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Semiclassical relativistic fluid theory for electrostatic envelope modes in dense electron–positron–ion plasmas: Modulational instability and rogue waves

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Abstract. A fluid model is used to describe the propagation of envelope structures in an ion plasma under the influence of the action of weakly relativistic electrons and positrons. A multiscale perturbative method is used to derive a nonlinear Schrödinger equation for the envelope amplitude. Criteria for modulational instability, which occurs for small values of the carrier wavenumber (long carrier wavelengths), are derived. The occurrence of rogue waves is briefly discussed.

1. Introduction

Dense plasmas are an exotic form of matter, occurring in ultra high densities and/or low temperatures. Such plasma configurations, tacitly believed to occur in superdense astrophysical objects, e.g. white and brown dwarfs, neutron stars, and magnetars, are nowadays also realized in the laboratory during ultra-intense laser-matter interaction experiments (Shapiro and Teukolsky 1983; Koester and Chanmugam 1990; Lai 2001; Hansen et al. 2004; Fortov 2009). According to the standard qualitative picture, matter in the interior of such astrophysical objects is compressed to such a high density that the thermal energy is negligible in comparison with the Fermi energy, which arises due to the overlap of fermion wavefunctions. In regions of high density and low temperature, therefore, pressure due to particle interactions is thus amplified by the degenerate fermion kinetic energy. This is a purely quantum-mechanical effect, and is thus not sensitive to the particle temperature, so pressure does not go down as the star cools. Electron degeneracy pressure thus sustains dense objects by balancing their own gravitational pull. Due to a very high particle number density, which may exceed 10^{30}cm^{-3}, the electron Fermi energy becomes comparable with the electron rest energy and the electron speed attains relativistic values. Consequently, the equation of state (EoS) changes from $P \sim n^{5/3}$ (non-relativistic expression) to $P \sim n^{4/3}$ (ultra-relativistic limit), for the pressure as function of the particle density n, making the white dwarf gravitationally unstable for masses roughly larger than $M_C = 1.4 \times M_\odot$, where $M_\odot$ represents the solar mass. The critical mass limit M_C is referred to as the Chandrasekhar limit (Chandrasekhar 1939, 1935), i.e. the mass threshold above which the star collapses.

High-density dynamical astrophysical environments, such as the interior of white dwarfs, are characterized by electron and positron coexistence, alongside a small fraction of ions also likely to be present (Lallement et al. 2011; Rahman et al. 2013a). As energy is gradually lost, stellar material cools down and ions may behave as quasiclassical particles, although still subject to the quantum features of the environment (e.g. Fermi–Dirac degeneracy effect of electrons). It is obvious that the ion component significantly modifies the response of electron–positron (*e-p*) plasmas, hence it may be associated with slow dynamical scales occurring in electron–positron–ion (*e-p-i*) plasmas. Collective interactions in dense *e-p-i* plasmas have been the focus of various studies in last few years (Haider et al. 2012; Zeba et al. 2012; Khan 2013; Rahman et al. 2013a).

Wave propagation in complex matter configurations, such as plasmas, is characterized by nonlinear amplitude modulation, in turn associated with harmonic generation and modulational instabilities in plasmas. This may occur due to either simple nonlinear self-interaction of the carrier wave, parametric wave coupling, or the interaction, e.g. between high- and low-frequency modes. The standard method employed to study this mechanism is a multiple space and time scale perturbation technique (Asano et al. 1969; Taniuti and Yajima 1969) which leads to a nonlinear Schrödinger-type equation (NLSE) describing the evolution of the wavepacket's envelope. Modulated waves are known to undergo Benjamin–Feir-type (modulational) instability (MI), referring to modulated envelope collapse due to small external perturbations (Dauxois and Peyrard 2005; Kourakis and

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Shukla 2005). This mechanism favors energy localization via the formation of envelope localized structures (envelope solitons), i.e. long-lived localized excitations, which are sustained by a mutual balance between dispersion and nonlinearity and can propagate in the medium over long distances, remarkably surviving impacts with each other. Let us add, for the sake of rigor, that we have chosen to neglect pair-annihilation (recombination) processes, for simplicity in the analysis, in agreement with earlier studies (Svensson 1982; Akbari-Moghanjoughi 2010; Rahman et al. 2013b), suggesting that recombination processes can be ignored for a range of density values of relevance in astrophysical plasmas.

