{(11)}. \tag{11}$$

A linear dispersion relation is obtained as a compatibility condition, from the first-order equations, in the form:

$$\omega^2 - k^2 = \frac{\omega}{\omega - \alpha\Omega}\,, \tag{12}$$

in perfect agreement with the so far established theory for left-handed (for $\alpha = +1$) or right-handed (for $\alpha = -1$) linear EM waves in magnetized plasmas [41]. Note that the unmagnetized result $\omega^2 = 1 + k^2$ (in scaled units) is readily obtained for $\Omega = 0$.

The above result is illustrated in Fig. 1, where we plot the frequency ω as a function of the wave number k, for three different values of Ω. Note that only one real branch for ω is obtained for RCP waves, while two branches exist for LCP waves (to be henceforth refereed to as the upper and the lower LCP branches).

In 2nd order in ϵ, the condition for annihilation of secular terms leads to $\partial \cdot /\partial t_1 + v_g \partial \cdot /\partial x_1 = 0$ (for $\cdot = p_{y/z}$ or $A_{y/z}$), implying that the envelope moves at the group velocity $v_g = \omega'(k)$:

$$v_g = \frac{2k(\omega - \alpha\Omega)^2}{2\omega(\omega - \alpha\Omega)^2 + \alpha\Omega}\,, \tag{13}$$

as physically expected.

3.2. Nonlinear analysis: evolution equation for the fundamental harmonic amplitude

In order ϵ^3, a compatibility equation is obtained, by imposing the annihilation of secular terms. This leads to the *nonlinear Schrödinger* equation:

$$i\frac{\partial\psi}{\partial\tau} + P\frac{\partial^2\psi}{\partial\xi^2} + Q|\psi|^2 = 0, \tag{14}$$

where ψ denotes the amplitude $A_y^{(11)}$, the (slow) time and space variables are $\tau = t_2$ and $\xi = x_1 - v_g t_1$, and the dispersion coefficient P and the nonlinear self-phase modulation (SPM) coefficient Q are respectively given by the (real) expressions:

$$P \equiv \frac{1}{2}\frac{\partial^2\omega}{\partial k^2} = \frac{v_g}{2k} + \frac{v_g^2}{\omega - \alpha\Omega} + \frac{v_g^3(3\omega - \alpha\Omega)}{2k(\omega - \alpha\Omega)} \tag{15}$$

$$Q = \frac{v_g}{k}(\omega^2 - k^2)^4\,. \tag{16}$$

We note that the coefficients P and Q are essentially real functions of the carrier wavenumber k [recall (12) and (13) above], in addition to their dependence on the magnetic field via the real parameter Ω.

It should be noted, for completeness, that Eq. (14) may be employed to investigate the modulational stability profile of CPEM waves, as well as the formation and dynamics of *envelope solitons* of the bright or dark type [19, 27]. These are reminiscent of optical solitons, yet in this context, would represent electromagnetic (field) pulses (bright) or

