

Electron-scale dissipative electrostatic solitons in multi-species plasmas

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The linear and nonlinear properties of small-amplitude electron-acoustic solitary waves are investigated via the fluid dynamical approach. A three-component plasma is considered, composed of hot electrons, cold electrons, and ions (considered stationary at the scale of interest). A dissipative (wave damping) effect is assumed due to electron-neutral collisions. The background (hot) electrons are characterized by an energetic (excessively superthermal) population and are thus modeled via a κ -type nonthermal distribution. The linear characteristics of electron-acoustic excitations are discussed, for different values of the plasma parameters (superthermality index κ and cold versus hot electron population concentration β). Large wavelengths (beyond a threshold value) are shown to be overdamped. The reductive perturbation technique is used to derive a dissipative Korteweg de-Vries (KdV) equation for small-amplitude electrostatic potential disturbances. These are expressed by exact solutions in the form of dissipative solitary waves, whose dynamics is investigated analytically and numerically. Our results should be useful in elucidating the behavior of space and experimental plasmas characterized by a coexistence of electron populations at different temperatures, where electron-neutral collisions are of relevance. © 2015 AIP Publishing LLC.

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I. INTRODUCTION

Dissipative nonlinear excitations (solitary waves), or *dissipative solitons*, have been studied in plasma physics^{1–4} and also in nonlinear optics.^{5–9} Solitons in fibre-optic communications are important, thanks to their stationary profile, so that they can be treated as a natural bit of information. To transmit information (e.g., in optical fibre communications), localized structures (solitons) have to survive for an extended period of time, even in the presence of dissipation in the medium. However, *dissipative solitons* do not possess a stationary profile, in contrast with solitonic solutions in energy-conserving systems. Their profile evolves while propagating, that is, they change in amplitude, width, and speed, and eventually diminish with time. As most of the systems are lossy by nature, solitons produced in such system continuously lose energy and thus need to be boosted by an external source of energy, so as to retain a stable profile. Solitary waves in plasmas may suffer dissipation/damping due to collisions between charged particles and neutrals. Dissipative/damping effects in plasmas may also arise due to the interparticle collisions to Landau damping or to kinematic fluid viscosity, e.g., due to shear stress of the inertial fluid motion. It was shown in Ref. 10 that the electron and ion elastic collisions with neutrals and dust, in addition to a dust-charging related mechanism, lead to dissipation. The authors in Ref. 11 have observed that Langmuir waves are damped due to electron collisions, where elastic Coulomb collisions of electrons with dust, electron-neutral collisions, dust-electron, and dust neutral collisions were taken into account.

Space plasma observations^{12–14} indicate the presence of excess energetic particles, at superthermal speeds, resulting in the distributions of thermodynamic equilibrium.^{14–16} Kappa (κ) type particle distribution functions¹⁷ efficiently describe such (nonthermal) effects. In agreement with these considerations, we consider here a three-component plasma consisting of ions, hot electrons, and cold electrons. Importantly, different time scales exist in such plasmas, not only due to the mass differences between electrons and ions but also due to the temperature differences between different electron components. Electron acoustic (EA) excitations are known to occur in this case,^{18–21,23,24} where the inertia is provided by the “cold” electrons and the restoring force is provided by the “hot” electrons. Earlier studies have been devoted to different types of localized structures (viz., solitons, shocks, vortices, etc.) in such three component plasmas.^{18,19,22,30–33}

The nonlinear propagation of electron-acoustic waves (EAWs) has been studied by various authors^{18–37} via different theoretical approaches. Singh and Lakhina²⁶ investigated large amplitude electron-acoustic structures in the framework of broadband electrostatic noise (BEN) emission observed in the auroral zone and in the Earth's magnetosphere.^{38–41} Non-Maxwellian plasma distribution of either vortex-like²⁵ or Cairns type²⁸ has also been addressed in the past, to investigate the nonlinear properties of EA solitary waves. Bright/dark-type envelope modes have also been studied from first principles in Refs. 27 and 30. In the presence of collisionality (energy dissipation), the modulational instability of electrostatic solitary structures has been investigated via a complex Ginzburg-Landau equation formulation,²⁹ and finite amplitude electron-acoustic shock waves³⁴

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have also been shown to occur. Dutta *et al.*^{35–37} have studied small-amplitude EA solitary waves^{35,37} and electron-acoustic cyclotron waves³⁶ in the presence of uniform magnetic field.