In this paper, we employ the generalized Chandrasekhar (1939) equation of state, combined with a fluid-dynamical formulation, to investigate the nonlinear dynamics of modulated ion-acoustic wavepackets. This is clearly a simplistic approach to the analytical modeling of dense relativistic plasma which, thanks to its analytical tractability and simplicity of formulation, provides some insight in the dynamics, compared with the classical, non-relativistic description. Influence is drawn from earlier work (Akbari-Moghanjoughi 2011), relying on a similar model to describe the dynamics via the Sagdeev pseudopotential method. We consider a dense plasma composed of a non-relativistic non-degenerate cold ion fluid and relativistically degenerate electrons and positrons. Our aim is to investigate, from first principles, the occurrence of modulational instability and the existence of envelope solitary structures associated with ion acoustic waves.

2. Quantum ion-fluid model

Ions are assumed to constitute a system of 'cold' particles with an individual charge of $Z_i e$ (Z_i denotes the ion charge state, while e is the electron charge), subject to the influence of the electrostatic potential ϕ. Adopting a one-dimensional fluid formulation, the evolution equations for the ion density n_i, fluid speed v, and electron/positron pressure $P_{e/p}$, in terms of ϕ, read as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v) = 0, \qquad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{Z_i e}{m}\frac{\partial \phi}{\partial x} = 0,$$

$$e n_e \frac{\partial \phi}{\partial x} - \frac{\partial P_e}{\partial x} = 0,$$

$$e n_p \frac{\partial \phi}{\partial x} + \frac{\partial P_p}{\partial x} = 0, \qquad \frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e - Z_i n_i - n_p). \tag{2.1}$$

Electrons and positrons are treated as a degenerate ensemble; we adopt (for species denoted by index $j = e, p$) the relativistic equation of state (Chandrasekhar 1939),

$$P_j = \frac{\pi m_j^4 c^5}{3h^3}\left[\eta_j\left(2\eta_j^2 - 3\right)\left(1+\eta_j^2\right)^{\frac{1}{2}} + 3\sinh^{-1}(\eta_j)\right], \tag{2.2}$$

where $\eta_j = p_{Fj}/m_j c = \sqrt{\gamma_j^2 - 1}$, $p_{Fj} = \sqrt{2 m_j E_{Fj}} = (3h^3 n_j/8\pi)^{\frac{1}{3}}$ is the Fermi momentum and $\gamma_j = \sqrt{1+\eta_j^2}$; $\sinh^{-1}$ denotes the inverse hyperbolic sine function.

The third and fourth equations in (2.1) can be integrated as $\frac{1}{n_{e,p}}\frac{\partial P_{e,p}}{\partial x} = \frac{mc^2 \eta_{e,p}}{(1+\eta_{e,p}^2)^{\frac{1}{2}}}\frac{\partial \eta_{e,p}}{\partial x}$. The number densities can then be expressed as functions of ϕ, viz.

$$n_{e,p} = \frac{8\pi m_{e,p}^3 c^3}{3h^3}\left[\frac{e^2\phi^2}{m_{e,p}^2 c^4} \pm \frac{2e\phi}{m_{e,p}c^2}(1+\eta_{e,p0}^2)^{\frac{1}{2}} + \eta_{e,p0}^2\right]^{\frac{3}{2}} \tag{2.3}$$

and therefore reduce the number of equations to three. In (2.3), $\eta_{j0} = (3h^3 n_{j0}/8\pi m^3 c^3)^{\frac{1}{3}}$ is the value of the relativity parameter, η_j, at equilibrium.

The model is rewritten in terms of dimensionless variables. The number densities are rescaled by their equilibrium values, n_{j0}. By L_0, t_0, and $V_0 = L_0/t_0$ we denote, respectively, the characteristic length, time, and speed scales. Finally, ϕ_0 is the scale parameter for the potential. The physical scales actually adopted need only be determined later, based on physical arguments and analytical convenience. The reduced system reads as follows:

$$\frac{\partial \tilde{n}_i}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{x}}(\tilde{n}_i \tilde{v}) = 0, \qquad \frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{x}} + a\frac{\partial \tilde{\phi}}{\partial \tilde{x}} = 0,$$

$$\frac{\partial^2 \tilde{\phi}}{\partial \tilde{x}^2} = b(\beta \tilde{n}_e - \tilde{n}_i - \alpha \tilde{n}_p). \tag{2.4}$$