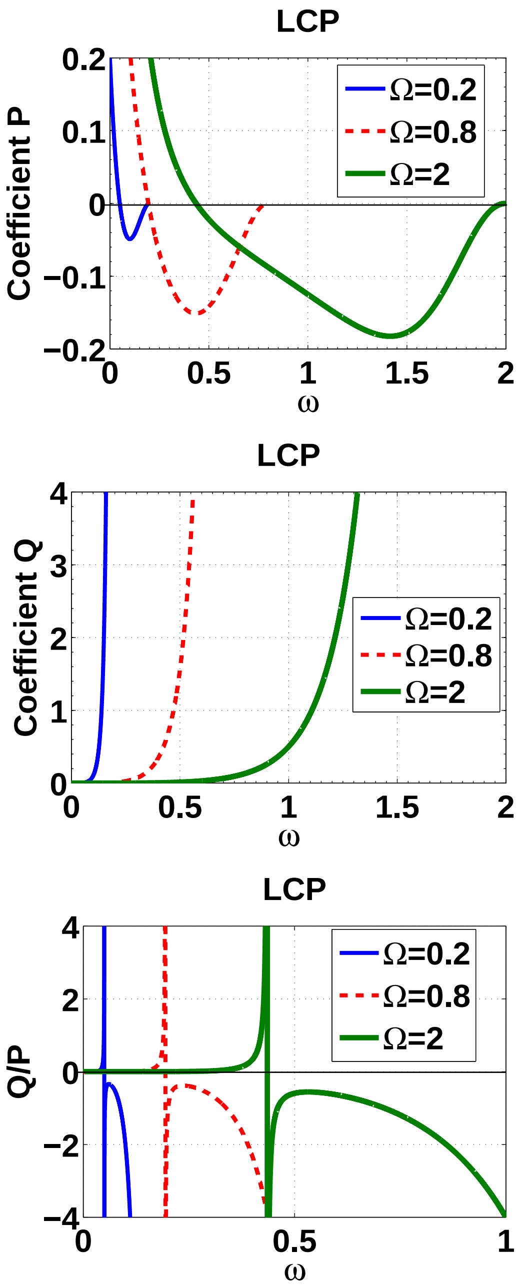


Figure 2. (Color online) The dispersion coefficient P (top panel), the nonlinearity coefficient Q (bottom panel) and the ratio Q/P (bottom panel) are depicted versus the frequency, in the *low* frequency band of LCP-waves (magnetic field values as in Fig. 1).

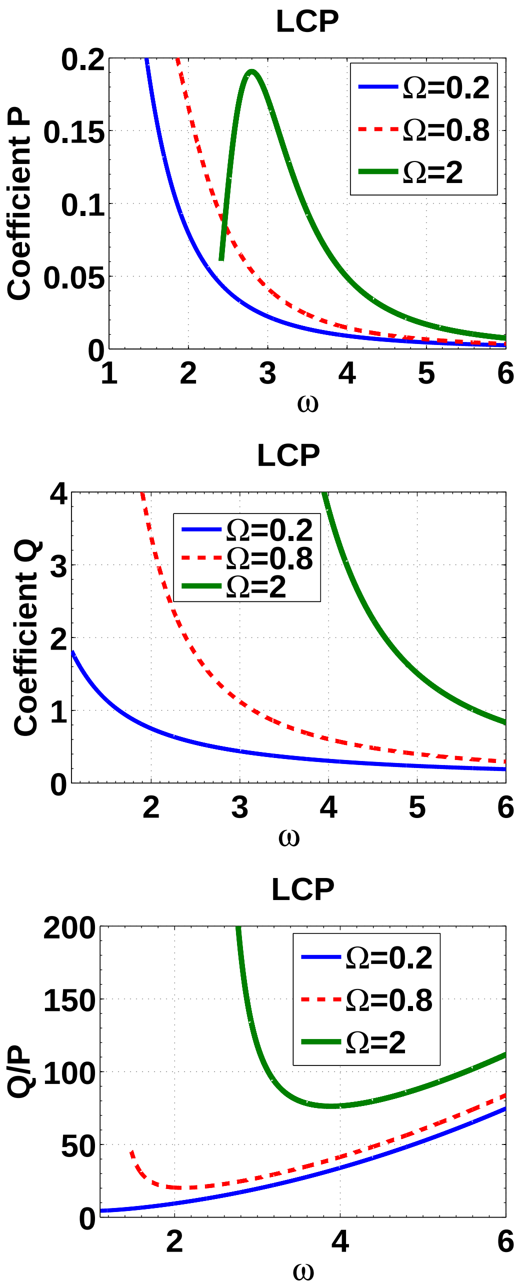


Figure 3. (Color online) The dispersion coefficient P (top panel), the nonlinearity coefficient Q (bottom panel) and the ratio Q/P (bottom panel) are depicted versus the frequency, in the *high* frequency band of LCP-waves (magnetic field values as in Fig. 1).