Electron-neutral collisions are often present in plasmas.^{10,11,42} Inspired by (and extending) the earlier studies of nonlinear dissipative structures in the presence of collisionality,^{29,34,43} we have undertaken an investigation of weakly nonlinear electron-acoustic solitary waves (EASWs) in a collisional three-component plasma comprising two different electron populations (here referred to as “hot” and “cold” electrons). We have investigated the occurrence of dissipative solitary structures, from first principles, and have studied their dynamics, in terms of intrinsic plasma (configurational) parameters.

The manuscript is arranged as follows. The basic formalism is presented in Section II. The linear properties of electron-acoustic excitations are discussed in Section III. An analytical and numerical investigation is carried out in Section IV. Finally, a short summary is given in Section V.

II. MODEL EQUATIONS

We consider a three-component unmagnetized plasma, consisting of inertial cold electrons (charge $q_c = -e$ and mass m_e), κ -distributed superthermal hot electrons (charge $q_h = -e$ and mass m_e), and stationary ions (charge $q_i = Z_i e$ and mass m_i). Quasi-neutrality is assumed to hold at equilibrium (only), i.e., $Z_i n_{i0} = n_{c0} + n_{h0}$; here, Z_i is the ion charge state and n_{s0} is the equilibrium number density of species s (here $s = h, c, i$ refer to the hot electrons, cold electrons, ions, respectively). We note that the phase speed of electron-acoustic waves is much smaller than the thermal speed of hot electrons but much higher than the thermal speed of both cold electrons and ions (i.e., adopting the ordering $v_{th,c} v_{th,i} \ll v_{ph} \ll v_{th,h}$). Accordingly, the hot electron inertia can be neglected, while the ions can be safely assumed to be immobile (simply maintaining the overall neutrality of the system). A one-dimensional (1d) geometry is adopted.

The dynamics of the cold electron fluid is governed by the continuity, momentum, and Poisson’s equation(s). The normalized (dimensionless) form of these equations reads

$$\frac{\partial n}{\partial t} + \frac{\partial(nu)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} - \nu u, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} \approx \beta(n - 1) + a\phi + b\phi^2, \quad (3)$$

where the cold electron number density n_c , velocity u_c , and electrostatic wave potential Φ are normalized by the unperturbed density n_{c0} , the hot electron thermal speed $C_0 = (k_B T_h / m_e)^{1/2}$, and $k_B T_h / e$, respectively. Here, k_B is the Boltzmann constant, e is the (absolute) electron charge, and T_h is the characteristic hot electron temperature. Space x and time t are normalized by the hot electron screening length $\lambda_h = (k_B T_h / 4\pi e^2 n_{h0})^{1/2}$ and the hot electron plasma period (inverse frequency) $\omega_{ph}^{-1} = (4\pi e^2 n_{h0} / m_e)^{-1/2}$, respectively.

We have also defined the ratios $\beta = n_{c0} / n_{h0}$ and $\nu = \nu_{cn} / \omega_{ph}$, where ν_{cn} is the cold electron-neutral collision frequency. The hot electrons are modeled by a κ -distribution; the (normalized) hot electron density therefore reads

$$n_h = \left(1 - \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + 1/2}, \quad (4)$$

where n_h is normalized to the equilibrium (hot electron) density n_{h0} . It is obvious that $\kappa > 3/2$ for a physically meaningful distribution. Recalling that the hot electrons, which follow κ -distribution, are not affected by the collisional dissipation as their inertia can be neglected (i.e., hot electrons-neutrals collision is neglected). The expansion coefficients appearing in Eq. (3) are related to the κ index as

$$a = \frac{\kappa - 1/2}{\kappa - 3/2}, \quad b = \frac{(\kappa - 1/2)(\kappa + 1/2)}{2(\kappa - 3/2)^2}. \quad (5)$$

As expected, the Maxwellian limit is recovered for $\kappa \rightarrow \infty$. It is noted that κ should take values $\kappa \geq 3$ in order to satisfy the expansion in Eq. (3) (via Eq. (4)) as the higher order terms are large compared to those of lower order terms and one cannot neglect them in the range $3/2 < \kappa \leq 3$. That is, for small amplitude solitary waves (studied via reductive perturbation approach), one should use $\kappa \geq 3$ to analyze the soliton properties.⁴⁴

III. LINEAR DISPERSION CHARACTERISTICS

We proceed by considering perturbations of the form $e^{i(kx - \omega t)}$. Linearizing the dimensionless system of evolution equations (1)–(3), we obtain a dispersion relation in the form

$$\begin{aligned} \omega(\omega + i\nu) &= \frac{k^2 \beta}{k^2 + a}, \\ \Rightarrow \omega &= -\frac{i\nu}{2} + \sqrt{\frac{k^2 \beta}{k^2 + a} - \frac{\nu^2}{4}}, \end{aligned} \quad (6)$$

relating the wave frequency ω and the wavenumber k .