The choice of scale(s) affects the fluid equations through parameters, a, b, α, and β, namely as: $a = \frac{Z_i e \phi_0}{m_i V_0^2}$, $b = \frac{4\pi Z_i e n_{i0} L_0^2}{\phi_0}$, $\alpha = \frac{n_{p0}}{Z_i n_{i0}}$, and $\beta = \frac{n_{e0}}{Z_i n_{i0}}$. A natural choice of scales, leading to $a = b = 1$, will be adopted: space and time are scaled by C_i/ω_{pi} and $\omega_{pi}^{-1} = \left(m_i/4\pi n_{i0} Z_i^2 e^2\right)^{1/2}$, respectively, while the electrostatic potential is scaled by $\phi_0 = \frac{E_{Fe0}}{Z_i e}$, and the ion fluid speed by $C_i = \left(E_{Fe0}/m_i\right)^{1/2}$.

Poisson's equation at equilibrium implies: $\beta = 1 + \alpha$. (The e-i plasma case is recovered for $\beta = 1$). If the system does not deviate too far from equilibrium, the electron and positron number densities can be approximated by the Taylor expansion, hence

$$\frac{\partial^2 \tilde{\phi}}{\partial \tilde{x}^2} + b(\tilde{n}_i - 1) \simeq c_1\tilde{\phi} + c_2\tilde{\phi}^2 + c_3\tilde{\phi}^3, \tag{2.5}$$

where the (constant) coefficients on the right-hand side are defined as

$$c_1 = \frac{3\beta b \gamma_{e0}}{2} + \frac{3\alpha b E_{Fe0}\gamma_{p0}}{2E_{Fp0}},$$

$$c_2 = \frac{3\beta b}{8}(2\gamma_{e0}^2 - 1) - \frac{3\alpha b E_{Fe0}^2}{8E_{Fp0}^2}(2\gamma_{p0}^2 - 1),$$

$$c_3 = \frac{\beta b \gamma_{e0}}{16}(2\gamma_{e0}^2 - 3) + \frac{\alpha b E_{Fe0}^3 \gamma_{p0}}{16 E_{Fp0}^3}(2\gamma_{p0}^2 - 3), \tag{2.6}$$

$E_{Fj0} = m_j c^2 \eta_{j0}^2/2$ and $\gamma_{j0} = \sqrt{1+\eta_{j0}^2}$. For the remainder of this work, the scaled variables will be used (dropping the tilde where obvious).

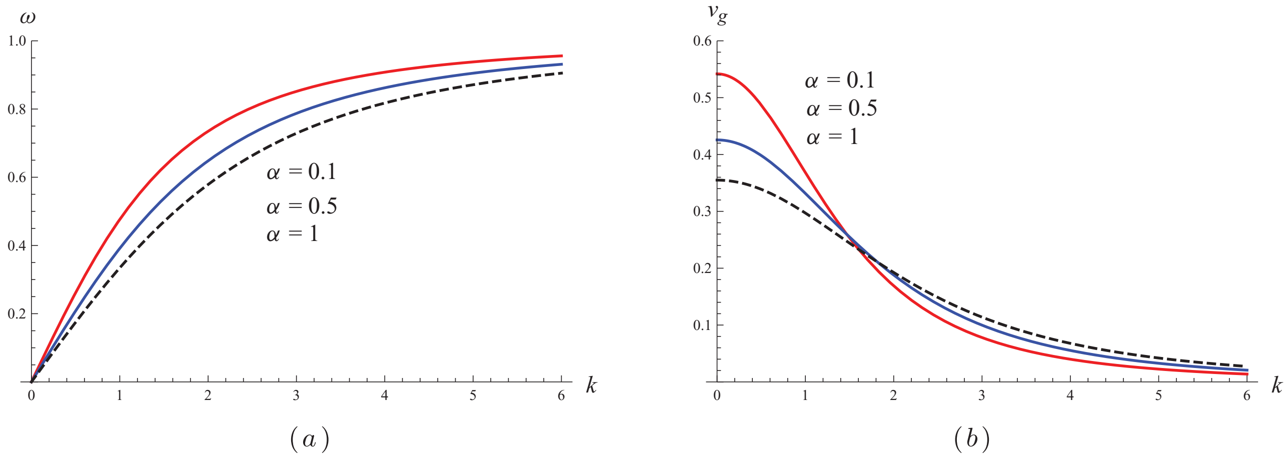


Figure 1. (Colour online) Plots of the angular frequency (ω) and of the group velocity (v_g) for $\eta_{e0} = 1.19$, corresponding to $n_{e0} \approx 10^{30}$ cm^{-3}.