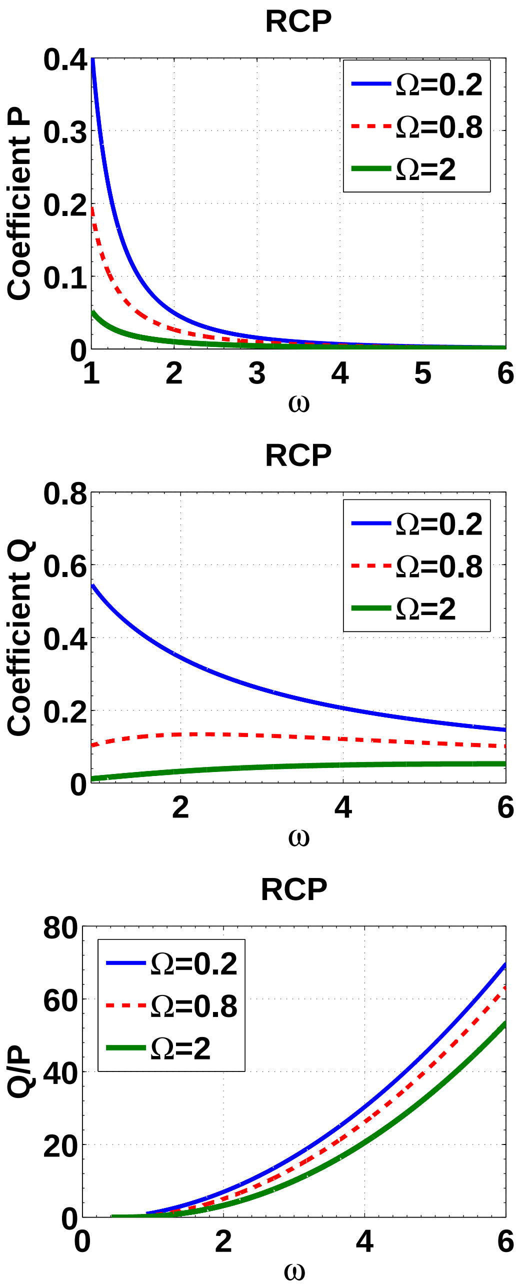


Figure 4. (Color online) (Color online) The dispersion coefficient P (top panel), the nonlinearity coefficient Q (bottom panel) and the ratio Q/P (bottom panel) are depicted versus the frequency, considering LCP-waves (magnetic field values as in Fig. 1).

voids (dark) propagating in (and coupled to) the plasma medium. The analytical formalism related to these phenomena is well-known (an interested reader is referred e.g., to [27] for details) We chose not to enter into detail along this direction, going beyond our well defined scope, for lack of space (the modulational profile of CPEM waves will be investigated elsewhere in extent [40]).

In order to gain some insight, we have depicted the coefficients P and Q, as well as their ratio Q/P, by considering a representative wavenumber $k = 0.5$ (in scaled units; see Table in the Appendix), for the same values of Ω as in Figure 1. The *low* LCP ("acoustic"-type), *high* LCP ("optic"-like) and RCP branches visible in Fig. 1 correspond to Figs. 2, 3 and 4, respectively. Recall that the ratio Q/P is an important quantity, as it determines:
— the stability profile of EM wavepackets ($Q/P < 0$ denotes stability, while for $Q/P > 0$, *modulational instability* occurs beyond a certain threshold [27]);
— the nature of envelope excitations (bright solitons for $Q/P > 0$, while dark solitons otherwise);
— the geometrical features of envelope excitations: $\psi_0 L \propto (Q/P)^{1/2}$ for envelope solitons of width (spatial extension) L and maximum amplitude ψ_0 [27].
Last but not least, the breather-like excitations presented in the next Section occur in the region $Q/P > 0$.

We shall now summarize in the next Section the existing analytical theories for rogue-wave like excitations, using as basic working horse the NLS equation (14), and will then proceed by investigating their characteristics under the effect of the magnetic field (via Ω).

4. Breather-type solutions of the NLSE as prototypical rogue waves

It was suggested [18] that certain (classes of) solutions of the nonlinear Schrödinger equation (14) are good candidates as analytical models for rogue waves, as they capture the essential physics and the qualitative features of these unique excitations. In plasma physics, this is still a practically unexplored area, as discussed above. In the following, we shall summarize the current state of the art, regarding analytical rogue-wave-like solutions of the NLSE (14), briefly discussing their relevance in our current context.

The Peregrine soliton. The Peregrine "soliton" [18, 20] appears to be a good qualitative candidate for a freak-wave-like behavior based on a NLS description. This solution has been successfully employed to fit experimental observations in nonlinear optics [6], in water basins [9] and in plasmas [25]. The Peregrine solution reads [20, 18, 21]:

$$\psi(\xi,\tau) = \left[1 - \frac{4(1+i2Q\tau)}{1+2Q\xi^2/P+4Q^2\tau^2}\right]\exp(iQ\tau) \tag{17}$$

The corresponding waveform decays to a plane wave asymptotic background for either large ξ or τ, but exhibits a non-trivial behavior over a small region of (ξ,τ). For our purposes, all physical information is contained within the coefficients P and Q in (17) (and in (14)) which are functions of relevant plasma parameters.

Akhmediev breather. The Akhmediev breather [18, 24] is given by

$$\psi(\xi,\tau) = \Big[1 + \frac{2(1-2a)\cosh(bQ\tau) + ib\sinh(bQ\tau)}{\sqrt{2a}\cos(w\sqrt{\frac{Q}{2P}}\xi) - \cosh(bQ\tau)}\Big]\exp(iQ\tau) \qquad (18)$$

where $\alpha \in (0, 1/2]\,, w = 2\sqrt{1-2\alpha}\,\text{and}\, b = \sqrt{8a(1-2a)}$ are interdependent real parameters. It is straightforward to see that this waveform is periodic in space only (and localized in time). Interestingly, the Peregrine solution is recovered if one takes the limit of an infinite spatial period.