We note the appearance of the collisionality parameter in (6). The linear wave properties are evidently influenced by the plasma configurational parameters (e.g., the superthermality index κ , the cold electron concentration via β , and the collisional effect via ν). The effects of these parameters on linear wave excitations are explored in Figs. 1 and 2.

In Fig. 1, we have depicted the real part of the wave frequency ω in Eq. (6) against the wavenumber k . The imaginary part, representing the wave damping rate, is independent of k . The real part of the wave frequency is found to decrease as ν increases (see Fig. 1). As analytically derived from Eq. (6), the wavenumber has a threshold value (k_1 and k_2 in Fig. 1 for $\nu = 0.02$ and $\nu = 0.05$, respectively), below which the wave is overdamped and would not propagate. This overdamping mechanism operates unless

$$|k| > \left(\frac{a\nu^2}{4\beta - \nu^2}\right)^{1/2} \equiv k_{od}, \quad (7)$$

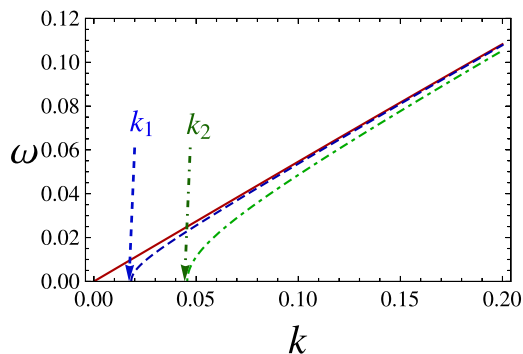
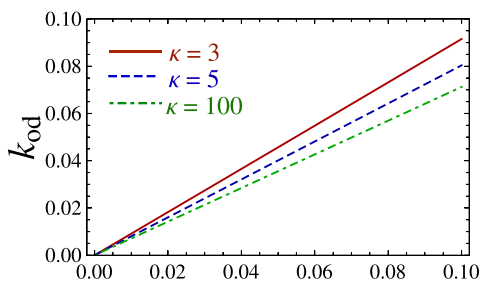


FIG. 1. The variation of the real part of the wave (angular) frequency ω versus the wavenumber k is depicted, for superthermality index $\kappa=3$, and assuming electron concentration (cold-to-hot electron ratio) as $\beta=0.5$. The k region below which the wave is overdamped is highlighted. The solid curve corresponds to $\nu=0$, the dashed curve to $\nu=0.02$, and the dotted-dashed curve to $\nu=0.05$.

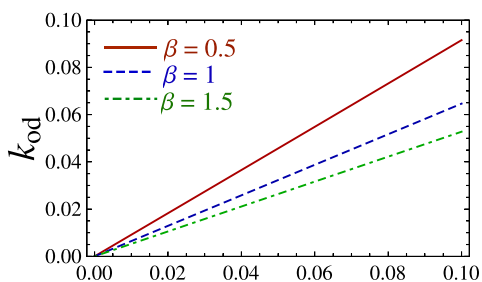
which defines the critical value k_{od} : no oscillation occurs below k_{od} , thus the frequency and its derivatives become imaginary. For smaller values of the collision frequency (i.e., $\nu \ll \omega_{ph}$), Eq. (7) becomes

$$|k| > k_{od} = \frac{\nu}{2} \sqrt{a/\beta}. \quad (8)$$

The variation of k_{od} with ν for different κ and β are shown in Figs. 2(a) and 2(b), respectively; this behavior will be discussed in a later section. In the long wavelength approximation, i.e., for $k \ll a$, the real part of Eq. (6) can be written as



(a)



(b)

FIG. 2. Variation of the wavenumber threshold k_{od} (given by Eq. (7)) versus the collision frequency ν : (a) For different superthermality parameter κ , where cold-to-hot electron population ratio $\beta=0.5$ and (b) for different β , where $\kappa=3$.

$$\omega = \sqrt{\frac{k^2 \beta}{a} - \frac{\nu^2}{4}}. \quad (9)$$

The phase speed of small amplitude waves in a collisional plasma thus reads

$$v_{ph} = \sqrt{\frac{\beta}{a} - \frac{\nu^2}{4k^2}}. \quad (10)$$

One can easily recover the κ -dependent phase speed

$$\sqrt{\frac{\beta}{a}} = \left(\frac{\kappa - 3/2}{\kappa - 1/2} \right)^{1/2}$$

for collisionless plasmas upon setting $\nu=0$ in Eq. (10), in agreement with Refs. 30 and 32. The phase speed vanishes, that is, wave propagation is not possible, for $k < k_{od}$. One should therefore restrict themselves in the region above k_{od} in the analysis of propagating structures.