3. Multiple scales perturbative analysis

Following the method by Taniuti and coworkers (Taniuti and Yajima 1969), we assume that each of n, v, and ϕ takes the form of a supersposition of phase harmonics. We assume that the carrier depends on (x, t), but the wave envelope depends on an infinite set of (slow) variables, $\{X_1, X_2, ..., T_1, T_2, ...\}$, where $T_r = \epsilon^r t$ and $X_r = \epsilon^r x$ (for $r = 1, 2, 3, ...$) and $\epsilon \ll 1$ is a free (real, small) parameter. Furthermore, the variables are expanded around their equilibrium values: $n \approx 1 + \epsilon n_1 + \epsilon^2 n_2 + \cdots$, $v \approx \epsilon v_1 + \epsilon^2 v_2 + \cdots$, and $\phi \approx \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots$. Each of these u_j (say, any of n_j, v_j, ϕ_j) is split into a sum of Fourier components: $u_j = \sum_{r=-j}^{j} u_j^{(r)} e^{ir(kx-\omega t)}$. The number densities, speed, and potential are real-valued quantities, so $u_j^{(-r)} = \bar{u}_j^{(r)}$ (the bar here denoting the complex conjugate). The stretched variables are treated as independent variables. With this in mind, the model equations are transformed into a series of coupled polynomials whose solutions provide expressions for state variables in terms of their harmonic amplitudes.

Linear analysis. The equations in the first order of ϵ can be expressed in the form of a singular matrix equation, the operator of the equation possessing a non-trivial kernel. The vanishing determinant of this operator forms the dispersion relation, $\omega^2 = \frac{k^2 ab}{c_1 + k^2}$. The dispersion relation is depicted in Fig. 1, choosing an arbitrary value for the electron density, $n_{e0} \approx 10^{30}$ cm^{-3}, of relevance to densities in the interiors of white dwarfs (Koester and Chanmugam 1990; Fortov 2009). The linear equations in ϕ_1, n_{i1}, and v_1 are under-determined, so letting the electric potential (amplitude) $\phi_1^{(1)} = \psi$ be a free variable, we obtain: $\phi_1 = \psi e^{i(kx-\omega t)} + \bar{\psi} e^{-i(kx-\omega t)}$, where $n_{i1} = \frac{k v_1}{\omega} = \frac{c_1 + k^2}{b}\phi$.

Nonlinear analysis and nonlinear Schrödinger equation. The equations for the first-harmonic components at second order are singular, forcing the following condition to be imposed for secular term annihilation: $\frac{\partial\psi}{\partial T_1} + v_g \frac{\partial\psi}{\partial X_1} = 0$, where $v_g = d\omega/dk = \left(abc_1/(c_1 + k^2)^2\right) k/\omega$, thus the envelope moves at the group velocity.

The first-harmonic components depend on $\partial\psi/\partial X_1$. The second-harmonic components are found to be proportional to ψ^2, while the zeroth-harmonics are proportional to $|\psi|^2$.

The solution has the form: $\phi \approx \epsilon\psi e^{i(kx-\omega t)} + \epsilon^2 \left(\frac{1}{2} C_{2_3}^0 |\psi|^2 + C_{2_3}^2 \psi^2 e^{2i(kx-\omega t)}\right) + c.c.$

The equations for the first-harmonic at the third order in ϵ require ψ to obey the nonlinear Schrödinger equation:

$$i\frac{\partial\psi}{\partial\tau} + P\frac{\partial^2\psi}{\partial\xi^2} + Q|\psi|^2\psi = 0, \tag{3.1}$$

where we have defined the (slow) moving coordinates $\xi = X_1 - v_g T_1 = \epsilon(x - v_g t)$ and $\tau = T_2 = \epsilon^2 t$. The dispersive coefficient, P, is equal to half the gradient of the group velocity. The nonlinearity coefficient, Q, is more complicated and is written below:

$$Q = \frac{\omega[2c_2(C_{2_3}^0 + C_{2_3}^2) + 3c_3]}{2(c_1 + k^2)} - k(C_{2_2}^0 + C_{2_2}^2) - \frac{\omega}{2}(C_{2_1}^0 + C_{2_1}^2). \tag{3.2}$$

They are depicted in Fig. 2 for different values of the parameter, α, and for fixed value(s) of η_{e0}. $C_{2\,l}^m$ are associated with the second- (for $m = 2$) and zeroth-harmonic (for $m = 0$) components of the ion-density, speed, and potential, respectively, for l =1, 2, 3. These are provided in the Appendix.