Kuznetsov-Ma breather. The Kuznetsov-Ma breather [22, 23] is given by

$$\psi(\xi,\tau) = \Big[\frac{\cos(\frac{1}{2}s'Q\tau - 2i\phi) - \cosh\phi\cosh(s\sqrt{\frac{Q}{2P}}\xi)}{\cos(\frac{1}{2}s'Q\tau) - \cosh\phi\cosh(s\sqrt{\frac{Q}{2P}}\xi)}\Big]\exp(iQ\tau) \qquad (19)$$

where $\phi \in \Re\,, s = 2\sinh\phi\,\text{and}\, s' = 2\sinh(2\phi)$ are real parameters. This waveform, which is periodic in time, yet localized in space, was recently detected in optical fibers [7].

5. Parametric analysis

It is obvious from the analytical expressions presented in the previous Section that the essential characteristics of rogue-wave like excitations, namely their magnitude and their periodicity in space or in time, will depend on the value(s) of P and Q, which in turn depend on the intrinsic parameters involved in a given problem: in our case, the ratio (Ω) between the cyclotron frequency ($\propto B$) and the plasma frequency. In the following, we shall briefly analyze the parametric dependence of these waveforms on Ω, which thus arises as a tunable parameter (assuming that such a high degree of sophistication is available in plasma diagnostics).

First of all, we have considered the *low* LCP frequency band (see the acoustic-like curve/s in Fig. 1a), corresponding to whistler waves [41]. Considering two representative values of $\Omega < 1$, we have depicted the Peregrine soliton in Fig. 5. It is clear that an increased value of Ω suppresses the extension of the excitation in the time domain, while it does not seem to affect its spatial size. A similar qualitative result is drawn for the Akhmediev-breather (see Fig. 6) and for the Kuznetsov-Ma breather (see Fig. 7).

In an analogous manner, we have also considered the *high* LCP frequency band (i.e., the upper curve/s in Fig. 1a), plotting the Peregrine soliton, the A-breather and the K-Ma breather in Figs. 8, 9 and 10 respectively. The qualitative effect of Ω is analogous in this case (cf. the previous paragraph), in that a higher magnetic field (i.e. a higher value of Ω) results in a more time-extended (less time-localized) waveform in all three cases. However, the effect is far more significant, since an increase of the magnetic field

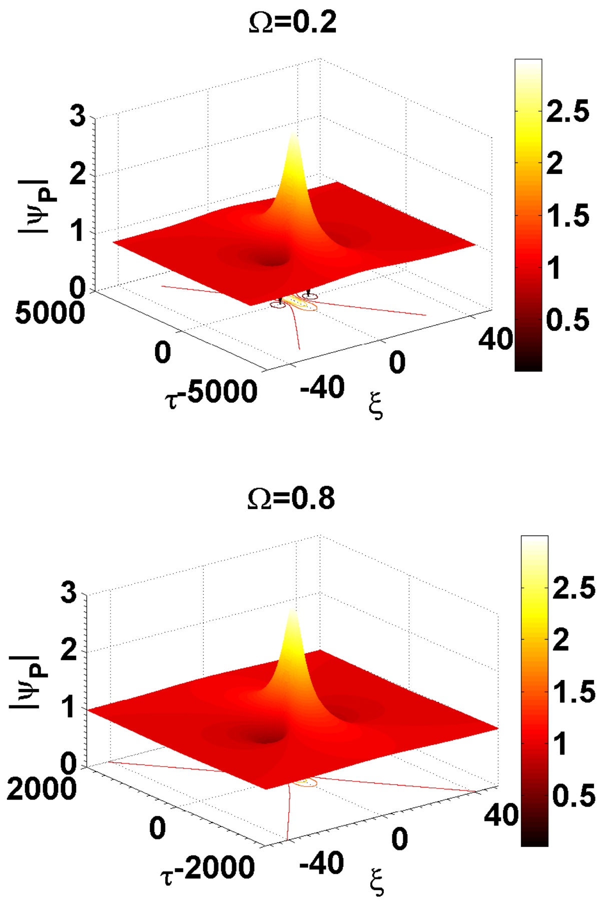


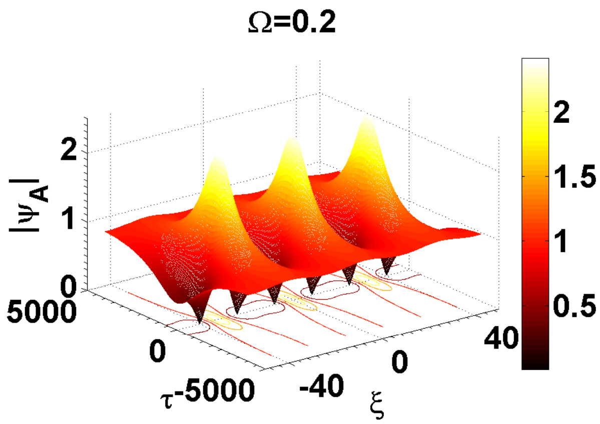
Figure 5. (Color online) Peregrine soliton for LCP waves,in the low frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$.

by a factor 4 results in time-localization being increased (i.e. duration reduced) by a factor 10 (for Peregrine, see Fig. 8 or for K-Ma, see Fig. 10) or 4 (for A-breathers: see Fig. 9). This is accompanied by an increased spatial localization, in the latter two cases (see Figs. 9 and 10).