IV. SOLITARY WAVES

In order to investigate the nonlinear propagation of electrostatic solitary structures, we adopt stretched coordinates⁴⁵ as

$$\xi = \epsilon^{1/2}(x - v_0 t), \quad \tau = \epsilon^{3/2} t, \quad (11)$$

where the small parameter ϵ ($\epsilon \ll 1$) measures the strength of nonlinearity and v_0 is the speed of the solitary wave front (normalized by the hot electron thermal speed C_0); the value of v_0 will later be determined by compatibility requirements. We also assume a weak damping, by taking $\nu = \epsilon^{3/2} \nu_0$. The dependent variables n , u , and ϕ are expanded around the unperturbed states as

$$\left. \begin{aligned} n &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \dots, \\ u &= \epsilon u_1 + \epsilon^2 u_2 + \dots, \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \dots. \end{aligned} \right\} \quad (12)$$

Substituting Eq. (12) into Eqs. (1)–(3) and combining the terms in $\epsilon^{3/2}$ from the first two and the term in ϵ^1 from the third equation, one gets

$$n_1 = -\frac{\phi_1}{v_0^2}, \quad u_1 = -\frac{\phi_1}{v_0}. \quad (13)$$

Substituting Eq. (13) into Eq. (3), one can obtain the

$$v_0 = \sqrt{\beta/a}, \quad (14)$$

which suggests that solitary waves will travel at (or rather, slightly above) the phase speed of (linear) electron-acoustic waves, which is precisely the acoustic speed defined above.

Considering the next order in ϵ and eliminating the second-order quantities in combination with (14), we obtain the equation

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + C \phi_1 = 0, \quad (15)$$

which describes the evolution of the leading-order electric potential (disturbance) ϕ_1 . The nonlinearity, dispersion, and damping terms appearing in the latter equation, respectively, read

$$A = -\frac{3}{2} \sqrt{\frac{a}{\beta}} - \frac{b}{a^{3/2}} \sqrt{\beta}, \quad B = \frac{\sqrt{\beta}}{2a^{3/2}} \quad (16)$$

and $C = \nu_0/2$.

It is anticipated that the mutual balance between nonlinearity and dispersion leads to the formation of coherent (soliton) structures. It is therefore appropriate to investigate the dependence of the nonlinearity and dispersion coefficients, A and B , on the superthermality index (via κ), and also on the cold-to-hot electron number density (via β). These are depicted in Figs. 3(a) and 3(b), respectively. We see that, for stronger superthermality (smaller κ), the nonlinear coefficient A increases in absolute value, whereas the dispersive coefficient B decreases, that is, superthermality (low κ) results in a loss of balance between dispersion and nonlinearity: assuming a soliton-shaped initial condition, a change in κ would result in loss of ability to sustain its shape, and energy breakdown to smaller structures or/and to random oscillations.^{32,33} On the other hand, smaller values of cold-to-hot electron number density ratio β result in smaller nonlinear coefficient A (in absolute value) but larger dispersive term B . That is, when a solitary pulse propagating from a lower β region to a higher β region, the increasing nonlinearity A dominates over dispersion, which may lead to breaking of the initial solitary pulse.

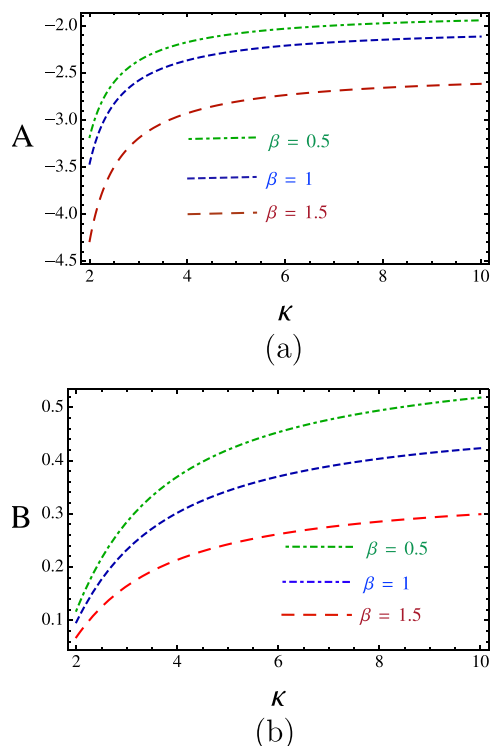


FIG. 3. Variation of (a) the nonlinear coefficient A and (b) the dispersion coefficient B with the spectral index κ , for different values of β .