4. Modulational stability profile and rogue waves

The stability of plane–wave solutions to periodic perturbation is determined by the relative signs of P and Q (Dauxois and Peyrard 2005): when they are of the same sign, i.e. for small value of k (see Fig. 3), the perturbation will either grow or collapse exponentially. This can be seen from the dispersion relation for the disturbance of a plane wave, $\rho_0 e^{iQ\rho_0^2\tau}$ by a periodic function of the

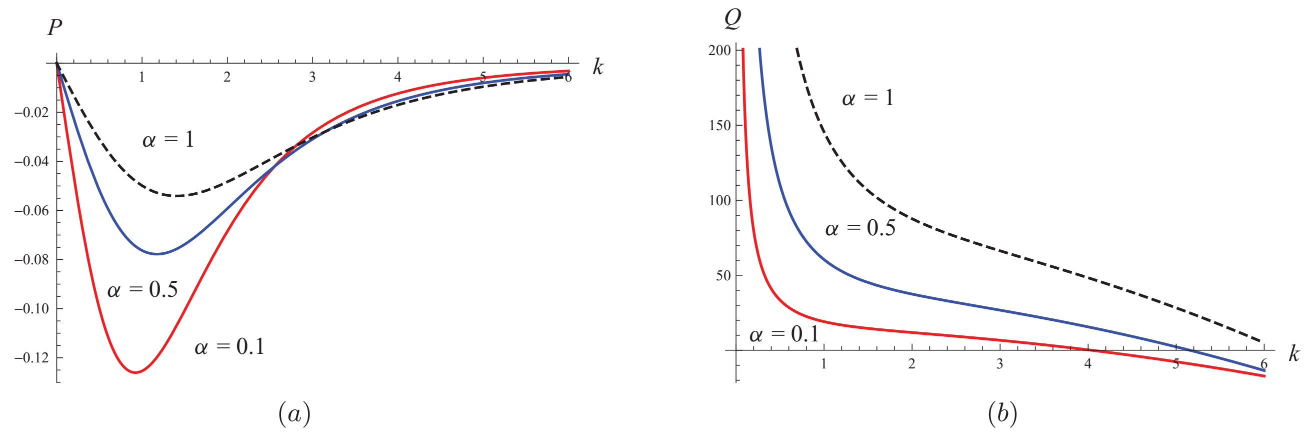


Figure 2. (Colour online) Plots of P $(=\frac{1}{2}\frac{d^2\omega}{dk^2})$ and of Q versus the wavenumber, k.