Finally, we have considered *right-hand* polarized (RCP) waves (see Fig. 1b): the corresponding Peregrine, A-breather and K-Ma breather forms are shown in Figs. 11, 12 and 13 respectively. The qualitative effect of Ω is reversed in this case (cf. the previous paragraph), in that a higher magnetic field (i.e. a higher value of Ω) results in a more time-extended (i.e. of longer duration) peak-shaped excitation in all three cases. No measurable effect is observed in the space domain though.

6. Soliton interaction

We have recently undertaken a systematic study of interacting standing solitary waves (soliton pulses) of the Esirkepov type [42], via fluid simulations [43]. A numerical integration of the fluid-plasma-Maxwell model has provided the evolution of an initial



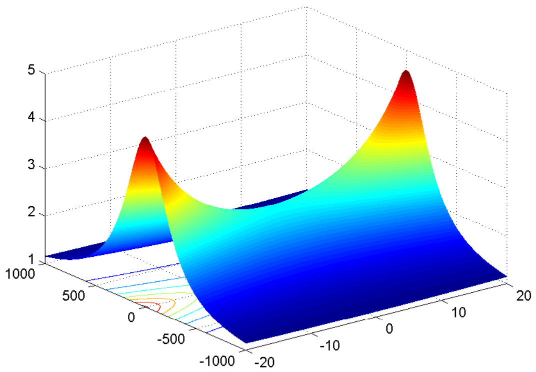


Figure 6. (Color online) Akhmediev breather for LCP waves, in the low frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter α takes the value $\alpha = 0.25$.

two-pulse state, in various regimes (in terms of the pulse separation and the relative amplitudes). A large-amplitude excitation was clearly observed as a transitive state during pulse interaction in those simulation; see Figure 14. Although this is still speculation, it appears that pulse interaction might arise as an alternative scenario for rogue-wave formation, under favorable circumstances. This path, which is qualitatively reminiscent of earlier studies based on nonlinear evolution equations, has never been investigated with respect to electromagnetic waves in plasmas. We have undertaken an investigation in this direction, and we hope soon to be able to characterize the emerging localized structures and describe their dynamics (it appears too early to report our preliminary results, however promising, at this stage).

7. Conclusions

We have presented a theoretical model, from first principles, for rogue waves (freak waves) associated with electromagnetic pulse propagation interacting with a magnetized

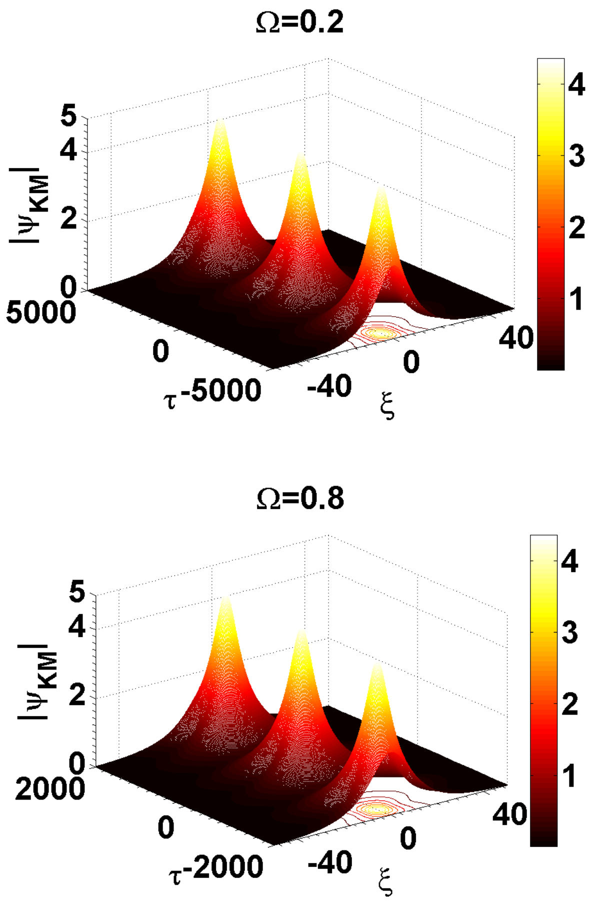


Figure 7. (Color online) Kuznetsov-Ma breather for LCP waves,in the low frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter s take the value $s = 0.7$.

plasma. Solving the fluid-Maxwell equations via a multiscale technique, we have derived a nonlinear Schrodinger type evolution equation for the amplitude of the vector potential, associated with the propagation of a circularly polarized electromagnetic wavepackets. A A set of non-stationary envelope solutions of the NLS equation were presented, based on the earlier observations of Dysthe and Trulsen [18], and were proposed as models for rogue waves in beam-plasma interactions.