A. Analytical approach

Assuming a weak value of the damping term in Eq. (15), thus to be treated as a small perturbation, one may consider as initial condition a known function of ζ , and study its evolution in time τ . A suitable pulse-shaped initial condition is analytically found to be $\phi_1(\zeta, 0) = \phi_0(0) \operatorname{sech}^2 \sqrt{\frac{A \phi_0(0)}{12B}} \zeta$. Following Refs. 43, 46, and 47, the time-dependent expression thus obtained (omitting details) reads

$$\phi_1(\zeta, \tau) = \phi_0(\tau) \operatorname{sech}^2 \sqrt{\frac{A \phi_0(\tau)}{12B}} \left(\zeta - \frac{A}{3} \int_0^\tau \phi_0(\tau) d\tau \right), \quad (17)$$

where the amplitude, velocity, and width of the pulse⁴³ are functions of time given, respectively, by

$$\phi_0(\tau) = \phi_0(0) \exp\left(-\frac{2\nu_0}{3} \tau\right), \quad (18)$$

$$v_0 = \frac{A \phi_0(0)}{3} \exp\left(-\frac{2\nu_0}{3} \tau\right), \quad (19)$$

$$L = \sqrt{\frac{12B}{A \phi_0(0)}} \exp\left(\frac{\nu_0}{3} \tau\right). \quad (20)$$

B. Numerical investigation

Our main interest now is to trace the effect of dissipation on the dynamics of pulse structures. To do so, we assume a negligible dissipative effect, by formally considering the limit $\nu_0 \rightarrow 0$. Equation (15) is then reduced to Korteweg de Vries (KdV) equation, which has a solitary wave solution in the form:

$$\phi_1(\zeta, \tau) = \hat{\phi}_0 \operatorname{sech}^2 \left(\frac{\zeta - U_0 \tau}{L} \right), \quad (21)$$

where $\hat{\phi}_0 = 3U_0/A$ is the solitary wave amplitude, $L = \sqrt{4B/U_0}$ is the width, and U_0 is the pulse speed (normalized by the hot-electron thermal speed).

The solitary wave solution given in Eq. (21) will be used as an initial condition to analyze the dissipative effect on pulse-type electron-acoustic solitary waves. We have analyzed the propagation of EA solitary structures by a numerical integration of the KdV equation, employing a Runge-Kutta 4 method. A time interval of 10^{-5} and a spatial grid size of 0.1 were considered.

We have first investigated the stability of a stable pulse propagating in strongly superthermal ($\kappa=3$) plasma. The outcome of our simulation is depicted in Fig. 4. In this case, we have considered the pulse soliton solution (21) for $\nu_0=0$, $\kappa=3$, and $\beta=0.5$ as initial condition, while considering the effect of dissipation in the dynamics, by adopting in the simulation the values $\nu_0=0.02$, $\kappa=3$, and $\beta=0.5$. The pulse amplitude decreases with time during the propagation (see Fig. 4). In the second simulation in Fig. 5, we have considered a stable pulse propagating in a moderately superthermal ($\kappa=6$) plasma where pulse soliton solution (21) for $\nu_0=0$, $\kappa=6$, and $\beta=0.5$ is assumed as initial condition. It is

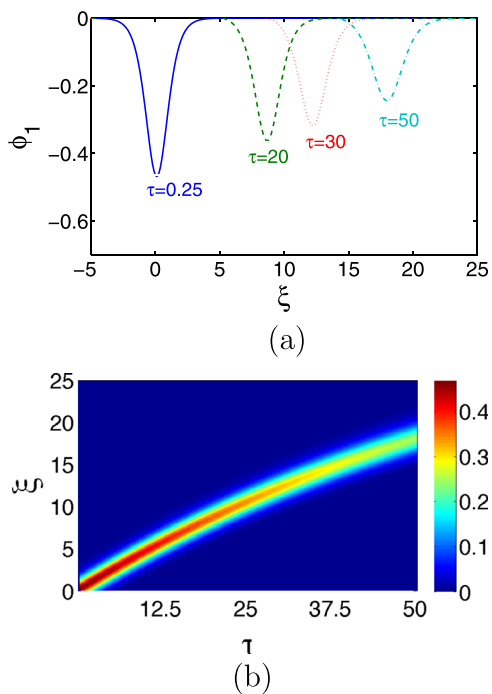


FIG. 4. (a) Evolution of electrostatic solitary structures (given in Eq. (15)) propagating in superthermal plasma ($\kappa=3$) with cold-to-hot electrons number density $\beta=0.5$. Equation (21) was considered as initial condition for $\beta=0.5$, $\kappa=3$, where $U_0=0.5$; and (b) showing the propagation in the space-time plane for same condition as panel (a).

expected like earlier case and also seen in numerical simulation that the soliton amplitude decreases with time, while it propagates in the dissipative plasma.