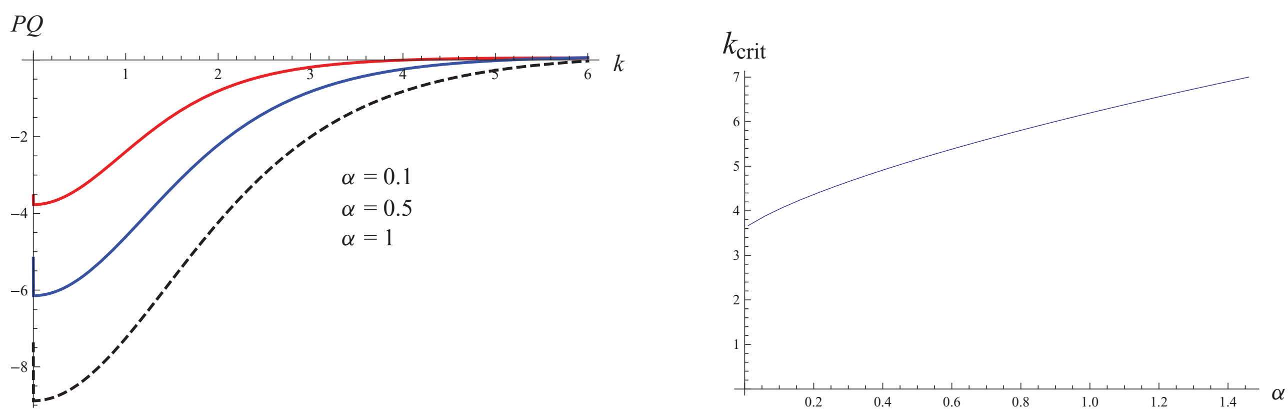


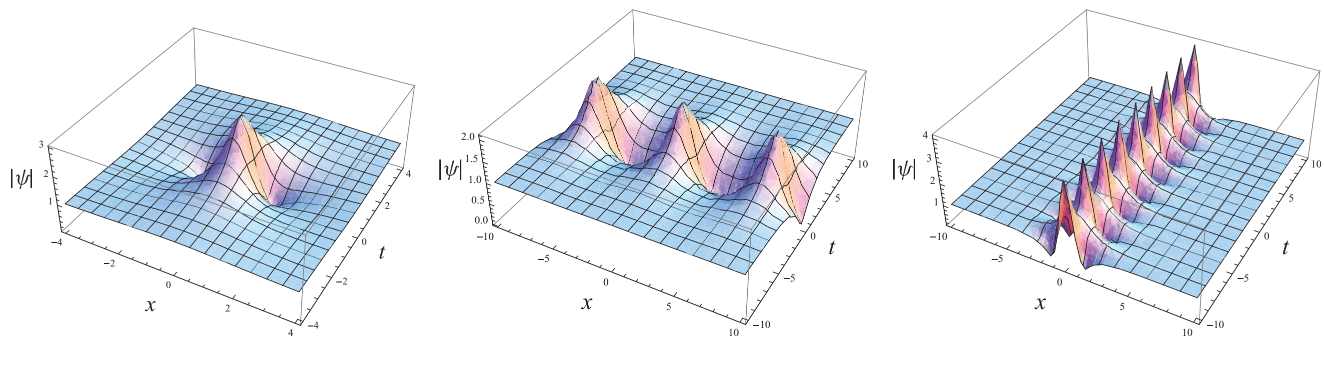
Figure 3. (Colour online) The product PQ is plotted against the wavenumber, k (left panel). The dependence of this critical wavenumber on the ratio $\alpha = n_{p0}/(Z_i n_{i0})$ is shown in the right panel, for fixed electron number density. Here we have taken $\eta_{e0} = 1.19$, corresponding to $n_{e0} \approx 10^{30}$ cm^{-3}.

variable $\kappa\xi - \Omega\tau$, namely $\Omega^2 = (P\kappa^2)^2\left(1 - \frac{2Q}{P\kappa^2}|\rho_0|^2\right)$ (Dauxois and Peyrard 2005; Kourakis and Shukla 2005). It is straightforward to see that the frequency Ω takes imaginary values if κ is *less than* $\left(\frac{2Q}{P}|\rho_0|^2\right)^{1/2}$. This instability does not occur if $PQ < 0$, i.e. for large carrier wavenumber values k (see Fig. 3). Note the difference between the wavenumbers k and κ, referring to the carrier wave and an external perturbation (e.g. turbulence, or noise), respectively. A wavenumber threshold is thus defined, which depends in fact on relevant plasma parameters (see Fig. 3b).

It might be appropriate to discuss the degeneracy effect, which may be traced in principle by investigating the impact of the relativistic parameter η_j, in fact a function of electron density. However, this task would require significant space here, and has thus been left for future work, i.e. a detailed study currently in preparation. On the other hand, the degeneracy effect can be qualitatively traced in the deviation from the classical results, e.g. the known wavenumber thresholds (for modulation instability) in classical plasmas. As a matter of fact, quantum statistics in our case increases the instability threshold, in comparison with the classical model. To see this, first recall that the instability (wavenumber) threshold is equal to a factor 1.47 times the inverse Debye length (λ_D^{-1}), for classical ion-acoustic waves (Kourakis and Shukla 2004), and note that here the wavenumber threshold is of the order of ∼4–7 times (see Fig. 3) the inverse Fermi length (recall the spatial scale definition following (2.4) above), which is $\gg \lambda_D^{-1}$, by a factor $(k_B T_e/E_F)^{1/2} \gg 1$.