The variation of the structural properties of these structures with the ratio between the (magnetic field dependent) cyclotron (gyro-) frequency and the plasma frequency was investigated. A set of qualitative predictions were presented for the (three) different EM modes of relevance in this problem. Though admittedly at a fundamental qualitative level, and certainly not exhaustive (an investigation should be undertaken, to span a wider region of Ω values), our study aims at setting a first framework for the investigation of extreme electromagnetic events in beam-plasma interactions. These findings would hopefully be confirmed by future experiments, if diagnostic techniques allow for a quantitative characterization of such highly localized structures. Further studies are in progress and will be reported in the future.

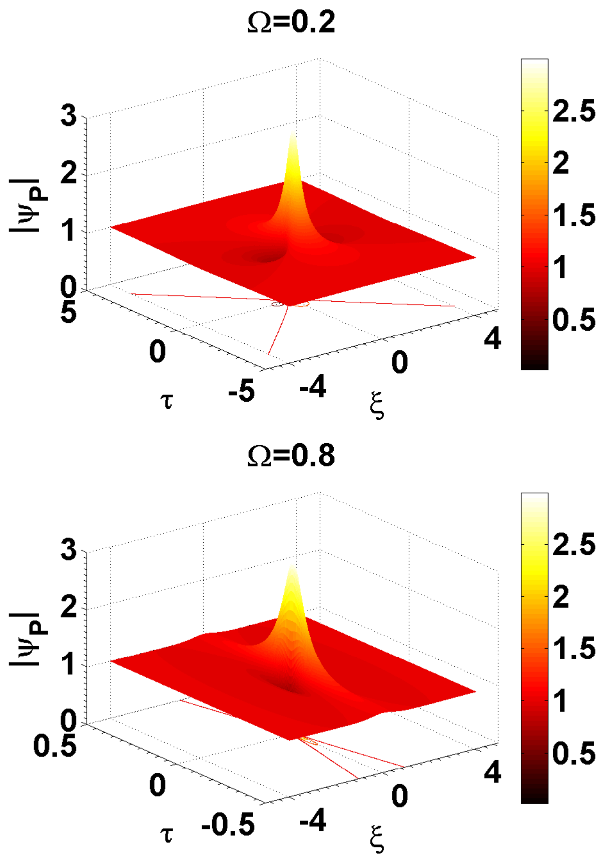


Figure 8. (Color online) Peregrine soliton for LCP waves,in the high frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$.

8. Acknowledgments

This article is dedicated to the memory of Padma Kant Shukla, who passed away while these lines were being written. To one of us (IK) he was a friend, a colleague and an influence never to be forgotten.

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Appendix A. Frequency values considered (corresponding to $k = 0.5$)

	$\Omega = 0.2$	$\Omega = 0.8$
RCP	1.0436	0.8796
LCP2	1.2038	1.5311
LCP1	0.0398	0.1485

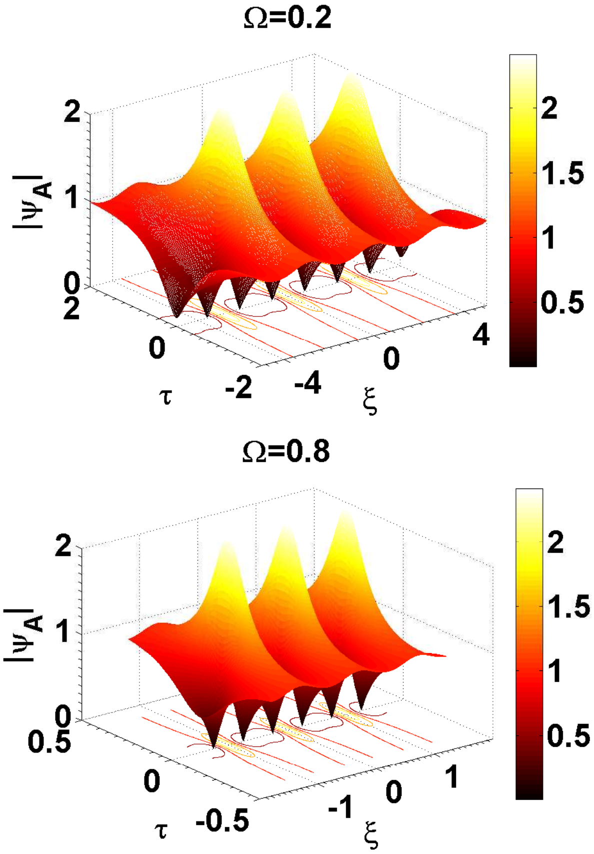

Figure 9. (Color online) Akhmediev breather for LCP waves, in the high frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter α takes the value $\alpha = 0.25$.