We have subsequently considered the same scenario, but now considering the pulse solution for a quasi-

Maxwellian plasma ($\kappa=100$), for $\nu_0=0$, $\kappa=100$, and $\beta=0.5$, as initial condition, while running the simulation for the values: $\kappa=100$, $\nu_0=0.02$, and $\beta=0.5$. The characteristics of the EA pulses are significantly modified, and the amplitude (width) is found to decrease (increase), while it propagates in the dissipative plasma, as shown in Fig. 6. The pulse in this (quasi-Maxwellian) case has larger amplitude (see Fig. 6) than its counterpart in superthermal ($\kappa=3$) plasma (see Fig. 4).

In Table I, we compare the amplitude values observed numerically for different time τ , in Fig. 4, for strongly superthermal plasma ($\kappa=3$), and also in Fig. 6 for quasi-Maxwellian plasma ($\kappa=100$), with the theoretically predicted results obtained via Eq. (18). The parameter values used are $\beta=0.5$, $U_0=0.5$, and $\nu_0=0.02$. The numerical results match the theoretical results almost perfectly, as shown in Table I.

In Fig. 7, we have considered a pulse (i.e., an exact soliton solution) obtained for a lower density cold electron plasma environment ($\beta=0.5$), assumed to enter in a plasma environment with higher cold-electron density ($\beta=1.5$). In our simulation (results depicted in Fig. 7), we have considered the exact soliton solution given in Eq. (21) for $\kappa=3$ and $\beta=0.5$ as initial condition, while the KdV equation integrated was considered for $\kappa=3$ and $\beta=1.5$. In the case depicted in Fig. 7, the initial pulse decomposed into a pair of daughter pulses: a dominant (thinner, steeper) fast pulse, followed by a smaller (and slower) one. This suggests that a delicate energetic balance was achieved, in that the initial condition provided the necessary energy for the formation of a two-pulse configuration, which appear to propagate with decreasing amplitude as time evolves.

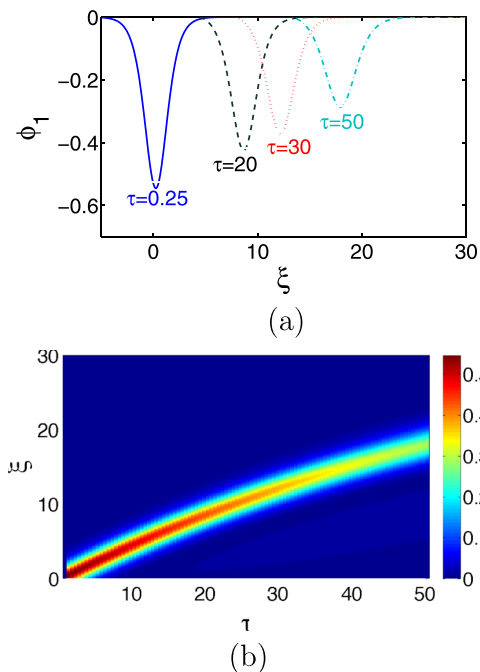


FIG. 5. (a) Evolution of electrostatic solitary structures (given in Eq. (15)) propagating in a moderately superthermal plasma ($\kappa=6$) with $\beta=0.5$. Equation (21) was considered as initial condition for $\beta=0.5$, $\kappa=6$, where $U_0=0.5$; and (b) showing the propagation in the space-time plane for same condition as panel (a).

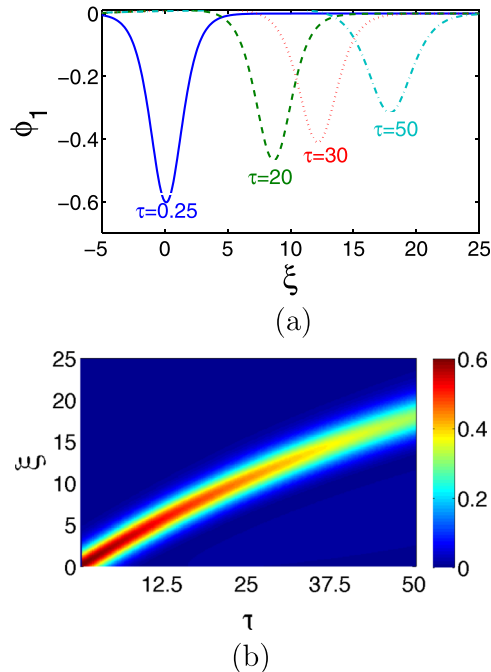


FIG. 6. (a) Evolution of electrostatic solitary structures (given in Eq. (15)) propagating in quasi-Maxwellian plasma ($\kappa=100$) with $\beta=0.5$. Equation (21) was considered as initial condition for $\beta=0.5$, $\kappa=100$, where $U_0=0.5$; and (b) showing the propagation in the space-time plane for same condition as panel (a).