Details on the Benjamin–Feir-type modulational instability mechanism are considered here, and its analytical description can be found elsewhere (Dauxois and Peyrard 2005; Kourakis and Shukla 2005) and is omitted here. Note that the instability threshold is expressed in terms of the perturbation wavenumber in this theory. Various instability scenarios have been discussed in the past. For a rigorous analysis of modulational instability mechanisms and associated instability thresholds, in particular described via a kinetic formulation, we refer the interested reader to Popel et al. (1995a, b) and Vladimirov and Popel (1995, 1996).

Rogue waves (or freak waves) are extreme events characterized by their sudden appearance from and subsequent disappearance into an oscillating background. During its brief display, the rogue wave attains a height



(a) (b) (c)

Figure 4. (Colour online) Plots of (a) Peregrine's 'soliton'; (b) Akhmediev's breather; (c) the Kuznetsov–Ma breather.

that exceeds the average turbulence by some margin (Peregrine 1983; Dysthe and Trulsen 1999; Akhmediev et al. 2009).

Three types of rogue waves are depicted in Fig. 4. The first is Peregrine's 'soliton', which first appeared in literature 30 years ago. This solution features only a single rogue structure whose amplitude decreases in both time and space. Its relevance in nonlinear optics has been established recently (Kibler et al. 2010). Akhmediev's breather is periodic in space, but highly localized in time. The Kuznetsov–Ma breather is highly localized in space, but periodic in time. The basic formulary for these solutions can be found in Veldes et al. (2013), so details are omitted here.

5. Conclusions

We have investigated, from first principles, the amplitude modulation of electrostatic wavepackets propagating in dense quantum plasmas, consisting of 'classical' ions and degenerate electrons and positrons. While the former (ions) were considered as inertial species, via a fluid model, a relativistic equation of state was adopted for the latter two species (electrons and positrons). The model was reduced to a nonlinear Schrodinger-type partial-differential equation, describing the evolution of the envelope. The analysis showed that the wave envelope is subject to modulational instability for small values of the carrier wavenumber (long carrier wavelengths). The occurrence of rogue waves was briefly discussed. It may be added, for the sake of rigor, that the model employed here is introduced as 'toy-fluid model', which reproduces the essential features of the physics of dense quantum plasmas in the presence of relativistic light charged particles. A more sophisticated approach for plasma modeling at highly relativistic conditions should be more elaborate (albeit presumably less tractable, analytically), yet goes beyond our scope here.

A more detailed study incorporating ion thermal effects and a detailed parametric analysis are in preparation.

Acknowledgements

This paper is dedicated to the memory of Padma Kant Shukla, who passed away earlier this year unexpectedly. In his person, one of us (Ioannis Kourakis) has lost an excellent collaborator, an inspiring mentor, and a good friend to be missed. Science is poorer without Padma. Michael Mc Kerr acknowledges support from the Department of Employment and Learning – Northern Ireland (DEL NI) in the form of a PhD studentship. Ata Ur-Rahman acknowledges support from the Higher Education Commission (HEC), Pakistan, in the form of a visiting research fellowship (held at Queen's University Belfast) under the International Research Support Initiative Program (IRSIP).

Appendix

The coefficients for the second-harmonic components are listed below. The indices in $C^{j}_{i\,k}$ denote the order of epsilon ($i = 0, 1, 2$), the harmonic ($j = 0, 1, 2$), and the variable ($k = 1, 2, 3$ for density, velocity, and potential, respectively).

$$C^0_{2_1} = \frac{c_1 \left(\frac{c_1+k^2}{b}\right)^2 \left(\frac{2v_g\omega}{k} + \frac{\omega^2}{k^2}\right) - \frac{2ac_2}{c_1}}{c_1 v_g^2 - ab},$$

$$C^0_{2_3} = \frac{b}{c_1} C^0_{2_1} - \frac{2c_2}{c_1},$$

$$C^0_{2_2} = v_g C^0_{2_1} - 2\left(\frac{c_1+k^2}{b}\right)^2 \frac{\omega}{k},$$

$$C^2_{2_3} = \frac{3\left(\frac{(c_1+k^2)^2}{2b}\right) - c_2}{3k^2},$$

$$C^2_{2_1} = \frac{c_1+4k^2}{b} C^2_{2_3} + \frac{c_2}{b},$$

$$C^2_{2_2} = \frac{\omega}{k} C^2_{2_1} - \left(\frac{c_1+k^2}{b}\right)^2 \frac{\omega}{k}.$$

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