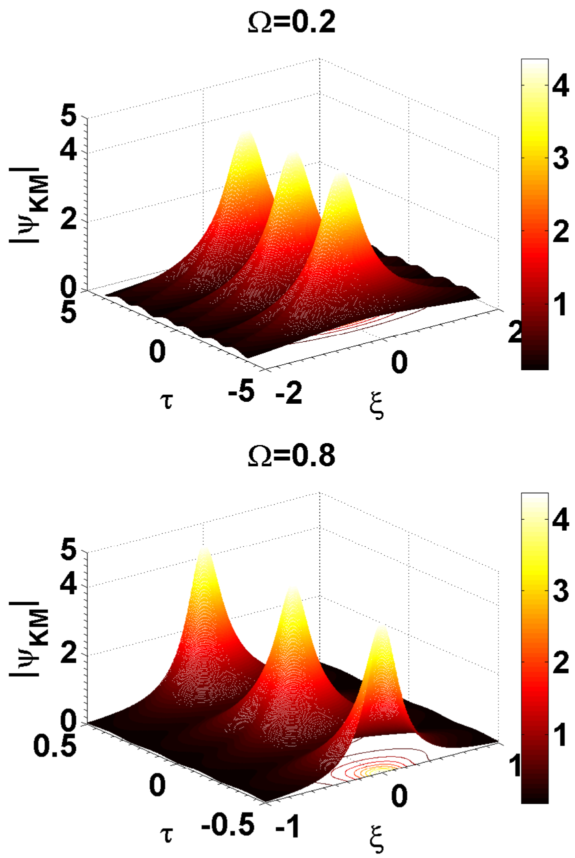

Figure 10. (Color online) Kuznetsov-Ma breather for LCP waves,in the high frequency band, for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter s take the value $s = 0.7$.

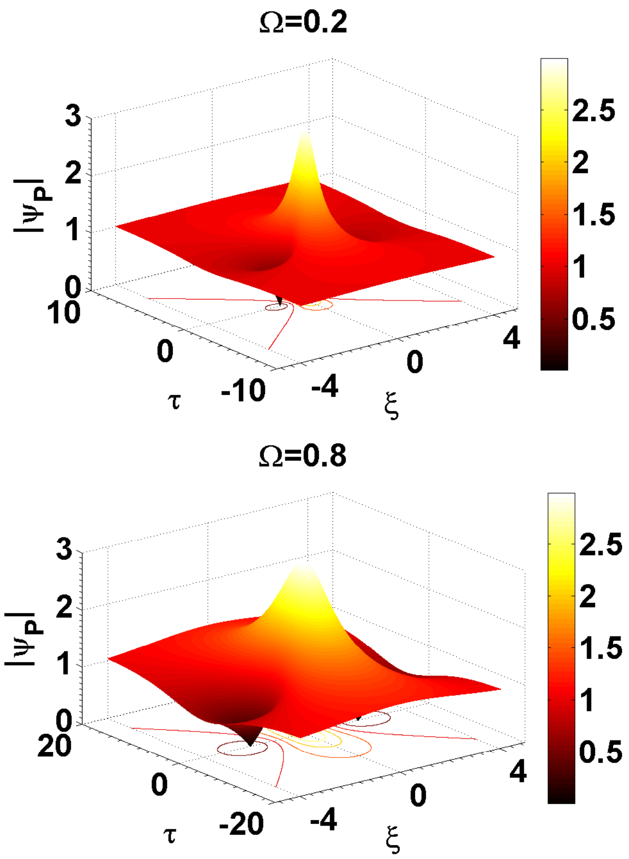

Figure 11. (Color online) Peregrine soliton for RCP waves for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$.