TABLE I. Comparison between theoretical and numerical values of amplitude (absolute value) for different time τ .

Superthermality index κ	Time τ	Theoretical value	Numerical value
3	0.25	0.44	0.45
	20	0.34	0.34
	30	0.29	0.30
	50	0.22	0.23
100	0.25	0.60	0.59
	20	0.46	0.46
	30	0.40	0.40
	50	0.31	0.31

We now depict the time dependent amplitude $\phi_0(\tau)$ given in Eq. (18) and width L given in Eq. (20) of the solitons profile in Fig. 8 for the different values of the superthermality index κ . Other parametric values that we have chosen are cold-to-hot electrons number density ratio $\beta=0.5$ and $U_0=0.5$. We have observed that a higher excess of electron superthermality (lower value of κ) leads to smaller amplitude (in absolute value) and narrower pulses than the ones sustained in quasi-Maxwellian plasma. In other words, both width and amplitude are lower for strongly superthermal plasma (solid curve) than in Maxwellian plasma (dotted curve), shown in Fig. 8. We have also found that the amplitude $\phi_0(\tau)$ decreases while the pulse width L increases with time as shown in Fig. 8. This was expected, as taller pulses are narrower, as indicated by the exact solution above.

Interestingly, the characteristic soliton “decay” time is independent from κ and only depends on the viscosity value

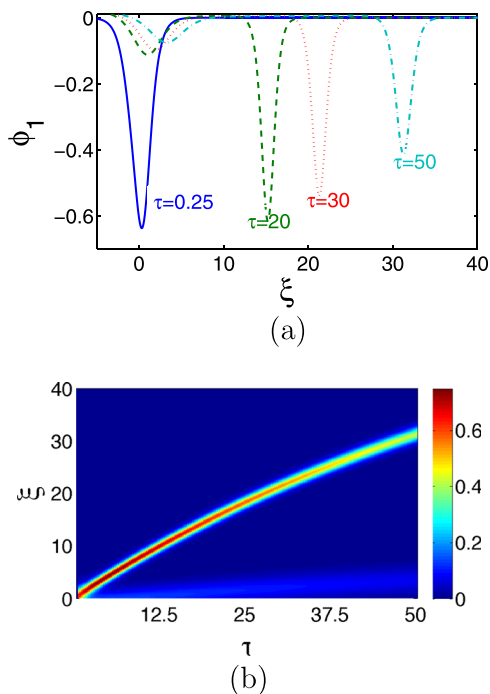


FIG. 7. (a) Evolution of electrostatic solitary structures (given in Eq. (15)) propagating in a superthermal plasma ($\kappa=3$) with $\beta=1.5$. Equation (21) was considered as initial condition for $\beta=0.5$, where $U_0=0.5$; and (b) showing the propagation in the space-time plane for same condition as panel (a).

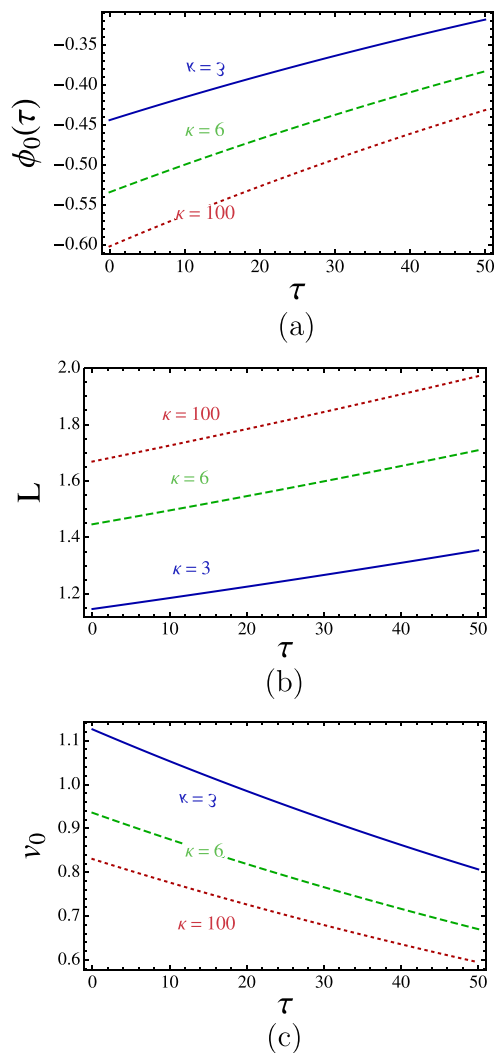


FIG. 8. We have depicted the variation of the time-dependent pulse characteristics, namely, (a) the pulse amplitude $\phi_0(\tau)$ versus time τ (given in Eq. (18)), (b) the pulse width L versus time τ (given in Eq. (20)), for different superthermality index κ , and (c) the pulse speed ν_0 , from Eq. (19). The remaining parameters are $\beta=0.5$, $U_0=0.5$, and $\nu_0=0.02$.

ν , as predicted analytically, and also confirmed numerically (see Fig. 9), where we have considered the numerical evolution of the analytical solution in the extreme cases $\kappa=3$ and $\kappa=100$; the amplitude decay rate seems to be practically identical in these case.

V. CONCLUSIONS

To summarize, we have considered the propagation of high frequency electrostatic excitations in a multi component, unmagnetized, collisional plasma whose constituents are superthermal hot electrons, inertial cold electrons, and stationary ions. We have mainly focused on the damping effect on the EA solitary waves whose phase speed lies between the hot electrons thermal speed and ion sound speed.

Our analysis suggests a dependence of the dispersion characteristics on the spectral index κ , on the cold-to-hot electron density ratio β and, finally, on the phenomenological damping rate ν . Since pulses are weakly superacoustic,

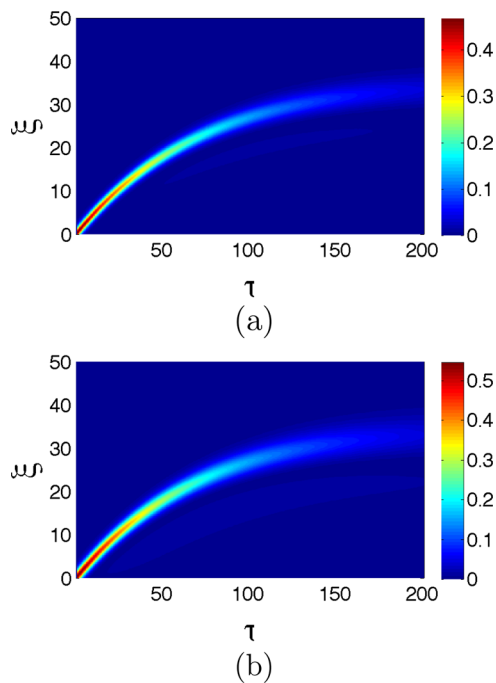


FIG. 9. (a) Evolution of electrostatic solitary structures (given in Eq. (15)) in the space-time plane (a) propagating in a superthermal plasma ($\kappa=3$), where Eq. (21) was considered as initial condition for $\beta=0.5$ and $U_0=0.5$ and (b) propagating in a Maxwellian plasma ($\kappa=100$), where Eq. (21) was considered as initial condition for $\beta=0.5$ and $U_0=0.5$.

when modeled via a KdV-type description, this fact affects the soliton propagation speed (which is κ dependent); to low-order, the latter reads $v_0 = [(\kappa - 3/2)/(\kappa - 1/2)]^{1/2}$.

Our numerical analysis reveals that, when a pulse-shaped initial condition propagates in a dissipative plasma, the amplitude decreases in time, as expected, under the effect of dissipation. This is true either for strongly superthermal or for quasi-Maxwellian plasmas (and any state in between). However, it is found that the characteristic decay time is independent from κ , and only depends on the dissipative term ν , as confirmed both analytically and numerically. We have also observed the spontaneous formation of multi-soliton configurations under certain conditions, when an initial solitary pulse is considered to propagate between different plasma environments (i.e., when passing from a low- β to a high- β region; recall that this implies that there are more cold electrons in the new region, since $\beta = n_{c0}/n_{h0}$). This situation may be the outcome of a delicate energy balance, as a particular plasma configuration (i.e., a given combination of β and κ values) supports soliton solutions of specific energy. Therefore, when a solitary pulse propagates from one plasma environment to another, it may lose energy, in order to adapt its form to a new environment; the energy remainder may then suffice to form a smaller pulse.

The (time dependent, in our case) pulse speed (and amplitude, since taller soliton solutions are faster) is higher in superthermal plasmas than in quasi-Maxwellian plasmas.

Our investigation may be useful in understanding some important features of electron-acoustic perturbations that

may propagate in laboratory as well Space plasmas, where different electron populations may coexist.

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