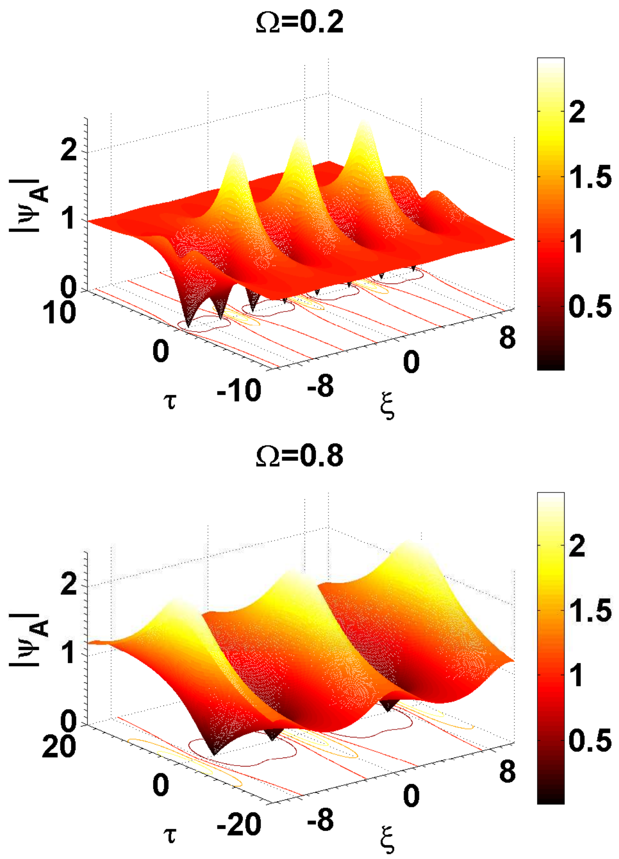

Figure 12. (Color online) Akhmediev breather for RCP waves for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter α take the value $\alpha = 0.25$.

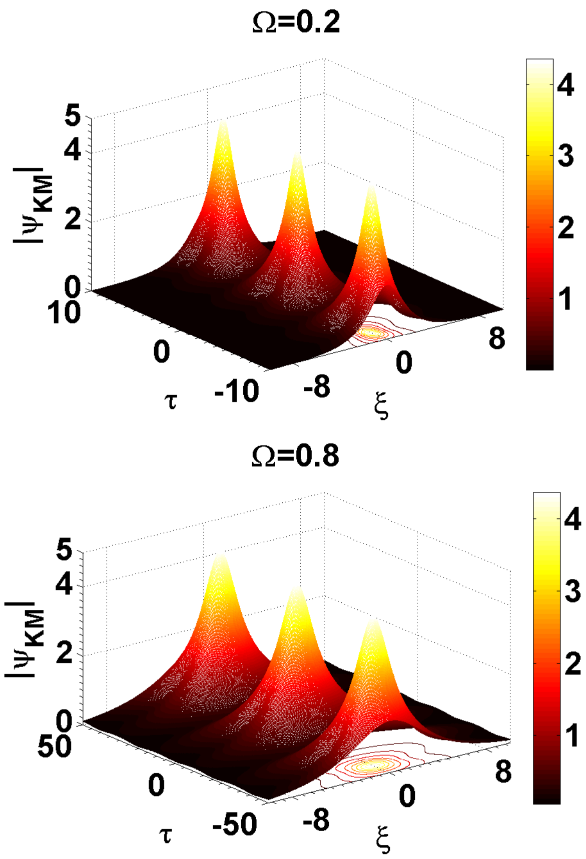

Figure 13. (Color online) Kuznetsov-Ma breather for RCP waves for different values of Ω, i.e. $\Omega = 0.2$ (top panel) and $\Omega = 0.8$ (bottom panel). The wavenumber k takes the value $k = 0.5$ and the parameter s take the value $s = 0.7$.

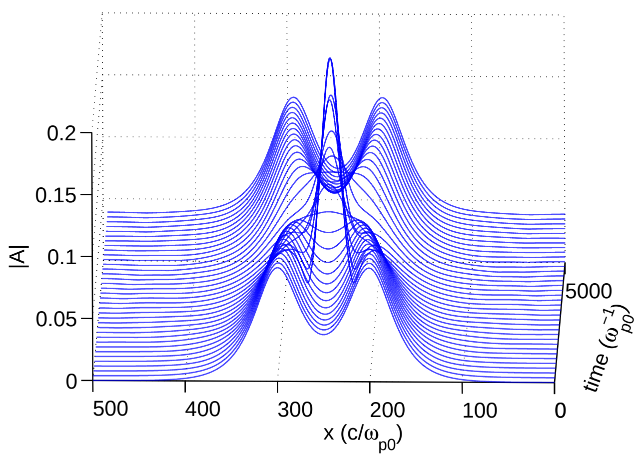

Figure 14. (Color online).

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(2.25), (2.29) and (2.33) therein); note that, in the expressions in above mentioned book, the contribution of the ions should be neglected, and the expressions should then be cast in normalized form, which then recovers exactly our relation (12).

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